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Review

Review on the Hydro- and Thermo-Dynamic Wakes of Underwater Vehicles in Linearly Stratified Fluid

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Abstract: Wakes produced by underwater vehicles, particularly submarines, in density-stratified fluids play a pivotal role across military, academic, and engineering domains. In comparison to homogeneous fluid environments, wakes in stratified flows exhibit distinctive phenomena, including upstream blocking, pancake eddies, internal waves, and variations in hydrodynamic performance. These phenomena are crucial for optimizing the operation of underwater vehicles. This review critically assesses the hydrodynamic and thermodynamic aspects of these wakes through an integration of theoretical, experimental, and numerical approaches. The hydrodynamic wake evolution, comprising near-wake, non-equilibrium, and quasi-two-dimensional regimes, is scrutinized. The underlying physics, encompassing energy transformation, vertical motion suppression, and momentum dissipation, are analyzed in detail. Special emphasis is placed on numerical methods, encompassing diverse approaches and turbulence models and highlighting their differences in fidelity and computational cost. Numerical simulations not only provide insights into the intricate interplay among various factors but also emerge as a crucial focal point for future research directions. In the realm of thermodynamic wakes, we delve into the thermal wake induced by the discharge of high-temperature cooling water and the cold wake resulting from the stirring of seawater. The generation, evolution, and ascent to the free surface of these wakes are explored. Additionally, this review identifies and analyzes current research shortcomings in each aspect. By systematically addressing existing knowledge gaps, our study contributes novel insights that propel academic progress and bear significant implications for submarine engineering. This work not only enhances our understanding of the intricate dynamics involved but also provides a foundation for future research endeavors in this critical field.

Keywords: hydrodynamic; thermodynamic; wake; underwater vehicle; stratified fluid

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1. Introduction

Underwater vehicles, particularly submarines, play a vital role in modern naval fleets due to their unique combination of stealth, agility, endurance, and minimal logistical needs. As the propagation distance of sound waves far surpasses that of visible light, radar, infrared, and other electromagnetic waves, sonar detection remains the most effective method for tracking underwater vessels. However, the relentless pursuit of quieter submarines necessitates the continuous evolution of detection systems aligned with the strategic objectives of wide-area surveillance (WAS), demanding long-range detection capabilities and comprehensive monitoring. This calls for a paradigm shift in submarine detection technology, including the exploration of non-acoustic detection methods [1].

The movement of underwater vehicles disrupts the sea surface flow field, causing wrinkles and ripples of varying roughness to appear. By capturing these changes in sea surface roughness signals, synthetic aperture radar (SAR) enables the detection and estimation of underwater objects [2,3]. Additionally, thermal signals play a crucial role. Surveillance satellites equipped with highly sensitive infrared CCD cameras can capture thermal images of underwater vehicles, permitting precise determination of their position, direction, and speed [4,5].

When traversing the open ocean, the generation and evolution of an underwater vehicle's wake are inextricably linked to the specific characteristics of the environment. A key feature of the marine environment is stratification, where seawater properties like temperature, salinity, and density vary with depth. Compared to uniform flow, an underwater vehicle navigating within a density-stratified flow experiences a more complex wake due to the influence of density gradients. This complexity can lead to unique phenomena such as upstream blocking [6], the generation of internal waves [7], and the formation of pancake eddies in the far-field late wake [8].

Therefore, the impact of stratification on underwater vehicle wake characteristics is highly significant and cannot be ignored. Numerous studies have been conducted to enhance an understanding of flow and wake characteristics, employing theoretical analyses [9], model experiments [10,11], and numerical simulations [12–14]. However, a significant portion of these studies neglect the influence of stratification. Research on the generation, evolution, and propagation characteristics of wakes from underwater vehicles in density-stratified flow is not only a pressing need in engineering practice, but also a key issue in current international hydrodynamics research.

2. Definition and Classification

The wake of an underwater vehicle is the complex, turbulent flow field generated around the moving vessel. This dynamic signature continuously evolves and propagates to distant regions, leaving a signature like the proverbial "trace" of a wild goose. Submarines and other submersibles inevitably disturb the surrounding water during their navigation, causing some fluid to adhere due to viscosity and the remainder to propagate outward as waves. This review focuses on the study of hydrodynamic and thermodynamic wakes produced by underwater vehicles, outlining their definitions and classifications.

2.1. Hydrodynamic Wakes of Underwater Vehicles

A submarine's hydrodynamic wake refers to the perturbation in the fluid flow pattern created by its movement. This manifests as alterations in velocity and pressure fields within the wake region, leading to the generation of vorticity and turbulence [15–18]. Understanding these wakes is crucial for assessing a submarine's stealth, maneuverability, and overall hydrodynamic performance. The formation process is intricate and involves various contributing factors, which are broadly categorized into three types: the Bernoulli hump, Kelvin wake, and internal wave.

2.1.1. Bernoulli Hump

The Bernoulli hump is a rise and fall in sea surface height above the submarine, most prominent when operating close to the surface [19]. This phenomenon plays a significant role in submarine-detection programs for world navies, as changes in the free surface caused by the hump can be tracked by various methods. The hump's height varies from tens of centimeters near the surface to a few millimeters at a depth of about 50 m, primarily influenced by the vessel's velocity, depth, and dimensions.

2.1.2. Kelvin Wake

One of the most studied surface waves, the Kelvin wake forms the distinct V-shaped pattern trailing behind a moving object [20]. While all objects moving on the free surface in deep water generate such a wave pattern, ship-generated Kelvin wakes have received extensive attention due to their contribution to resistance forces. In contrast, thorough research on submarine-generated Kelvin wakes, including the behavior of wave height and wake angle with respect to Froude number and submergence depth, is lacking [21].

2.1.3. Internal Wave

Unlike surface waves, internal waves propagate within a stratified fluid medium, typically beneath the surface, driven by oscillations and movement along density interfaces [22]. These waves can travel horizontally along these interfaces, their behavior influenced by the fluid's stratification. The ocean's pycnocline, a sharp density gradient between layers, plays a crucial role in their generation and propagation. Internal waves represent the dynamic interplay between gravity and buoyancy forces in a stratified environment, significantly impacting vertical mixing of properties within the fluid.

2.1.4. Wake Behavior in Stratified Fluids

In a uniform fluid, inertial and viscous forces primarily influence the wake and near-field fluid structure of underwater vehicles. These momentum wakes dissipate rapidly due to viscous effects. However, in stratified fluids, buoyancy becomes a crucial factor in determining the momentum wake and flow structure, alongside inertial and viscous forces. Due to the vertical density gradient, turbulent fluctuations in the wake are restricted in their vertical motion, forming large-scale coherent structures [23].

2.2. Thermodynamic Wakes of Underwater Vehicles

The thermodynamic wakes associated with submarines encompass alterations in temperature and energy distribution surrounding the submerged vessel. This transformation results from the exchange of heat between the submarine and its environment, giving rise to temperature gradients and energy dissipation. The investigation of these wakes yields valuable insights into the thermal signature and environmental impact of submarines, which is particularly pertinent for naval operations and environmental monitoring.

2.2.1. Thermal Wake Due to Cooling Water Discharge

Submerged vehicles, especially those utilizing nuclear propulsion, release heated cooling water that rapidly elevates the surrounding seawater temperature. The reduced density of the heated water causes it to ascend to the sea surface, forming a distinct "thermal signal" [24].

2.2.2. Cold Wake Due to Stirring of Stratified Water

As an underwater vehicle navigates through stratified seawater with varying temperature and density, its hull and propellers disrupt the stable structure. This disturbance brings colder seawater to the free surface, creating a discernible "cold signal" on the sea surface [25].

These thermal and cold signals, manifested on the sea surface through the movement of underwater vehicles, play a crucial role in comprehending their overall environmental impact and susceptibility to detection.

Underwater vehicle wakes in stratified flows involve complex hydrodynamic and thermodynamic processes, presenting a challenging multi-field coupling problem with interdisciplinary intersections and multifactor interferences. As depicted in Figure 1, the sketch vividly captures the key flow features and wake categories discussed earlier. The illustration portrays submarine wakes traversing through stratified fluids, giving rise to a diverse array of vortices and wakes that propagate downstream. Phenomena distinctive to stratified fluids are accentuated with yellow lettering, underscoring their appearance and significance within the depicted context. By examining the research methods, current status, and outcomes from various perspectives, this review aims to provide a comprehensive overview of underwater vehicle wakes and pave the way for future research directions.

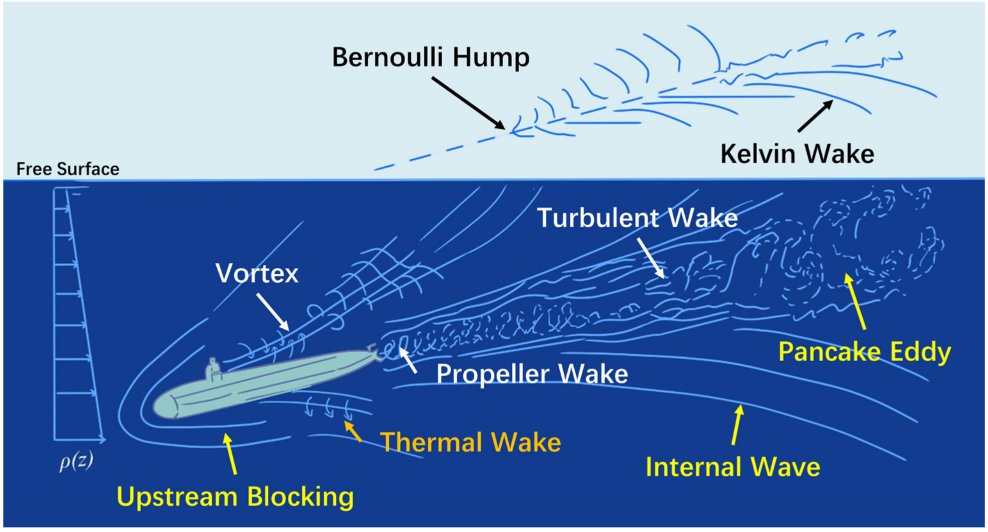

Figure 1. Sketch of submarine wakes in stratified fluids.

3. Hydrodynamic Wakes in the Stratified Fluid

The formation of wakes has always been a matter of great concern for scholars and engineers engaged in fluid dynamics, as it is a fundamental flow phenomenon generated when fluid flows past obstacles. The formation of wakes is closely related to the Reynolds number, as shown in Equation (1) [26].

$$Re = \frac{UL}{\nu} \quad (1)$$

where U represents the flow velocity, L is the characteristic length, and v is the kinematic viscosity of the fluid.

When the Reynolds number gradually exceeds a critical value, the wake begins to become unstable and exhibits a regular flow pattern, such as the Karman vortex street behind a cylinder. As the Reynolds number continues to increase, the regular flow pattern is disrupted, leading to the formation of turbulence [27].

In comparison to uniform fluids, in stratified fluids with density and temperature, the wake is not only related to the Reynolds number but also closely associated with the internal Froude number Fr, as shown in Equation (2) [28].

$$Fr = \frac{U}{NL} \quad (2)$$

where N is the Brunt-Väisälä frequency, defined as [29]

$$N = -\sqrt{\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}} \quad (3)$$

where ρ_0 is the undisturbed original density, z is the vertical coordinate, and g is the gravitational acceleration. The Brunt–Väisälä frequency is commonly used to indicate the oscillation frequency of fluid particles under infinitesimal disturbances, serving as a fundamental characteristic of stratified fluids. In recent years, researchers from various fields have studied the hydrodynamic wakes of objects in linearly stratified fluids from three perspectives: theoretical, experimental, and computational fluid dynamics (CFD).

3.1. Theoretical Methods

Theoretical investigations of hydrodynamic wakes in linearly stratified fluids with density and temperature gradients have initially focused on simplified models like point sources. Hudimac [30] modeled a perturbation source in stratified flow as a stationary moving point, applying Fourier transforms to derive theoretical models for surface and interfacial waves after perturbation. This enabled qualitative analysis of the relationship between transverse and dispersive wave systems and the point source velocity. Further contributions to this field were made by Keller and Munk [31], who derived wake equations for wave sources moving in anisotropic media, extensively studying near- and far-field wake behavior. Their work, primarily aimed at internal wave analysis, demonstrated that steady-state linear internal waves generated by submarines exhibit vertical propagation, intensifying their convergence and divergence on the free surface. A consolidation of previous research on Green's function in stratified fluids was provided by Voisin [32,33], who offered analytical expressions for pulsating point sources in unbounded stratified fluids with and without the Boussinesq assumption.

However, these early studies have dealt with infinite flow fields, neglecting the free surface's influence. Recognizing this limitation, Zhu et al. [34] incorporated the impact of the free surface into their model. They established a mathematical model for pulsating point source internal waves in continuously stratified fluids and used Fourier transforms and contour integration to derive analytical expressions for the Green's function under specific frequency conditions. This allowed them to explore the generation mechanism, propagation characteristics, free surface wave characteristics, and influencing factors of such waves in these fluids.

Focusing on simpler geometries, Miles [35], Miles and Huppert [36,37], Huppert and Miles [38], and Long [39–41] have used asymptotic analysis to study the formation of lee waves around cylinders under different internal Froude numbers, building upon existing dispersive wave models. Moving closer to realistic scenarios, Tuck [42] employed integration to calculate the velocity field and far-field waveform of the internal wave wake generated by a moving submarine. Similarly, Motygin and Kuznetsov [43] applied the potential flow method to quantify the increased resistance effect of a moving body in stratified fluid. Radko [44] has further utilized perturbation theory to derive the far-field waveform generated by an object moving in a multilayer fluid system. Their findings suggest that the number of layers has a minimal impact on the wake waveform, while the waveform near the free surface and internal interfaces resembles classical Kelvin waves.

Stefanick [45] pioneered the use of near-field theory to estimate the height of the Bernoulli hump produced by an Ohio-class submarine under varying submersion depths and speeds. His calculations revealed that the Bernoulli hump's shape is largely independent of submersion depth but decreases rapidly with distance from the submarine and quadratically with increasing speed. It also decreases quadratically with increasing submersion depth but typically remains below one submarine length in height.

3.2. Experimental Methods

While theoretical studies provide valuable insights into wake behavior, experiments play a crucial role in validating and enriching our understanding. Initially, researchers have focused on simple geometries like spheres, employing flow visualization techniques, hot-wire anemometry, shadowgraphy, and particle image velocimetry (PIV) to unravel the complexities of wake evolution in stratified fluids. These studies, spanning a wide range of Reynolds (*Re*) and internal Froude (*Fr*) numbers, have revealed a fascinating story of wake development under the influence of stratification. Afanasyev [46], Chomaz et al. [47], and Lin et al. [48] have investigated wakes generated by spheres of different diameters dragged at varying speeds under diverse stratification intensities. These experiments, covering Reynolds numbers (*Re*) in the range of [100, 10^4] and internal Froude numbers (*Fr*) between [0.1 and 100], have revealed crucial insights into the behavior of stratified wakes.

As Figure 2 demonstrates, high *Fr* wakes exhibit three distinct regimes: near-wake (NW), non-equilibrium (NEQ), and quasi-two-dimensional (Q2D). Within the NW regime, the wake spreads uniformly and turbulence resembles that in a homogeneous fluid. Meunier and Spedding [49], Spedding et al. [50,51], and Spedding [52] have identified the NEQ regime as the onset of stratification effects, marked by the conversion of potential energy to kinetic energy and anisotropic horizontal and vertical motions. Finally, the Q2D regime is characterized by vertically suppressed two-dimensional eddies known as "pancake vortices".

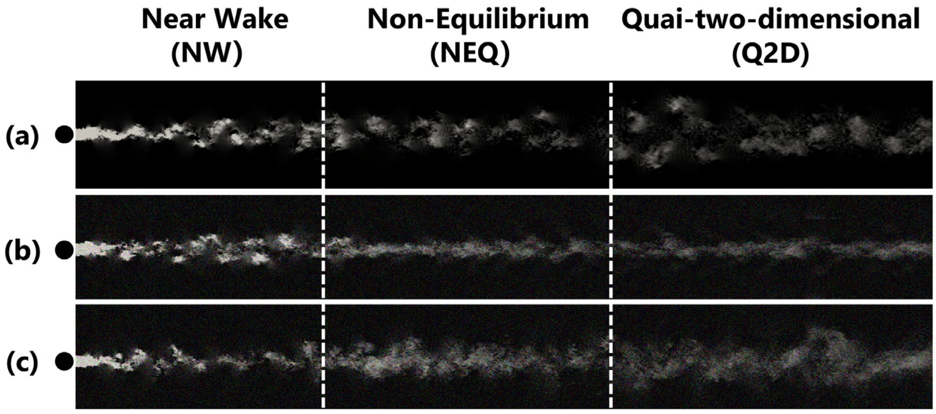

Figure 2. Flow regimes past a bluff body in stratified fluids, streamwise velocity is depicted for three scenarios: (**a**) unstratified wake; (**b**) stratified wake in the vertical cut; (**c**) stratified wake in the horizontal cut.

According to the different *Re*, in the near wake, the flow structure includes stable two-dimensional attached vortices, unstable attached vortices, two-dimensional shedding vortices, unstable symmetrical lee waves, non-axisymmetric attached vortices, symmetrical shedding vortices, asymmetrical shedding vortices, and turbulent wakes. Additionally, Kelvin–Helmholtz (K-H) and spiral instability structures may also appear. Measurements of the frequency characteristics of the near-wake flow field have found two frequency modes in the near-field wake: a low-frequency mode with a corresponding Strouhal number (*St*) of approximately 0.18, and a high-frequency mode with *St* increasing with *Re*. For a more in-depth exploration of wake evolution dependent on *Re*, additional insights can be gleaned from the works of Derakhshandeh and Alam [53] and Thompson et al. [54]. These comprehensive reviews delve into the dynamics of wakes past bluff bodies in homogeneous flow, providing valuable perspectives on the influence of *Re* on the evolving wake structures.

After the near wake undergoes evolution over a buoyancy period $T = 2\pi/N$, the vertical spreading will reach its maximum. The buoyancy effect in stratified fluids inhibits the vertical movement, leading to the collapse of the wake. During this phase, the vertical velocity and thickness of the wake gradually decrease, while the horizontal velocity and width increase. According to results from PIV experiments, the flow energy within the momentum wake dissipates rapidly [55–57]. The horizontal average velocity v_i of fluid particles within the momentum wake is proportional to x^{-1}, where x is the horizontal distance between the velocity measurement point and the trailing sphere. This behavior is consistent with the situation of dragging a sphere in a uniformly dense fluid. However, in the collapse stage, it is observed that the average velocity of the momentum wake in fluids exhibiting linearly stratified density and temperature does not diminish with increasing x distance. This suggests that in this stage, in addition to energy dissipation, there is also energy conversion [58].

After the vertical velocity of the wake decreases to near zero, the late wake begins. In this stage, the average velocity of fluid within the momentum wake begins to decrease again with the increase in x, and it is proportional to $x^{-0.38}$. Obviously, the rate of energy decay of the momentum wake at this stage is much slower than in a uniform fluid or in the near wake. In the late wake, vertical motion almost completely disappears, and the wake evolves only in the horizontal plane. Pancake vortices gradually appear at the edges of the wake on both sides and expand gradually. After a certain moment, these pancake vortices evolve into a set of vortex pairs with opposite rotation directions. Ultimately, in the entire late wake region, a Q2D large-scale vortex street structure is formed, and the rotation direction of the vortices is consistent with the Karman vortex street, also known as the positive Karman-type dipole vortex street [47,59–61].

Regarding the formation mechanism of this Q2D vortex structure, there are currently two viewpoints. One viewpoint suggests that this large-scale coherent vortex street is actually the product of the amplification of near-field shedding vortices of the dragging sphere under the influence of the buoyancy effect of stratified fluids, causing them to be "flattened" [62]. The other viewpoint suggests that the formation of the Q2D vortex street structure is unrelated to the near-field characteristics of the dragging sphere but is caused by the shear instability of the momentum wake edge interacting with the background fluid [55,63].

After decades of research, there is now a relatively comprehensive understanding of the evolution of large-scale vortex structures related to the momentum wake of a spherical body in stratified fluids. Various aspects, such as the characteristics of the momentum wake in the near-field, collapse, and far-field stages, energy transport characteristics during wake evolution, and the formation mechanisms and characterization rules of large-scale vortex streets, have been well understood. However, as research continues to advance, researchers have gradually realized that, for underwater moving objects, the jet momentum wake generated by the propulsion system is equally important as the momentum wake caused by the resistance of the hull.

To study the generation and evolution of momentum wakes, the traditional approach has been to simplify the effects of the propulsion system, such as jetting, by equivalent representation as jet flow through a nozzle. In early studies, a linearly stratified environment was created in a density-stratified water tank, and the motion of dye-colored liquid was used to simulate the effects of a moving jet. Under the appropriate conditions of jet momentum flow, it was found that a pair of oppositely rotating dipole vortices could be easily generated. This is an early discovered Q2D structure present in density-stratified fluids. Numerous theoretical, numerical simulation, and experimental results have been obtained regarding the generation mechanism and evolutionary characteristics of these dipoles. These dipole vortices can interact with each other or with solid structures, leading to many meaningful phenomena [64,65].

In experiments conducted in density-stratified water tanks using the method of moving a dye-colored liquid to simulate the effects of a moving jet, it was demonstrated that both steady and unsteady moving jet flows in density-stratified fluids can lead to the formation of large-scale quasi-two-dimensional dipole vortex street structures [66]. The rotation direction of these vortices is consistent with the Karman vortex street, also known as the positive Karman-type dipole vortex street. The actual oceanic scale of these large-scale dipole vortex street structures can reach 1 to 2 km, and the decay time required can be several days. Additionally, significant vortex flow features can be observed on the free surface [67]. Relevant research indicates that although the structures of jet momentum wakes and resistance momentum wakes differ in the near-field, and the induced velocity distributions are inconsistent, and the evolution characteristics of the two wakes have similarities. The buoyancy effect of stratified fluids has a similar inhibitory effect on the vertical motion of both types of momentum wakes, and the energy dissipation patterns within the wakes during the collapse and late wake also exhibit similarities [50,68].

In recent years, the evolution characteristics of the self-propelled momentum wake of underwater moving bodies have been increasingly emphasized. This is because, in real-world scenarios, underwater moving bodies generate both resistance momentum wakes from their hulls and jet momentum wakes from their propulsion systems. The interaction between these two types of momentum significantly influences the formation of large-scale vortex structures in self-propelled bodies.

3.3. Computational Fluid Dynamics

Numerical simulations have emerged as a powerful tool for studying stratified wakes, offering detailed insights beyond the limitations of experiments. Unlike experiments, numerical methods can readily handle high Reynolds numbers and complex geometries, enabling a deeper understanding of wake dynamics.

Early studies have focused on spheres, highlighting the significant impact of initial conditions on near-wake characteristics. However, the late wake, characterized by large-scale dipole vortex streets, was found to be less sensitive, behaving similarly to a turbulent water mass. Comparing homogeneous and stratified fluids, researchers have observed a striking difference in energy dissipation. In homogeneous fluids, the wake's energy rapidly dissipates, while in stratified fluids, it remains significantly higher, highlighting the stabilizing effect of stratification.

Gourlay et al. [69] have paved the way for direct numerical simulation (DNS) of turbulent wakes. Later, large eddy simulation (LES) was employed by Dommermuth et al. [70] and Diamessis et al. [71] to successfully capture the late-wake structures, confirming their agreement with experimental findings. Further intriguing findings came from Menuier et al. [72], who have identified an additional viscous three-dimensional wake stage following the near-wake, collapse, and late-wake stages. Numerical simulations excel in their ability to tackle high Reynolds numbers, exceeding the capabilities of experiments. Diamessis et al. [73] have reported simulations at $Re = 10^5$, an order of magnitude higher than experiments. Bonnier et al. [55] have demonstrated that late-wake dynamics are independent of initial conditions, allowing researchers like Riley and Debruynkops [74], Waite and Bartello [75], Brethouwer et al. [76], Deloncle et al. [77], Augier and Billant [78], and Augier et al. [79] to study wake evolution at higher *Re* values by simply increasing computational costs.

To alleviate computational burdens, researchers have introduced the temporal evolution model [80–82]. This approach utilizes the stabilized wake flow field as an initial condition, effectively skipping the early stages of wake evolution. However, Redford et al. [83] have revealed its sensitivity to initial conditions, impacting velocity decay patterns and potentially leading to inaccurate results. Recognizing the limitations of the time model, some researchers have opted for direct simulations. This approach avoids the need for initial conditions but requires more cells, making it computationally expensive for high-*Re* and low-*Fr* conditions. Pasquetti [84] and Vandine et al. [85] have proposed a hybrid approach, directly simulating the near wake and employing a time model or spatially evolving model for the far wake.

Direct simulations of spheres at low *Re* have only recently become feasible. Orr et al. [86] have simulated conditions with $Re = 1000$, and Pal et al. [87] have simulated wakes at low Reynolds numbers ($Re = 3700$) and low Froude numbers ($Fr < O(1)$). These simulations have captured patterns of vortex shedding from the object, wake oscillations modulated by lee waves, wake fluctuations activated in low Froude number conditions, and unique phenomena of vortex dynamics in density-stratified flows. These features are evidently challenging to capture using a time model. Through the combined study of experiments and numerical simulations, researchers have clarified the entire process of the evolution of wakes behind a sphere in density-stratified flows across various stages.

Recently, there has been significant progress in numerical modeling for linearly stratified fluids based on density and temperature. Magnaudet and Mercier [88] and Zhang et al. [89] have developed their proprietary code, JADIM, which discretizes governing equations using the finite volume method and directly solves Navier–Stokes equations. The code includes boundary-fitted methods and a fixed grid approach, enabling it to handle rigid body motion and interface topological deformation. It is suitable for DNS simulations of two-phase and three-phase flows. Brucker and Sarkar [80], de Stadler and Sarkar [81], Pal et al. [87], and Ortiz-Tarin et al. [90] have used a direct simulation approach to simulate the turbulent flow around spheres and ellipsoids in stratified flows. They have mainly analyzed the effects of Reynolds number, internal Froude number, and geometric factors on turbulent wakes. Their results indicate that density gradients significantly influence the evolution of separated shear flows, recirculation, and turbulent wakes. It was demonstrated that even for weakly stratified flows, the late wake morphology tends to $Fr = O(1)$, implying that density stratification leads to a larger spreading and longer duration of turbulent wakes. Diamessis et al. [73], Abdilghanie and Diamessis [82], and Zhou and Diamessis [91] have introduced the spectral multidomain penalty method (SMPM) model, which is an implicit LES method for solving incompressible Navier–Stokes equations. SMPM is primarily used to simulate local stratified turbulence in high-Reynolds number conditions.

The studies mentioned above have often focused on simple geometries like spheres, and DNS and LES methods were commonly employed for turbulent flow simulations. While these methods offer high computational accuracy and a detailed capture of flow phenomena, they come with high computational costs, making them almost impractical for engineering applications in high-Reynolds number conditions. For example, in simulating the turbulent wake behind a sphere using LES, when $Re = 4 \times 10^5$ and $Fr = 4$, the required number of grids reached $1024 \times 512 \times 1106 = 579{,}862{,}528$, and the computational time was 5×10^6 CPU/h. Such computational costs make it nearly impossible to simulate the hydrodynamic wakes of underwater vehicles in practical engineering applications. Thus, even in the design phase, there is limited consideration for the hydrodynamic performance of underwater vehicles in realistic stratified ocean environments, and the prediction and assessment of complex turbulent flows are rarely addressed. This discrepancy with the actual operating environment prevents an accurate prediction of the hydrodynamic performance in real ocean conditions.

In recent years, researchers in the field of ocean engineering have made initial explorations into simulating the wake fields of underwater vehicles in linearly stratified fluids with density and temperature variations. Esmaeilpour [92], Liu et al. [93], and Li et al. [94] have simulated near wakes of underwater vehicles and surface ships, including bubbly mixed flow, highlighting the substantial impact of linear stratification on hydrodynamic vessel performance. Ma et al. [95,96] have employed a hybrid grid and DES model to study underwater vehicles in linearly stratified fluids, examining the influence of parameters like speed, depth, density gradient, and motion attitude on turbulent wakes at the free surface. Cao et al. [97–99] and Huang et al. [100] have proposed a thermocline model based on Boussinesq approximation, simulating vortical structures and wakes of sphere and a generic submarine model in homogeneous and stratified environments. Anisotropy of the surface wave crest and trough, along with a larger influencing zone of Kelvin waves, was evident in the stratified case (Figure 3).

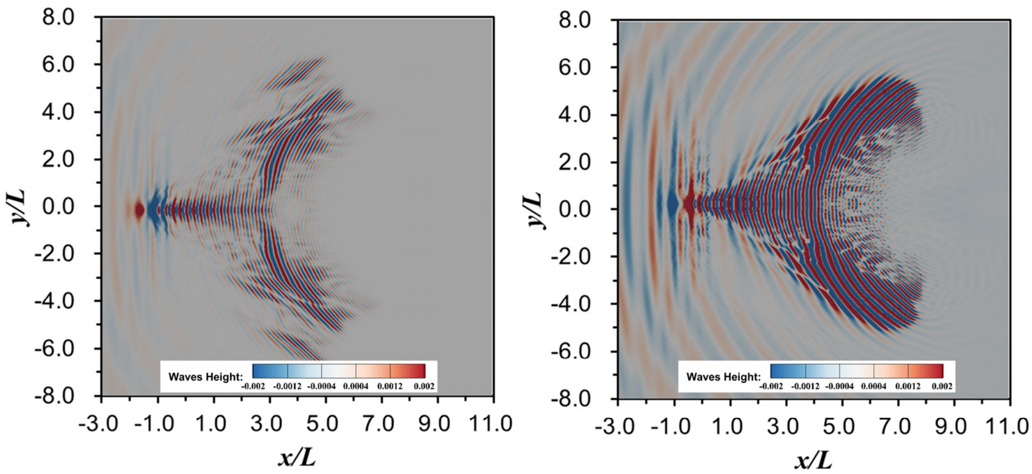

Figure 3. Comparison of the influence zone on the free surface of a submarine in homogeneous and stratified fluids.

Shi et al. [101] have studied the wave-making wake of maneuvering underwater vehicles in density-stratified fluid using RANS simulation, revealing Kelvin wave system characteristics and notable hydrodynamic performance changes with increasing drift angle. Jones and Paterson [102] have investigated swirl's impact on the near wake of a self-propelled axisymmetric vehicle with three propulsor schemes, noting reduced potential energy in the CRP configuration. Wang et al. [103] and Li et al. [104] have used CFD and the modified Korteweg-de Vries theory to simulate the interaction between internal solitary waves (ISWs) and submerged bodies, discussing forces and moments on submarine models at different submergence depths [105]. He et al. [106] have proposed a new numerical scheme for ISW generation with a current, demonstrating the interaction between a moving submarine and ISWs under varying pycnocline thicknesses and speeds.

In summary, the numerical simulation work on the hydrodynamic wakes of underwater vehicles in linearly stratified fluids, as mentioned above, has some limitations and shortcomings. Firstly, most of the current work focuses on simple geometric shapes such as spheres, ellipsoids, disks, plates, etc. The turbulent wakes of these geometric shapes exhibit strong regularity, and there is a relatively clear understanding of the wakes formed in linearly stratified fluids. However, the shapes of most marine and ocean engineering models are highly complex, and their turbulent characteristics vary widely. Therefore, corresponding numerical methods must be developed for the study of actual oceanic vehicles. Secondly, the Reynolds numbers in the mentioned research are generally low, typically ranging from Re = [100 to 10^5], while the Reynolds numbers of ships and marine engineering models are mostly above 10^6, and the Reynolds numbers at actual scales can reach 10^9, with more prominent instantaneous and turbulent characteristics. The flow phenomena are significantly different at these higher Reynolds numbers. Finally, the aforementioned works mainly focus on the model scale of underwater vehicles, with almost no simulations conducted at the actual scale of underwater vehicles. Currently, there is no relevant research on the similarity principles of flow fields in linearly stratified fluids, making it difficult to obtain an accurate overview of the hydrodynamics and flow fields of underwater vehicles at high Reynolds numbers and actual scales. Therefore, it is essential to conduct an analysis of the hydrodynamics of underwater vehicles in linearly stratified fluids and their free surface characteristics.

4. Thermodynamic Wakes in Stratified Fluid

When a submerged body is in motion underwater, especially the rotation of the stern propeller and the generation of specific spatiotemporal scale wake vortices, it stirs the surrounding fluid, thereby disturbing the originally stable stratified structure. This disturbance leads to the excitation of internal waves, disrupting the original density and temperature distribution characteristics. In addition, the rotation of the hull and propeller generates heat due to the impact friction with seawater. Submerged bodies, especially those with nuclear power systems releasing cooling water, can cause a rapid increase in the surrounding seawater temperature. The corresponding reduction in the density of some seawater during the diffusion process gradually causes it to rise.

Stefanick [5] has pointed out that a nuclear-powered submarine with a power of 190 MW and a speed of 5 knots would release about 45 million calories of heat energy per second into the ocean, or 45.4 °C cubic meters of hot water. The emitted heat energy causes the seawater behind the underwater vehicle to immediately rise by 0.2 °C. Although submerged vehicles continuously release heat, the emitted heat energy is quickly diluted by the surrounding seawater. About 1 km downstream of a normally traveling underwater vehicle, the temperature change in the seawater may be only about one hundredth of a degree Celsius. The attenuation of the thermal wake temperature difference signal intensity is relatively fast in the initial stage, but over a long distance with the rise in height, the weakening of the thermal signal is extremely slow. Even after rising several tens of meters, the thermal wake still shows temperature changes in the order of 0.1 °C. During the wake's ascent, its width keeps increasing, and the thermal trace signal width of a typical underwater vehicle on the water surface can reach from a dozen to several dozen meters.

Only when changes in seawater temperature are reflected on the sea surface can they be detected by the thermal imaging system of an infrared detector. Currently, airborne and spaceborne high-sensitivity infrared detectors can sense temperature differences as small as 0.001 °C on the sea surface [107]. By detecting temperature imaging over a wide range of sea areas, it is possible to monitor the thermal wake of underwater vehicles in all weather conditions and analyze their heading, speed, and depth characteristics. However, detecting the thermal wake of underwater vehicles from the vast ocean is not easy; the temperature change caused by underwater vehicles on the sea surface must be at least 0.005 °C or more to be detectable. This is because, except for the Arctic region, the average temperature gradient of seawater on Earth is 0.005 °C/m [108].

4.1. Theoretical and Experimental Methods

As early as the 1950s, researchers discovered that the cooling water discharged by surface and underwater vessels leaves behind a "thermal wake" on the sea surface. The Naval Air Development Center of the United States proposed a detection technology project for submarine wakes as early as 1957, defining two types of submarine wakes: "thermal wakes" related to changes in sea surface temperature, and "turbulent wakes" generated by submarine turbulence [109]. In 1959, Moser [107] pointed out in a technical report for the U.S. military that a P2V-5 aircraft, flying at an altitude of approximately 150 m with a speed of 18 knots and equipped with a thermal sensor with a sensitivity of 0.0016 K, could monitor the "thermal wake" of a submarine at a depth of 50 m in all weather conditions. The results showed that submarines caused a sea surface temperature change of 0.04 °C (night) to 0.1 °C (day), and that the width of the "thermal wake" impact range could be up to 100 to 120 m.

In 1984, Gebhart et al. [110] simplified the process of submarine power plant coolant discharge to a small pipe thermal jet. They established a mathematical model that could predict the trajectory and attenuation of the thermal jet in uniform and stratified fluids through theoretical modeling and experimental verification. In the 1980s, experimental research on the "thermal wake" of surface vessels began to emerge. For example, Garrett and Smith [111] and Peltzer et al. [112] used infrared remote sensing on an aircraft to analyze the distribution of the thermal wake of a 75 m catamaran. They pointed out that although the wake was called "thermal surface characteristics," the measurement results showed that the actual surface wake was a cold wake, meaning that the wake temperature was lower than the temperature of the ambient water. In the experiment, to make the wake signal more clearly detected by infrared remote sensing, oil alcohol was sprayed behind the hull, and the wake with chemical substances mixed in showed more pronounced temperature difference signals, more persistent characteristics, and a larger impact range. In this case, the maximum temperature difference of the wake can reach 0.8 °C, and the impact range can extend to about 3 km behind the hull. However, searching for targets in the vast ocean through the temperature difference of the wake without knowing the specific location of the vessel is challenging. Schwartz and Priest [113] have pointed out that the temperature difference between the ship's wake and the background water shown in infrared remote sensing images may be due to reflection phenomena rather than actual temperature differences. If observed from different angles, even if the wake and the background water have the same temperature, the infrared remote sensing image may erroneously show a temperature difference. Therefore, he corrected the infrared contrast between the ship's wake and the rough sea surface in the calculation model, correcting the temperature deviation in the infrared remote sensing image.

However, due to the complexity of the ocean itself and limitations in experimental conditions, most of the experiments mentioned above have focused on only studying the ascent and diffusion patterns of thermal wakes emitted by underwater vehicle models. It is challenging to directly conduct experimental research on the process of full-scale underwater vehicles emitting thermal wakes. Yang et al. [114], based on the principle of similarity, have designed a scaled-down model infrared imaging experimental test system using geometric similarity, temperature field similarity, drained heat flux similarity, and density field similarity. The system systematically investigated the infrared characteristics of submarine thermal wakes under two different conditions: stationary thermal plumes and towed thermal plumes.

4.2. Computational Fluid Dynamics

The emission, diffusion, and ascent process of thermal wakes from underwater vehicles are fundamentally a heat and mass transfer process into the seawater. The entire process is highly complex, and relying solely on experiments makes it difficult to fully uncover the mysteries of the thermodynamic wake generation and propagation of underwater vehicles. It is challenging to conduct experiments specifically for full-scale underwater vehicles. Experimental research is costly and time-consuming, and the accuracy is constrained by the precision of measurement methods and instruments.

With the development of computer technology, researchers have both domestically and internationally begun using numerical methods to simulate the emission process of thermal wakes from underwater vehicles. Similar to experimental approaches, numerical simulation studies often start with simpler objects. For example, Smith et al. [115] and Judd et al. [116] have studied underwater vortex pairs and thermal plumes, respectively, analyzing their thermal characteristics after interacting with the free liquid surface. Moody et al. [117] have calculated the structure and intensity of the wake generated by a cylinder moving in stratified fluid and analyzed the thermal trace on the free surface.

Due to secrecy concerns, there is not much publicly available literature on the numerical simulation of naval thermal wakes. Only a few articles describe relevant research on ship thermal wakes. Schwartz and Priest [113] have proposed a corrected model to predict the spreading process of ship wakes in the background sea. Voropayev et al. [118] have simulated the main processes of ship-generated thermal wakes. They pointed out that when a ship travels in the ocean, the propeller and hull will mix the surface seawater with the colder water below, causing a temperature difference of approximately 1 to 2 °C.

The issue of thermal wakes from underwater vehicles not only involves the emission process of underwater thermal wakes but also includes the detection of surface thermal signals. Therefore, capturing surface thermal signals is also an important consideration in the computational process. In recent years, as CFD has matured, researchers have gained the capability to thoroughly investigate the entire process of thermal wakes induced by the discharge of cooling water from realistic submarine models. Luo et al. [119] and Li et al. [120] have used overlapping grids to more accurately simulate the motion of submarine models. They have discovered that compared to traditional inflow methods, the results obtained using overlapping grids have high accuracy and can fully reflect the ascent and spreading process of thermal wakes, especially the vortex characteristics.

In summary, the generation of the thermal wake of an underwater vehicle is typically coupled with the creation of its hydrodynamic wake. The mechanisms involved in their generation and propagation are highly intricate. However, most current research primarily focuses on the emission and ascent-spreading processes of underwater vehicle thermal wakes, with limited attention given to the simultaneous propagation of thermal wakes and hydrodynamic wakes. Additionally, experimental and numerical simulation efforts related to the emission process of underwater vehicle thermal wakes are often conducted at a model scale. When studying the influence of factors such as velocity, submersion depth, and the linear stratification of density and temperature on thermal wake patterns, there is often a lack of effective establishment of scaled ocean stratification environments, similarities in the excitation of internal waves by underwater vehicles, and correspondence between the model scale and the actual scale in the wake propagation process. Consequently, there is an insufficient understanding of the ascent and spreading processes of actual underwater vehicle thermal wakes. Therefore, it is crucial to conduct simulations of the emission, ascent, spreading, and propagation of actual-scale underwater vehicle thermal wakes to the free surface in linearly stratified fluid, as well as to analyze the characteristics of the free surfaces.

5. Conclusions

This review focuses specifically on the hydrodynamic and thermodynamic wakes of underwater vehicles in stratified fluids, providing a comprehensive overview of the current research landscape from theoretical, experimental, and computational perspectives. The key conclusions drawn are as follows:

1. After decades of research, there is now a thorough understanding of the evolution of wakes generated by simple geometric bodies, such as spheres and cylinders, in stratified fluids. Clear insights into the flow characteristics during the near-wake, non-equilibrium, and far-field developmental stages of wakes have been gained. Additionally, the understanding of the energy transport during wake evolution and the formation mechanisms and features of large-scale vortex streets has been greatly enhanced.
2. In recent years, there has been a growing focus on the evolution characteristics of wakes generated by underwater vehicles with complex shapes. Changes in the hydrodynamic performance of underwater vehicles in linearly stratified flows of density and temperature, along with the extensive generation, evolution, and flow characteristics of hydrodynamic and thermodynamic wakes, have gradually become subjects of study. Comparisons with uniform flows demonstrate that the wakes of underwater vehicles in stratified flows exhibit a larger propagation range and a longer duration.
3. Due to the complexity of the marine environment and limitations in experimental conditions, research on underwater vehicle wakes in stratified flows is often conducted at model scales. Currently, there is a lack of research on similarity criteria for flow fields in stratified fluids. In the study of the impact of speed, depth, and stratification on wakes, an effective establishment of a scaled ocean stratified environment, a similar relationship for the excitation of internal waves, and a correspondence between model-scale and full-scale wake propagation processes have not been established.
4. With the advent of CFD technology, research methodologies have evolved from theoretical analysis and model experiments to predominantly numerical simulations, expanding toward complex geometries and high Reynolds numbers. Numerical simulations, relative to model experiments, provide more comprehensive information about the flow field.
5. The discharge of cooling water from underwater vehicles is often coupled with the generation of hydrodynamic wakes, creating highly intertwined mechanisms for their generation and propagation. However, most current studies focus solely on either the hydrodynamic or thermodynamic wakes from underwater vehicles, with little research on the mixed or coupled propagation process.

The comprehensive exploration of wakes generated by underwater vehicles in density-stratified fluids has laid a solid foundation for future investigations. As we move forward, there are several promising directions for further research.

1. Extending the current understanding of wake evolution to encompass more complex geometries, especially those representing realistic underwater vehicles, will be crucial.
2. Addressing the transition from model-scale experiments to full-scale wakes and establishing similarity criteria for flow fields in stratified fluids will enhance the applicability of findings to real-world scenarios.
3. The increasing reliance on numerical simulations opens avenues for exploring complex geometries and high Reynolds numbers. Future research could delve deeper into refining numerical methods, focusing on enhancing fidelity while managing computational costs.
4. Efforts should be directed towards bridging the gap between hydrodynamic and thermodynamic wakes, considering their intertwined nature during underwater vehicle operations.

5. The mixed or coupled propagation processes resulting from the discharge of cooling water need more attention, as existing studies often focus on either hydrodynamic or thermodynamic aspects.

By addressing these areas, future research endeavors can contribute to a more holistic understanding of wakes in density-stratified fluids, with direct implications for both academic progress and practical submarine engineering.

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Nomenclature

g	Gravitational acceleration
L	Length
N	Brunt–Väisälä frequency
T	Time
U	Characteristic velocity
v	Velocity
x	Longitudinal coordinate
y	Lateral coordinate
z	Vertical coordinate
Fr	Internal Froude number
Re	Reynolds number
St	Strouhal number
ρ	Density
ν	Kinematic viscosity

Abbreviations

CCD	Charge-coupled device
CFD	Computational fluid dynamics
CPU	Central processing unit
CRP	Contra-rotating propellers
DES	Detached eddy simulation
DNS	Direct numerical simulation
ISW	Internal solitary wave
JADIM	In-house computational code
K-H	Kelvin–Helmholtz
LES	Large eddy simulation
MW	Megawatt
NEQ	Non-equilibrium
NW	Near wake
PIV	Particle image velocimetry
Q2D	Quasi-two-dimensional
RANS	Reynolds-averaged Navier–Stokes
SAR	Synthetic aperture radar
SMPM	Spectral multidomain penalty method
U. S.	United States
WAS	Wide-area surveillance

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