Wake Signature of a Submarine in Density Stratified Fluid

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1. Introduction
2. Numerical Method and Validation
3. Results and Discussion
4. Conclusions and future work
1 Introduction
2 Numerical Method and Validation
3 Results and Discussion
4 Conclusions and future work
1.1 Background

The mysterious “dead water” phenomenon

- In 1893, the Norwegian explorer Fridtjof Nansen experienced a strange phenomenon when he was travelling north of Siberia.

- Nansen wrote afterwards: “Fram appeared to be held back, as if by some mysterious force, and she did not always answer the helm. We made loops in our course, turned sometimes right around, tried all sorts of antics to get clear of it, but to very little purpose.”

Fridtjof Nansen and his ship Fram
1.1 Background

The mysterious “dead water” phenomenon

- In 1904, the Swedish physicist and oceanographer **Vagn Walfrid Ekman** showed in a laboratory that waves formed under the surface at the interface between the salt water and freshwater layers that form the upper portion of this area of the Arctic Ocean interact with the ship, generating drag.

- This phenomenon is seen in all seas and oceans where density stratification exits (because of salinity or temperature).
1.1 Background

What is the stratified fluid?

- Stratified fluid is a fluid with density variation. It is an important feature of geophysical flows such as those occurring in the oceans and the atmosphere. Expect at very small scales, all fluids in the nature are stratified.
- Variations in density can occur in any direction, but only vertical stratification is stable in the absence of external forces.

The three zones in the ocean

- **Surface zone (Mixed layer)**: In this zone, both temperature and salinity are approximately constant with depth because of mixing by wind and waves.
- **Thermocline/Pycnocline**: Density changes rapidly with depth due to extreme environmental changes.
- **Deep Layer**: This layer has relatively constant temperature and salinity distributions.
A stratified wake always differs from its unstratified counterpart as a sufficient downstream distance has passed; Once buoyancy effects have set in, it start suppressing vertical motions and wake height stops increasing; At further downstream, a quasi-two-dimensional regime where buoyancy forces dominate the flow, and the flow organizes into pancake eddies.
Most of the past work on stratified flow past bodies were the wake of bluff bodies, the Reynolds number is smaller than $10^5$ and lack of the free surface.

**Numerical Simulation**

<table>
<thead>
<tr>
<th>Homogeneous</th>
<th>Stratified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instantaneous vorticity $\omega_z$ in the horizontal plane</td>
<td>Instantaneous vorticity $\omega_y$ in the vertical plane</td>
</tr>
</tbody>
</table>

Objective of present study

- Develop a numerical method to simulate linearly stratified fluid past a bluff body with complex-shaped geometry at high Reynolds number
- Analyze the differences of wake signature between the stratified and unstratified conditions for a submarine
- Explore geometry variations’ effect on wakes and free surface signatures
  - Appendages
  - Propeller
  - Slenderness
1 Introduction
2 Numerical Method and Validation
3 Results and Discussion
4 Conclusions
2.1 Numerical Method

Methodology

Pycnocline refers in general to density, but corresponding nomenclature can be alternatively used to indicate a change due to salinity (halocline) or temperature (thermocline).

In open sea, the variation of salinity can be ignored when refer to the fluctuation of temperature. Therefore, density $\rho$ can be simply specified as a function of temperature $T$.

For the top 40~500m of the sea, we can describe density $\rho$ as a function of temperature $T$ in the form of a polynomial expression. While the temperature ranges as a function of the depth $z$ below the free surface.

$$
\begin{align*}
T &= f(z) = T_0 + k \cdot z \\
\rho &= \rho(T) = g(z) = \rho_0 - c \cdot z
\end{align*}
$$
2.1 Numerical Method

Governing Equations

\[ \nabla \cdot \mathbf{U} = 0 \]

\[ \frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot (\mathbf{U} \mathbf{U}) = -\frac{1}{\rho_0} \nabla p + \nu \nabla^2 \mathbf{U} + \frac{\rho g}{\rho_0} \]

Under Boussinesq approximation

Buoyancy

\[ \frac{\partial T}{\partial t} + \mathbf{U} \cdot \nabla T = \kappa \nabla^2 T + \frac{J}{\rho C_\rho} \]

\[ \rho = f(T) \]

2.1 Numerical Method

Turbulence Modelling (RANS $k$-$\omega$ SST)

\[
\frac{\partial}{\partial t} (\rho k) + \nabla \cdot (\rho k \mathbf{u}) = \nabla \cdot \left[ \left( \mu + \sigma_k \mu_t \right) \nabla k \right] + P_k - \rho \beta^* f_\beta \left( \omega k - \omega_0 k_0 \right) + S_k
\]

\[
\frac{\partial}{\partial t} (\rho \omega) + \nabla \cdot (\rho \omega \mathbf{u}) = \nabla \cdot \left[ \left( \mu + \sigma_\omega \mu_t \right) \nabla \omega \right] + P_\omega - \rho \beta f_\beta \left( \omega^2 - \omega_0^2 \right) + S_\omega
\]

VOF

\[
\frac{\partial}{\partial t} \int_V \alpha_i dV + \oint_A \alpha_i \mathbf{u} \cdot d\mathbf{a} = \int_V \left( S_{\alpha_i} - \frac{\alpha_i}{\rho_i} \frac{D \rho_i}{Dt} \right) dV - \int_V \frac{1}{\rho_i} \nabla \cdot (\alpha_i \rho_i \mathbf{u}_{d,i}) dV
\]

Depending on the value of the volume fraction, the presence of different phases in a cell can be distinguished:

\[
\begin{align*}
\alpha_i &= 0 \quad \text{the cell is completely void of phase air} \\
\alpha_i &= 1 \quad \text{the cell is completely filled with phase water} \\
0 < \alpha_i < 1 & \quad \text{indicate the presence of an interface between air and water}
\end{align*}
\]
2.2 Validation

Stratified fluid past a sphere

- Ensure the method is effective;
- Low Froude number:


Numerical

Experimental

Vorticity contours in the horizontal plane ($z=0$)

$Fr = 0.05$  
$Re = 180$

Vorticity contours in the vertical plane ($y=0$)

$Fr = 0.06$  
$Re = 180$

Streamline in the vertical plane ($y=0$)

$Fr = 0.05$  
$Re = 180$
2.2 Validation

Stratified fluid past a sphere

- Ensure the method is effective;
- High Froude number:


<table>
<thead>
<tr>
<th></th>
<th>Numerical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fr = 0.94</td>
<td>Re = 1120</td>
<td></td>
</tr>
<tr>
<td>Fr = 2.6</td>
<td>Re = 3810</td>
<td></td>
</tr>
</tbody>
</table>

Streamline in the horizontal plane (z=0)
Fr = 0.68  Re = 480

Streamline in the vertical plane (y=0)
Fr = 0.68  Re = 480

Vorticity contours in the vertical plane (y=0)
2.2 Validation

Analysis the free surface effects in the homogeneous fluid

- Ensure a proper grid size;
- Capture correct characteristics of the free surface;
- \( v = 1.201 \text{ m/s}, \ z/L = 0.205, \) both in consistent with Torunski.

Torunski, B. Computational analysis of the free surface effects on a BB2 submarine undergoing horizontal maneuvers[D]. The university of New Brunswick, 2018.
3 Results and Discussion
3.1 Linearly stratified fluid past a submarine

**Stratification setup**

- Initial density and temperature gradient: $\Delta T/\Delta z = 1.5 \, ^\circ\text{C}/\text{m}$, $\Delta \rho/\Delta z = 18.3 \, \text{kg/m}^4$

![Diagram of stratification setup]

The middle vertical plane ($y=0$)
3.1 Linearly stratified fluid past a submarine

**Computational Model and Grid Generation**

- Scaling ratio $\lambda = 18.348$
- Computational Domain: $-4L < x < 20L$, $-7L < y < 7L$, $-3L < z < 3L$
- $v = 1.201\text{m/s}$ $\frac{z}{L} = 0.25$ $\frac{\Delta T}{\Delta z} = 1.5 \degree\text{C/m}$ $\frac{\Delta \rho}{\Delta z} = 18.3 \text{kg/m}^4$

**Table:**

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Full (m)</th>
<th>Model (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>$L$</td>
<td>70.20</td>
<td>3.8260</td>
</tr>
<tr>
<td>Beam</td>
<td>$B$</td>
<td>9.60</td>
<td>0.5232</td>
</tr>
<tr>
<td>Draft to duck</td>
<td>$D$</td>
<td>10.60</td>
<td>0.5777</td>
</tr>
<tr>
<td>Draft to sail top</td>
<td>$D_{sail}$</td>
<td>16.20</td>
<td>0.8829</td>
</tr>
<tr>
<td>Wetted surface area</td>
<td>$S_{wa}$</td>
<td>2142.3</td>
<td>6.3635</td>
</tr>
</tbody>
</table>

The Joubert BB2 geometry
3.1 Linearly stratified fluid past a submarine

Linearly stratified fluid past the Joubert BB2 submarine

- Temperature distribution in different slices through $x$ direction
3.1 Linearly stratified fluid past a submarine

Linearly stratified fluid past the Joubert BB2 submarine

- Density distribution in different slices through $x$ direction
3.1 Linearly stratified fluid past a submarine

Density distribution in different slices through $x$ direction

- (a) $x/L = 0$
- (b) $x/L = 0.5$
- (c) $x/L = 1.0$
- (d) $x/L = 1.5$
- (e) $x/L = 2.0$
- (f) $x/L = 2.5$
- (g) $x/L = 3.0$
3.1 Linearly stratified fluid past a submarine

Density fluctuations of lines at different locations

- At $x/L=0$, the density profile behind the hull has a large fluctuation, which gradually decreases as the distance from the hull increases.
3.2 The free surface comparison

Free surface waves comparison

Homogeneous case

Stratified case
### 3.2 The free surface comparison

The waveform of the free surface in the stratified case has a longer and wider propagation distance in the $x$ and $y$ directions.

<table>
<thead>
<tr>
<th>Description</th>
<th>Unit</th>
<th>Homogeneous</th>
<th>Stratified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal propagation distance</td>
<td>m</td>
<td>9.52$L$</td>
<td>10.22$L$</td>
</tr>
<tr>
<td>Transverse propagation distance</td>
<td>m</td>
<td>5.81$L$</td>
<td>6.56$L$</td>
</tr>
<tr>
<td>The maximum wave trough value</td>
<td>m</td>
<td>0.0117</td>
<td>0.0206</td>
</tr>
<tr>
<td>The maximum wave crest value</td>
<td>m</td>
<td>0.0144</td>
<td>0.0174</td>
</tr>
</tbody>
</table>

- **Homogeneous case**:  
  - Wave height: -0.014 to 0.01
  - Wave height: -0.016 to 0.02

- **Stratified case**:  
  - Wave height: -0.007 to 0.002
  - Wave height: -0.008 to 0.004

+74%  +12.9%  +76.1%  +20.8%
3.2 The free surface comparison

Specific free surface profile analysis

- $y=0\text{m}$
- $y=0.1308\text{m}$
- $y=0.2616\text{m}$
- $y=0.3924\text{m}$
- $y=0.5232\text{m}$
3.3 The influence of geometry variations

Computational Model and Grid Generation

- Scaling ratio $\lambda = 18.348$
- Computational Domain: $-4L < x < 20L$, $-7L < y < 7L$, $-3L < z < 3L$
- $v = 1.201\text{m/s}$  $z/L = 0.25$  $\Delta T/\Delta z = 1.5 \, ^\circ\text{C/m}$  $\Delta \rho/\Delta z = 18.3 \, \text{kg/m}^4$

<table>
<thead>
<tr>
<th>Description</th>
<th>Full (m)</th>
<th>Model (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>70.20</td>
<td>3.8260</td>
</tr>
<tr>
<td>Beam</td>
<td>9.60</td>
<td>0.5232</td>
</tr>
<tr>
<td>Draft to duck</td>
<td>10.60</td>
<td>0.5777</td>
</tr>
</tbody>
</table>

The Joubert BB2 geometry (bare hull)
3.3 The influence of geometry variations

Computational Model and Grid Generation

- Scaling ratio $\lambda = 18.348$
- Computational Domain: $-4L < x < 20L$, $-7L < y < 7L$, $-3L < z < 3L$
- $v = 1.201\text{m/s}$, $z/L = 0.25$, $\Delta T/\Delta z = 1.5 \, ^\circ\text{C/m}$, $\Delta \rho/\Delta z = 18.3 \, \text{kg/m}^4$

<table>
<thead>
<tr>
<th>Description</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades</td>
<td>4</td>
</tr>
<tr>
<td>direction</td>
<td>Right-handed</td>
</tr>
<tr>
<td>Propeller diameter</td>
<td>227.27</td>
</tr>
<tr>
<td>Hub diameter $D_H$ (mm)</td>
<td>45.53</td>
</tr>
<tr>
<td>Pitch ratio $P/D_p$ (mm)</td>
<td>1.1</td>
</tr>
<tr>
<td>Propeller position $x/L$</td>
<td>0.982</td>
</tr>
<tr>
<td>Duck diameter $D_1/D_2$ (mm)</td>
<td>128/118</td>
</tr>
<tr>
<td>Duck position $x/L$</td>
<td>0.979</td>
</tr>
</tbody>
</table>

The Joubert BB2 geometry (with propeller)

Computation grid (33 million)
3.3 The influence of geometry variations

The influence of appendages

- Density distribution in different slices through $x$ direction

Bare hull

Joubert BB2

Joubert BB2 with ducted propeller
### 3.3 The influence of geometry variations

#### The influence of appendages

<table>
<thead>
<tr>
<th>Descriptions</th>
<th>Bare hull</th>
<th>Joubert BB2</th>
<th>Joubert BB2 with propeller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal propagation distance</td>
<td>9.89$L$</td>
<td>10.22$L$</td>
<td>10.38$L$</td>
</tr>
<tr>
<td>Changes (%)</td>
<td>-</td>
<td>$+3.34$</td>
<td>$+4.95$</td>
</tr>
<tr>
<td>Transverse propagation distance</td>
<td>5.94$L$</td>
<td>6.56$L$</td>
<td>6.64$L$</td>
</tr>
<tr>
<td>Changes (%)</td>
<td>-</td>
<td>$+10.44$</td>
<td>$+11.78$</td>
</tr>
<tr>
<td>The maximum wave trough value (m)</td>
<td>0.0131</td>
<td>0.0206</td>
<td>0.0201</td>
</tr>
<tr>
<td>Changes (%)</td>
<td>-</td>
<td>$+57.25$</td>
<td>$+53.44$</td>
</tr>
<tr>
<td>The maximum wave crest value (m)</td>
<td>0.0124</td>
<td>0.0174</td>
<td>0.0174</td>
</tr>
<tr>
<td>Changes (%)</td>
<td>-</td>
<td>$+40.32$</td>
<td>$+40.32$</td>
</tr>
</tbody>
</table>
3.3 The influence of geometry variations

The influence of appendages

- The generation of internal waves and downstream anisotropic turbulence provides a significant contribution to the higher wave velocity in the free surface.
3.3 The influence of geometry variations

The influence of body slenderness

- Scaling ratio $\lambda = 20$
- Computational Domain: $-3L < x < 10L$, $-5L < y < 5L$, $-3L < z < 3L$
- $v = 1.201 \text{m/s}$, $z/L = 0.25$, $\Delta T/\Delta z = 1.5 \, ^\circ\text{C/m}$, $\Delta \rho/\Delta z = 18.3 \, \text{kg/m}^4$

<table>
<thead>
<tr>
<th>Body slenderness</th>
<th>$L$(m)</th>
<th>$D$(m)</th>
<th>Displacement (t)</th>
<th>Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>135</td>
<td>19.286</td>
<td>32824</td>
<td>+5.4%</td>
</tr>
<tr>
<td>8.0</td>
<td>145</td>
<td>18.125</td>
<td>31140</td>
<td>-</td>
</tr>
<tr>
<td>9.0</td>
<td>155</td>
<td>17.222</td>
<td>30054</td>
<td>-3.5%</td>
</tr>
<tr>
<td>10.0</td>
<td>165</td>
<td>16.500</td>
<td>29365</td>
<td>-5.7%</td>
</tr>
<tr>
<td>11.0</td>
<td>175</td>
<td>15.909</td>
<td>28954</td>
<td>-7.0%</td>
</tr>
</tbody>
</table>

The geometry of different body slenderness
3.3 The influence of geometry variations

The influence of body slenderness

<table>
<thead>
<tr>
<th>Body slenderness</th>
<th>The maximum wave trough value (m)</th>
<th>Changes (%)</th>
<th>The maximum wave crest value (m)</th>
<th>Changes (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>0.0941</td>
<td>+76.5</td>
<td>-0.0624</td>
<td>+22.1</td>
</tr>
<tr>
<td>8.0</td>
<td>0.0533</td>
<td>-</td>
<td>-0.0511</td>
<td>-</td>
</tr>
<tr>
<td>9.0</td>
<td>0.0223</td>
<td>-58.2</td>
<td>-0.0413</td>
<td>-19.2</td>
</tr>
<tr>
<td>10.0</td>
<td>0.0232</td>
<td>-56.5</td>
<td>-0.0352</td>
<td>-31.1</td>
</tr>
<tr>
<td>11.0</td>
<td>0.0309</td>
<td>-42.0</td>
<td>-0.0322</td>
<td>-37.0</td>
</tr>
</tbody>
</table>
3.3 The influence of geometry variations

The influence of body slenderness

Select a small range \((0 < x < 2L, -2L < y < 2L)\) on the free surface to analyze variances of wave height.

<table>
<thead>
<tr>
<th>Body slenderness</th>
<th>Variances of wave height</th>
<th>Changes (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>(7.24 \times 10^{-4})</td>
<td>+52.7</td>
</tr>
<tr>
<td>8.0</td>
<td>(4.74 \times 10^{-4})</td>
<td>-</td>
</tr>
<tr>
<td>9.0</td>
<td>(1.37 \times 10^{-4})</td>
<td>-71.1</td>
</tr>
<tr>
<td>10.0</td>
<td>(1.42 \times 10^{-4})</td>
<td>-70.0</td>
</tr>
<tr>
<td>11.0</td>
<td>(8.85 \times 10^{-5})</td>
<td>-81.3</td>
</tr>
</tbody>
</table>
3.3 The influence of geometry variations

The influence of body slenderness

<table>
<thead>
<tr>
<th>Body slenderness</th>
<th>Kelvin wedge angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>26</td>
</tr>
<tr>
<td>8.0</td>
<td>25</td>
</tr>
<tr>
<td>9.0</td>
<td>24</td>
</tr>
<tr>
<td>10.0</td>
<td>23</td>
</tr>
<tr>
<td>11.0</td>
<td>22</td>
</tr>
</tbody>
</table>
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4. Conclusions and Future work

Conclusions

- A numerical method was proposed and verified. It can be used to deal with complex-shaped geometry at high Reynolds number in density stratified fluid.
- The wake signature in the stratified fluid has a longer propagation distance both in $x$ and $y$ directions compared to that of unstratified case.
- The existence of density stratification provides a significant contribution to the anisotropy of the wave height on the free surface and the greater wave amplitude.
- Submarine hull appendages, especially the propeller, will promote a remarkable increase to the propagation distance and wave velocity for the wakes.
- The slimmest body ($L/D=11$) has the best performance (smallest fluctuation) on the free surface.
4. Conclusions and Future work

Future work

- Advanced turbulence modelling (DES)

Density profile

Vortex structures (Q=5)

Density fluctuation

Internal waves
4. Conclusions and Future work

Future work

- Advanced turbulence modelling (DES)
4. Conclusions and Future work

Future work

- Advanced turbulence modelling (DES)

Density profile in the middle vertical plane ($y=0$)
Thank you