Vortex Identification and Visualization for Complex Ship and Ocean Engineering Flows

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Outline

• Motivation

• Overview of methods
  • Vorticity
  • Methods based on velocity gradient decomposition

• Applications
  • Propeller open water test
  • JBC hull drag simulation
  • Vortex-induced motion of a semi-submersible

• Summary
Motivation

- Marine hydrodynamic involves with high-Re flow and complex geometries
- These flows include complex and numerous vortex structures
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• Summary
Overview of methods

1. Vorticity

Defined as the curl of velocity $\omega = \nabla \times \mathbf{u}$


- Cannot distinguish between swirling motions and shearing motions
Overview of methods

2. Methods based on velocity gradient decomposition

\[ D = S + \Omega \]

**Velocity gradient tensor**

\[ D = \nabla u \]

**Symmetric, strain rate tensor**

\[ S = \frac{1}{2} (\nabla u + \nabla u^T) \]

**Skew-symmetric, vorticity tensor**

\[ \Omega = \frac{1}{2} (\nabla u - \nabla u^T) \]

Characteristic equation:

\[ \lambda^3 + P \lambda^2 + Q \lambda + R = 0 \]

**First invariant**

\[ P = -\text{tr}(D) \]

**Second invariant**

\[ Q = \frac{1}{2} (\text{tr}(D)^2 - \text{tr}(D^2)) \]

**Third invariant**

\[ P = -\text{det}(D) \]
Overview of methods

2. Methods based on velocity gradient decomposition

- Q-criterion: second invariant of velocity gradient tensor

\[
Q = \frac{1}{2} \left( tr(D)^2 - tr(D^2) \right) = \frac{1}{2} \left( \|\Omega\|^2 - \|S\|^2 \right)
\]

Defines a vortex as a “connected fluid region with a positive second invariant of \(\nabla u\)”

\[Q > 0\]
Overview of methods

2. Methods based on velocity gradient decomposition

- $\lambda_2$-criterion: second eigenvalue of $A = \Omega^2 + S^2$

Defines a vortex as a "a connected region with two negative eigenvalues of $\Omega^2 + S^2$"

$\lambda_2 < 0$

Vortex of wind turbine
Overview of methods

2. Methods based on velocity gradient decomposition

- **New Omega** (Liu et al., 2016):

  \[ \Omega = \frac{b}{a + b + \varepsilon} \]

  Where,

  \[ a = \text{tr}(S^T S) \]
  \[ b = \text{tr}(\Omega^T \Omega) \]
  \[ \varepsilon = 0.000001(b - a)_{\text{max}} \]

  - Connected regions where \( \Omega > 0.5 \) are identified as vortex
  - Normalized between 0 and 1, recommend value for iso-surface: 0.52

Overview of methods

2. Methods based on velocity gradient decomposition

Rortex/Liutex (Liu et al., 2018):

First, get the direction of Rortex/Liutex vector. Transfer velocity gradient to a local frame XYZ in which the new Z axis coincident with rotation axis

\[ \nabla U = Q \nabla u Q^T = \begin{bmatrix} \frac{\partial U_X}{\partial X} & \frac{\partial U_X}{\partial Y} & 0 \\ \frac{\partial U_Y}{\partial X} & \frac{\partial U_Y}{\partial Y} & 0 \\ \frac{\partial U_Z}{\partial X} & \frac{\partial U_Z}{\partial Y} & \frac{\partial U_Z}{\partial Z} \end{bmatrix} \]

Overview of methods

2. Methods based on velocity gradient decomposition

Rortex/Liutex (cont.)(Liu et al., 2018):

Then, obtain the strength (magnitude) of Rortex/Liutex vector

\[ R = \begin{cases} 
2(\beta - \alpha), & \alpha^2 - \beta^2 < 0 \text{ and } \beta > 0 \\
2(\beta + \alpha), & \alpha^2 - \beta^2 < 0 \text{ and } \beta < 0 \\
0, & \alpha^2 - \beta^2 \geq 0
\end{cases} \]

Where,

\[ \alpha = \frac{1}{2} \sqrt{\left( \frac{\partial U_Y}{\partial Y} - \frac{\partial U_X}{\partial X} \right)^2 + \left( \frac{\partial U_Y}{\partial X} + \frac{\partial U_X}{\partial Y} \right)^2} \]

\[ \beta = \frac{1}{2} \left( \frac{\partial U_Y}{\partial X} - \frac{\partial U_X}{\partial Y} \right) \]

Finally, Rortex/Liutex vector is defined as: \( \mathbf{R} = R \mathbf{r} \)

Overview of methods

2. Methods based on velocity gradient decomposition

- Normalized Rortex/Liutex (Dong et al., 2019):

\[
\tilde{\Omega}_R = \frac{\beta^2}{\beta^2 + \alpha^2 + \varepsilon}
\]

Where,

\[
\alpha = \frac{1}{2} \sqrt{\left(\frac{\partial U_Y}{\partial Y} - \frac{\partial U_X}{\partial X}\right)^2 + \left(\frac{\partial U_Y}{\partial X} + \frac{\partial U_X}{\partial Y}\right)^2}
\]

\[
\beta = \frac{1}{2} \left(\frac{\partial U_Y}{\partial X} - \frac{\partial U_X}{\partial Y}\right)
\]

- \(\varepsilon\) is a small number to avoid divided by zero

Overview of methods

2. Methods based on velocity gradient decomposition

➤ (Modified) Normalized Rortex/Liutex (Liu and Liu, 2019):

\[
\tilde{\Omega}_R = \frac{\beta^2}{\beta^2 + \alpha^2 + \lambda_{cr}^2 + \frac{1}{2} \lambda_r^2 + \varepsilon}
\]

Where,

➤ \( \lambda_r \) is the real eigenvalue of velocity gradient tensor

➤ \( \lambda_{cr} \) is the real part of conjugate complex eigenvalue of velocity gradient tensor

➤ \( \varepsilon \) is a small number to avoid divided by zero

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• Summary
1. Propeller open water test

- ONRT propeller from Tokyo2015 CFD workshop
- $k - \omega$ SST turbulence model
- Dynamic overset grid (1.13 million in total)
- Single-run for all advance ratios
Applications

1. Propeller open water test

$Q = 5$

$Q = 100$

$Q = 500$

$Q = 1000$
Applications

1. Propeller open water test

\[ \lambda_2 = -5 \]

\[ \lambda_2 = -100 \]

\[ \lambda_2 = -500 \]

\[ \lambda_2 = -1000 \]
Applications

1. Propeller open water test

\[ \tilde{\Omega}_R = 0.52 \]

\[ \tilde{\Omega}_R = 0.54 \]

\[ \tilde{\Omega}_R = 0.56 \]

\[ \tilde{\Omega}_R = 0.58 \]
2. JBC hull drag simulation

- JBC model from Tokyo2015 CFD workshop
- $k - \omega$ SST DDES turbulence model
- Stationary mesh (19.8 million)

Ref: https://t2015.nmri.go.jp/jbc.html
2. JBC hull drag simulation

\[ Q = 5 \]

\[ Q = 50 \]
Applications

2. JBC hull drag simulation

\[ \tilde{\Omega}_R = 0.52 \]

\[ \tilde{\Omega}_R = 0.60 \]
Applications

2. JBC hull drag simulation

\[ Q = 10 \]

\[ \tilde{\Omega}_R = 0.52 \]
Applications

3. Vortex-induced motion of semi-submersible

- Paired-Column Semi-submersible
- $k - \omega$ SST DDES turbulence model
- Dynamic overset mesh (2.53 million)
3. Vortex-induced motion of semi-submersible

\[ Q = 1 \]

\[ \tilde{\Omega}_R = 0.52 \]
3. Vortex-induced motion of semi-submersible

\[ Q = 1 \]

\[ \tilde{\Omega}_R = 0.52 \]
3. Vortex-induced motion of semi-submersible
Summary

- Various kinds of vortex identification (VI) methods have been applied to complex ship and ocean engineering flows.
- Vorticity cannot distinguish between swirling and shearing motions, thus cannot represent vortex.
- Traditional eigenvalue-based VI methods (such as Q and lambda2) cannot distinguish shearing motions from rotation and the vortex structures depend on the threshold value.
- The (modified) normalized Rortex/Liutex VI method simplifies the VI procedures in the following aspects:
  - Normalized, threshold for iso-surface is always \(~0.52\)
  - Identified vortex excludes shearing motions and shear boundary from vorticity.
  - Capture strong and weak vorticities simultaneously.
Future work

- Apply Rortex/Liutex to more marine hydrodynamic problems
- Determination of vortex core center lines

Thank you!