Available online at https://link.springer.com/journal/42241
http://www.jhydrodynamics.com
Journal of Hydrodynamics, 2021
https://doi.org/10.1007/s42241-021-0049-1

Applications of Liutex-based force field models for cavitation simulation *

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(Received May 7, 2021, Revised June 9, 2021, Accepted June 10, 2021, Published online June 28, 2021)

Abstract: When studying the flow dynamics and the characteristics of fluid motion, vortex structure with their interactions is the key issue. In the present work, a vortex control method is investigated based on the vortex identification system of Liutex. The numerical study is carried out in OpenFOAM by directly adding a source term to the Navier-Stokes equations, which is called the centripetal force model in Liutex method. A 2-D test case is examined to justify the proposed method in cavitating flow around Clark-Y hydrofoil, the simulation results show that the improved Liutex solver is feasible. Methodologies of controlling the rotation strength of vortices are able to change the flow field and suppress the cavitation. The applicability of vortex-based control method in 3-D flow field is also studied. The results show that cavitation surrounded by particular vortex can be effectively influenced.

Key words: Liutex force model, cavitation, vortex control

Introduction

Cavitation occurs when the liquid pressure drops below the saturated vapor pressure, which is a common physical phenomenon developed from the tiny gas core. With the continuous improvement of equipment operation speed, the frequency of cavitation is also increasing. Cavitation will not only affect the hydrodynamic performance of equipment, but also produce vibration, noise and material erosion. Therefore, cavitation research is becoming more and more important in the field of ocean engineering hydrodynamics. Cavitation flow contains many complex components, such as turbulence, two-phase flow, which also makes the study of cavitation more difficult.

In recent years, with the rapid development of computer technology, the research of cavitation characteristics based on numerical simulation method has gradually become the mainstream. In the design stage of hydraulic machinery, the results of numerical simulation generally replace the experimental results and become an important Ref. [1] Huang et al.[2] deeply analyzed a series of changes in the process of hydrofoil cavitation, including the influence of re-entrant jet and shock wave on cavitation and how vorticity and velocity vector change in the process of cavitation development. Large scale vortex structures are also well observed. There has been research about how to optimize and improve the geometric characteristics to achieve the purpose of influencing cavitation and alleviating pressure fluctuation in the design stage of propellers. Timoshevskiy's[3] research about cavitating flow around a 2-D hydrofoil has reference value in the verification and optimization of existing and newly developed numerical models. An improved multi-scale two-phase flow solver is realized by Zhang et al.[4] to simulate bubbly flow, which has a good application prospect in cavitation research.

On the other hand, further research on the mechanism of cavitation[5-6], and the strategy of cavitation suppression are also the focus of research. Considering the role of engineering application, the possibility of cavitation suppression by using vortex-based force field model is studied.

Vortex is very common in the field of fluid mechanics, and research on vortex is in constant development and change. The first generation of

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* Projects supported by the National Key Research and Development Program of China (Grant Nos. 2019YFC0312400, 2019YFB1704200), the National Natural Science Foundation of China (Grant Nos. 51879159, 51909160).
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vortex identification method is vorticity-based[7], which can be traced back to a hundred years ago. In recent decades, in order to solve the problems of the first-generation methods, researchers have developed many vortex identification criteria[8-10]. In fact, all these methods can be classified into the second-generation vortex identification method for they are based on the Cauchy-Stokes decomposition and/or eigenvalues of the velocity gradient tensor. The second-generation methods also have some problems that cannot be ignored. For example, as a scalar vortex identification method, the iso-surface method is used to represent the vortex structure of the flow field, and the artificial threshold is needed; vortex motion has rotation axis, and several scalar vortex identification methods give (local/global) rotation axis, etc.. The shortcomings of the previous methods also stimulate the progress in the research and development of new vortex recognition methods. Thus a Liutex vector was proposed to provide a mathematical definition of the local rigid rotation part of the fluid motion.

For a vortex definition method, there are six aspects to judge the reliability of it. In Liutex system, the six core elements of vortex, the absolute strength, the relative strength, the local rotation axis, the vortex rotation axis, the vortex core size and the vortex boundary can be provided. In recent years, the research on Liutex identification method has also made progress.

The research on vortex structure and wake of a sphere in stratified fluid with uniform density is the first step for vortex wake prediction of fully attached submarine in real marine environment[11]. Wang et al.[12] have studied the essence of vortex, their research shows that an open area for vorticity line penetration rather than a tube of vorticity lines without any leak. More importantly, the DNS observation and other results show that most of the vortices ended inside the flow field, which is contrary to the conclusion in some textbooks. On the basis of previous research, Liu et al.[13] continued to analyze from the level of mathematical model and put forward a new definition of vortex vector called Rortex. Through comparative analysis, it can be seen that this new method can identify both the swirling strength and the rotational axis, meaning that this is a better method to study vortex dynamics and turbulence.

Based on the definition of Liutex, a new criteria of vortex identification is proposed by Dong et al.[14]. Such method has been proved to be more advantageous in capturing vortex structures and distinguishing the rotational vortices from the shear layers and other non-physical structures. Xu et al.[15]. went a step further in the study of vortex identification. They developed a technique that can automatically identify vortex cores, which is very suitable for turbulence research. With the general development of computer, this technology will have a broad application prospect in the future.

Compared with the failure and inaccuracy of the first and second-generation methods, the Liutex system can give complete and accurate information of all six core elements, and the discovery of Liutex similarity might promote the utilization of Liutex system in the study of turbulence mechanism[16]. Zhao's[17] research on the turbulent flow around a circular cylinder shows that the Liutex method can simulate the flow field well and capture the change of vortex structure even without adding objects.

An alternative eigenvector-based definition of Rortex is introduced by Gao and Liu[18], and the systematic interpretation of scalar, vector and tensor versions of Rortex is presented. Zhao et al.[19] analyzed and compared the results of different eddy recognition methods in marine engineering. The first two methods have some limitations. The Liutex method has the advantages in visualization, and he also summarizes the problems to which attention should be paid in the specific application.

In the field of the combination of vortex dynamic method and cavitation research, some researchers have made very innovative progress. The research carried out by Wang et al.[20] has made a breakthrough in studying vortex dynamics and obtaining efficient vortex-based control strategies. The centripetal force model and the counter-rotation force model applied both have good simulation results in cavitation suppression. Yu et al[21]. realized the combination of Liutex method and cavitation model in OpenFOAM. The obtained results in the simulation of 2-D Clark-Y hydrofoil cavitation are satisfying. The influence of Liutex on cavitation and the change of minimum pressure in vortex center are obvious.

In this study, we first realized the combination of Liutex method and cavitation model in OpenFOAM, and verified the reliability of the solver with a 2-D hydrofoil example. Then, we use the solver to simulate the 3-D hydrofoil cavitation flow field, and discuss the applicability of the Liutex method in 3-D problems.

1. Computational methods

The vector definition method in Liutex, like most vortex identification methods, is also based on the velocity gradient tensor, which is determined by rotating the velocity gradient tensor to a special coordinate system where the rotation axis coincides with the local Z axis. This is also better to describe the local structure of the flow field topology. Liutex contains the direction information of the vortex, that is, the local rotation axis. Because the other two eigenvalues are complex conjugate, the rotation axis is the real eigenvector of the velocity gradient tensor. A simplified explicit expression is as follows:

$$\boldsymbol{a}=\frac{\boldsymbol{R}}{2}\times\left(\frac{\boldsymbol{R}}{2}\times\boldsymbol{l}\right)=\frac{1}{4}\boldsymbol{R}\times\left(\frac{\boldsymbol{R}}{2}\times\boldsymbol{l}\right) \tag{1}$$

$$\boldsymbol{R}=R\boldsymbol{r}=\left[\boldsymbol{\omega}\cdot\boldsymbol{r}-\sqrt{(\boldsymbol{\omega}\cdot\boldsymbol{r})^2-4\lambda_{ci}^2}\right]\boldsymbol{r} \tag{2}$$

where $\boldsymbol{R}$ is the Liutex vector, R is the magnitude of Liutex vector, r is the real eigenvector, ω is the vorticity vector, and λ_{ci} is the imaginary part of the complex eigenvalue. In Liutex system, $\boldsymbol{l}$ in Eq. (1) is a rotation vector starting at a random field point P and ending at the vortex core centre point P_0. P_0 is the local minimum pressure point.

In order to control vortices, the method used in this paper is to add a source term to Navier-Stokes equation, and the expression is as follows:

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{u})=0 \tag{3}$$

$$\frac{\partial\rho\boldsymbol{u}}{\partial t}+\nabla\cdot(\rho\boldsymbol{u}\boldsymbol{u})=-\nabla p+\nabla\cdot\tau+\rho\boldsymbol{g}+\boldsymbol{T}_\sigma\pm c\boldsymbol{a} \tag{4}$$

The above expression is momentum equation of Navier-Stokes equation in incompressible flow, in which $\boldsymbol{u}$ is the velocity vector field, p is the pressure, $\tau=\mu[\nabla\boldsymbol{u}+(\nabla\boldsymbol{u})^{\mathrm{T}}]$ is shear stress tensor, ρ is fluid density, and μ represents kinematic viscosity. In particular, c is a coefficient that flexibly controls the magnitude or strength of the force field. While $\boldsymbol{a}$ is the Liutex-based force field source term to be structured.

After the improved content is to add the source term to the momentum equation according to the previous formula, in order to solve this problem, we first need to calculate the Liutex vector of each element in the current time step, and then calculate the centripetal acceleration vector. Then we solve the modified Navier-Stokes equation with additional centripetal acceleration source term to obtain the velocity field and pressure field in the next time step, and then iterate repeatedly.

In another model, a parameter related to time scale is introduced, which can relax the time and make the rigid rotation of fluid disappear gradually.

Generally speaking, the selection strategy of two models of Liutex force model centripetal force model and counter rotating force model, is that if the main focus of the research is to alleviate the pressure minimum and control cavitation, the former will be chosen. The application of the centripetal force model can relieve the minimum pressure and attenuate the tension between adjacent fluids. The centripetal force required for fluid rotation is now provided by the force model instead of the pressure gradient.

In the research of using Liutex method to control vorticity to suppress cavitation, simulation has been carried out and satisfying results have been achieved by Yu[21] who applied the solver by Laboratory for Advanced Simulation of Turbulence (LAST) in Tsinghua University, including a novel cavitation model called nonlinear dynamic cavitation model (NDCM). In this study, the two-phase flow solver in OpenFOAM is improved. According to the method mentioned above, the NS equation is modified, the source term is added, and the centripetal force part is added to achieve the purpose of vortex control.

In the feasibility study of using Liutex force field to control the flow, considering the general situation, the definition of length in the force field model is simplified, which is a little different from the centripetal force model proposed by Yu and Wang[21], Wang et al.[20], and is the same as the method used by Zhao et al.[17], whose study focus on single-phase flow and does not involve gas-liquid two-phase flow. The main goal of this research idea is to study the vortex dynamics in two-phase flow, by analysing some responses and changes of the flow field, and to observe whether the vortex control method can change the vortex intensity in a specific region.

2. Simulation of Clark-Y hydrofoil cavitation

2.1 *Case setup*

In order to verify the availability of the improved solver, the cavitation suppression of 2-D hydrofoil is simulated. The example configuration used in the simulation is the same as that used in Yu[21] and Wang's[20] study. The chord length of the hydrofoil is 0.07 m, the incoming flow velocity is 10 m/s, the cavitation number is 0.8, and the grid is three-level refinement. The grid layout is shown in Fig. 1.

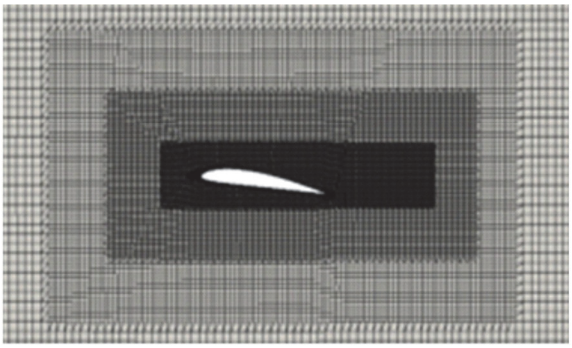
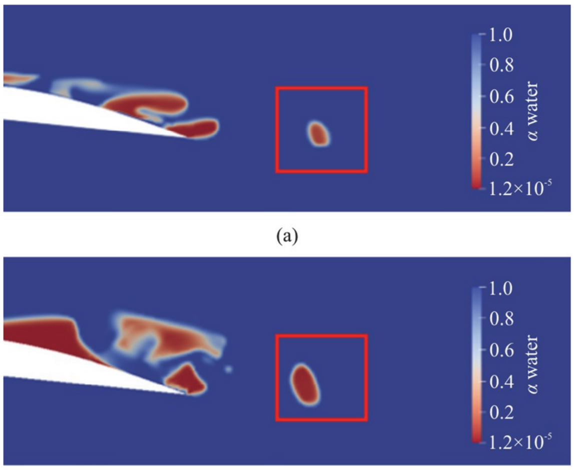


Fig. 1 The grid layout of Clark-Y hydrofoil

2.2 *Numerical results*

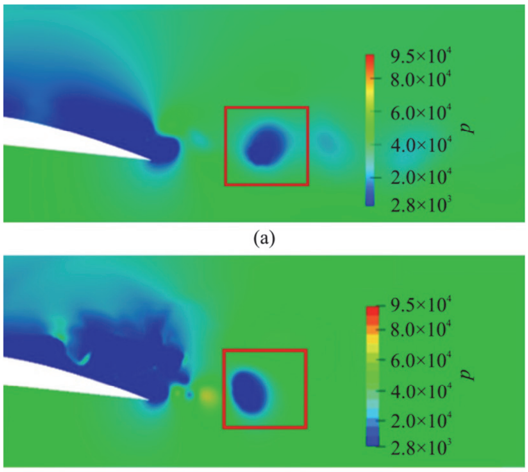
The simulation results of 2-D Clark-y hydrofoil cavitation flow are shown in the Figs. 2-4. The figures below show cavitation flow field, pressure field and Liutex vorticity field respectively. The pressure gradient toward the centre of rotation provides centripetal force for fluid rotation, and there is a local pressure reduction centre in the centre of vortex. And the centripetal force coefficient in the control model is taken as 1. In Figs. 2-4, (a) represents the simulation results using Liutex force model, while (b) represents the uncontrolled case.



(a)

(b)

Fig. 2 (Color online) The contours of volume of fraction of water

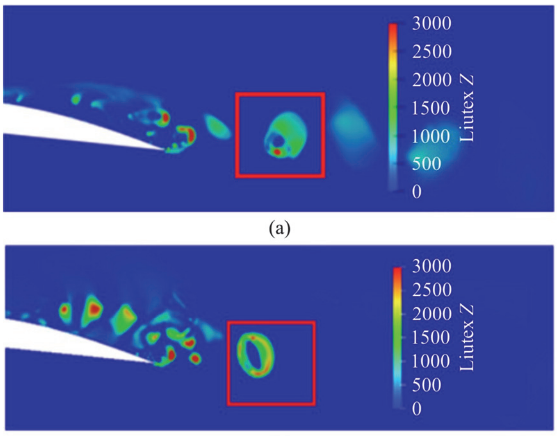


(a)

(b)

Fig. 3 (Color online) The contours of pressure

It can be seen that there is an obvious correlation between the pressure field and the cavitation distribution on the hydrofoil surface. The area where the pressure drops most is the area where cavitation occurs, which is also because the cavitation itself requires the liquid pressure to drop below the saturated vapor pressure. Without using the Liutex method to add the centripetal force source term, the flow field above the hydrofoil surface is more complex.



(a)

(b)

Fig. 4 (Color online) The distribution of Liutex component Z

The region surrounded by a red border is located in the wake region of hydrofoil, where high shear stress has little effect. When the centripetal force model is used to add the source term to exert control, the minimum pressure in the area surrounded by the red border is relieved, the pressure value increases, and the cavitation area also decreases, which means the cavitation is suppressed.

This phenomenon is consistent with the research conclusion of Yu and Wang[21], Wang et al.[20]. It can be seen from the comparison of Figs. 4(a), 4(b) that whether the centripetal force model is used or not has a certain influence on the vorticity field. The vortex in the area surrounded by the red border has little change. But the Liutex vorticity above the hydrofoil is obviously different. When the simplified centripetal force model is not used to control the vorticity, the vorticity above the hydrofoil is obviously larger in the positive direction, in contrast, when the Liutex force model is used, the originally small vortices become smaller and more fragmented.

The Liutex force model with centripetal force source term can achieve the goal of controlling vorticity, relieving the minimum pressure at the centre of vortex and suppressing cavitation. It shows that the method of modifying two-phase flow solver in OpenFOAM is feasible.

3. Simulation of NACA0009 hydrofoil cavitation

3.1 *Case setup*

The simulation results of 2-D hydrofoil cavitation flow field prove the feasibility of the modified solver. In this section, the 3-D flow field is studied and analysed in order to explore whether the vortex control method can achieve the effect of suppressing cavitation in 3-D situation. The Liutex solver based on OpenFOAM needs to be adjusted to adapt to 3-D problems. In the example, the 3-D NACA0009 hydrofoil is used. The computational domain is arranged as shown in Fig. 5, and the grid around the hydrofoil is refined by three levels.

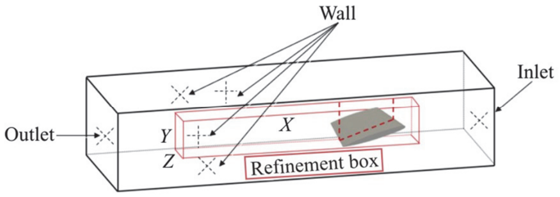


Fig. 5 (Color online) Computational domain and refinement layout

3.2 *Numerical results*

Figure 6 shows the iso-surface of NACA00009 hydrofoil 3-D cavitation image rendered with pressure field It can be seen that the volume of tip vortex cavitation formed on one side of the hydrofoil edge is significantly reduced. In Figs. 6, 7, (a) represents the simulation results using Liutex force model, while (b) represents the uncontrolled case.

The results show that the cavitation volume of the region extending from the middle and rear to the wake region of the 3-D hydrofoil surface is significantly larger when the Liutex force model is not used, after adding the centripetal force source term to suppress cavitation, it is obvious that the cavitation volume decreases and the cavitation region shrinks.

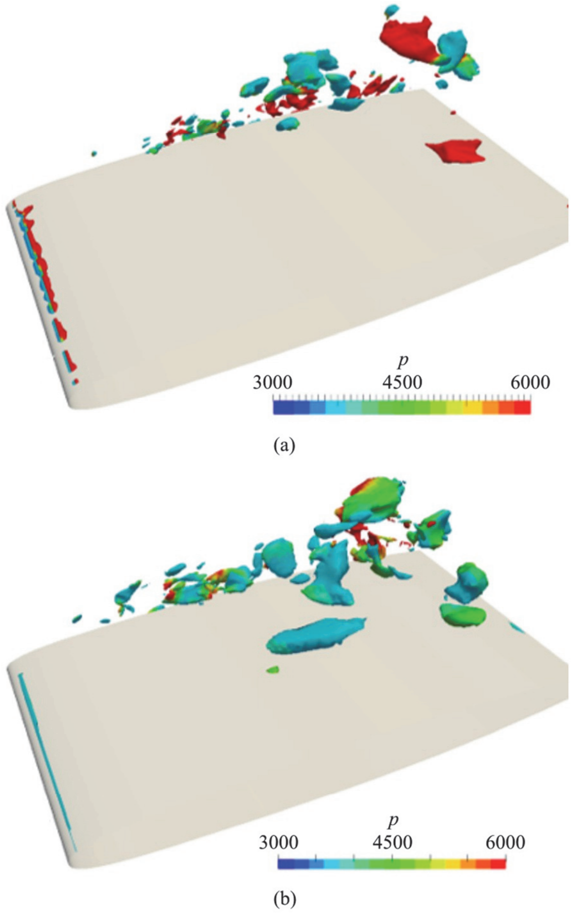
At the same time, the cavitation cloud image rendered by pressure also shows that the pressure increases and the cavitation is restrained. Compared with the Liutex vorticity, it can be seen that the vorticity on the hydrofoil surface tends to break up and decrease after using the Liutex force model to suppress cavitation, which is consistent with the conclusion of the 2-D case.

4. Conclusions

Based on the theory and idea of the Liutex force model, the modification and compilation of the vorticity controlled two-phase flow solver are implemented in OpenFOAM. Through the simulation results of 2-D and 3-D hydrofoil cavitation flow, the following conclusions can be drawn:

In the simulation of 2-D hydrofoil cavitation flow field, the simplified centripetal force model can achieve the goal of relieving the minimum pressure centre and suppressing cavitation. In the 3-D hydrofoil example, the cloud cavitation on the hydrofoil surface is also restrained.

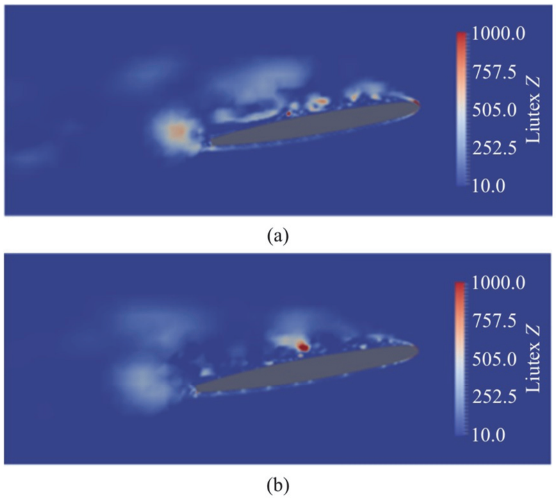
In the 2-D and 3-D examples, the Liutex vorticity field will also change, which shows that the simplified model has more influence on the flow field than the original centripetal force model.



(a)

(b)

Fig. 6 (Color online) The contours of volume of fraction of water



(a)

(b)

Fig. 7 (Color online) The distribution of Liutex component Z

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