

The effect of Wave Steepness on Wave Breaking Properties over Submerged Reef

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Abstract

Wave turning and breaking is a common phenomenon in epicontinental sea which is prominent subject in coastal and marine engineering that deserve investigated in detail. In this paper, a viscous flow solver (naoe-FOAM-SJTU) which is developed and based on the popular open source toolbox OpenFOAM is presented. The solver is adopted to study the effect of wave steepness on wave breaking properties over submerged reef with a 2D simulation. Wave steepness will be considered and the asymmetry properties and cavity properties that induced in the process of wave over-turning will be investigated in detail.

Keywords: submerged reef; wave breaking; wave steepness; asymmetry characteristic

Introduction

Wave breaking as a common phenomenon is an important element in many oceanographic and offshore engineering [1]. It is a two phase flow phenomenon involving air and water, and it strongly influences the air-sea interaction by enhancing mass, momentum and energy transfer between the phases [2]. Due to the nonlinear effect, the wave crests tend to sharpen while the troughs tend to flatten with decreasing of water depth [3]. According to the dispersion relation, the wave velocity and the wave length decrease with the reduction of the water depth in the process of wave propagation shoreward. Hence, the velocity of the front part is smaller than the rear part, which related to the wave energy accumulate upward with the shorter wave length, and the wave height increase which leads to the wave evolution in this process. When the steepness (H/L) is too large and exceed the limitation, the wave breaking occurs.

A relationship reported by Kjeldsen and Myrhaug (1978) [4] firstly that exists between the asymmetric parameters and the breaker type. They introduced steepness and asymmetry parameters to describe the asymmetry of the wave profile: crest front steepness (ϵ), crest rear steepness (δ), the vertical asymmetry factor (λ) and the horizontal asymmetry factor (μ) as depicted in Figure 1. Parameterization of the skewness and asymmetry of waves is an efficient way to describe the wave evolution process and also meaningful to the application and development of physical model. Hence, these parameters are always chosen to descript the transformation of waves [5-8]. Additionally, the geometric properties of breaking waves can be related to the breaker type, which plays a key role in estimation of breaking wave forces on marine structures [2].

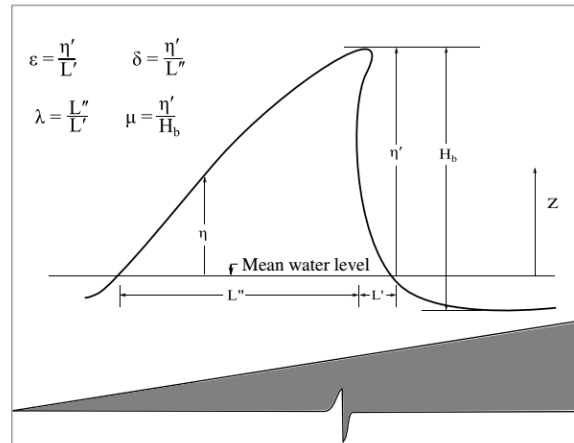


Figure 1. Definition of local wave profile asymmetry (Kjeldsen and Myrhaug, 1978)

Numerous studies have attempted to explain the wave over-turning and breaking process and their characteristics. The detailed literature review on wave breaking in deep and shallow water can be found in Cokelet (1977), Peregrine (1983), Basco (1985), Banner and Peregrine (1993), and Perlin et al. (2013) [9-13]. In the case of submerged terrain and reefs, wave breaking is strongly influenced by the local environmental parameters, such as water depth (d) and sea bed slope (m). This has been studied in laboratory experiments by Blenkinsopp and Chaplin (2008) [14]. In order to research the nonlinear phenomenon, several surface wave theories have been investigated and proposed to resolve the wave breaking issues. Meanwhile, many theories were put forward to describe the wave breaking, and most studies in the field of submerged breakwater structures have only focused on the prediction of the reflection and transmission characteristics of waves for a given environmental condition. Ting and Kim (1994) [15] investigated the wave transformation over a submerged structure and concluded that potential theory cannot be applied to model the flow process such as flow separation and energy dissipation. However, the breaking process and generation and dissipation of vortices are created by rotational flow [16]. Numerical modeling of wave breaking becomes challenging due to the intricacy in describing the physical processes involved such air-sea interaction, vorticity generation, overturning motion and the air entrainment. Hence, a straightforward approach to describing the breaking process numerically is applied to solve the fundamental fluid dynamic equations with CFD (Computational Fluid Dynamics) method. Chella et al. (2015) [2] did the investigation about the characteristics and profile asymmetry properties of wave breaking over an impermeable submerged reef by the CFD method, and the capture of free surface is conducted by the level set method. The numerical result showed great correlation to the experimental results which is just the contribution of consideration of vortex and viscosity.

Despite a considerable number of experimental, numerical and theoretical studies and field observations have been carried out to investigate the process, the wave breaking mechanism is not completely understood. A comprehensive examination of breaking wave properties is inevitable to understand the mechanism of wave breaking and thus the description of the breaking process. Wave breaking over a submerged reef primarily depends on the tidal level and the characteristics of the incident waves. Moreover, an accurate description of waves breaking over submerged structures has always been a central issue in estimation of hydrodynamic loads on marine structures. So that, in this paper, a viscous flow solver (naoe-FOAM-SJTU) which is developed and based on the popular open source toolbox OpenFOAM is presented. The solver is adopted to study the effect of wave steepness on wave breaking properties over submerged reef in 2D simulation. The asymmetry properties and cavity

properties that induced in the process of wave over-turning will be investigated in detail, and the numerical results will be compared with the experimental data and other numerical data so that to validate the accuracy of the solver and simulation. Some regularity about the wave characteristic of the over-turning wave were found in this work.

Numerical Methods

The present solver naoe-FOAM-SJTU [17] adopted for numerical simulation is based on a built-in solver in OpenFOAM named interDyFoam, which can be used to solve two-phase flow which is incompressible, isothermal and immiscible. To deal with common air-sea interaction and wave evolution problems in coastal and offshore engineering, wave generating module was adopted in this work. SST K- ω model is carried out in all the calculations. Mathematical formulae related to the solver are described as follows in detail.

Governing Equations

For transient, incompressible and viscous fluid, flow problems are governed by Navier-Stokes equations:

$$\nabla \cdot \mathbf{U} = 0 \quad (1)$$

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla(\rho(\mathbf{U} - \mathbf{U}_g)\mathbf{U}) = -\nabla p_d - \mathbf{g} \cdot \mathbf{x} \nabla \rho + \nabla(\mu \nabla \mathbf{U}) + \mathbf{f}_\sigma \quad (2)$$

Where \mathbf{U} and \mathbf{U}_g represent velocity of flow field and grid nodes separately; $p_d = p - \rho \mathbf{g} \cdot \mathbf{x}$ is dynamic pressure of flow field by subtracting the hydrostatic part from total pressure p ; \mathbf{g} , ρ and μ denote the gravity acceleration vector, density and dynamic viscosity of fluid respectively; \mathbf{f}_σ is surface tension which only takes effect at the free surface and equals zero elsewhere. The Laminar model means that the Navier-Stokes equation will be solved directly and the turbulence model is not been considered in the calculation.

Wave Generation and Relaxation Zone

Wave generation is a vital part for the investigation of floating offshore structures and wave evolution. The wave generation and wave damping work are implemented by an open-source toolbox for CFD library: waves2foam. The wave was generated by modification of the velocity boundary condition and the phase boundary condition. In this study, Stokes 2nd wave theory was adopted in the generation of the wave according to the calculated wave cases. The equation of Stokes 2nd wave theory was below:

$$\eta = \eta_1 + \eta_2 \quad (3)$$

$$\eta_1 = \frac{H}{2} \cos \omega t \quad (4)$$

$$\eta_2 = \frac{\pi H^2}{8L} \frac{\cosh kd}{\sinh^3 kd} (2 + \cosh 2kd) \cos 2\omega t \quad (5)$$

In which, η is the wave elevation of free surface in certain point, and H is wave height of the generated wave, k is the wave number and d is the water depth at the local position.

In this wave maker module, relaxation zones are implemented to absorb the incident wave that keeps mass conservation and avoids reflection of waves from outlet boundaries at the same time and what else to avoid waves reflected internally in the computational domain to interfere with the floating structure and wave maker boundaries. The former obviously contaminates the results, and the latter is found to create discontinuities in the surface elevation at the wave making boundary, which leads to divergent solutions [18]. A relaxation function:

$$\alpha_R(\chi_R) = 1 - \frac{\exp(\chi_R^{3.5}) - 1}{\exp(1) - 1} \quad \text{for } \chi_R \in [0, 1] \quad (6)$$

is applied inside the relaxation zone in the following way

$$\phi = \alpha_R \phi_{\text{computed}} + (1 - \alpha_R) \phi_{\text{target}} \quad (7)$$

in which ϕ is either velocity or phase indices. The definition of χ_R is such that α_R is always 1 at the interface between the non-relaxed part of the computational domain and the relaxation zone, as illustrated in Figure 2.

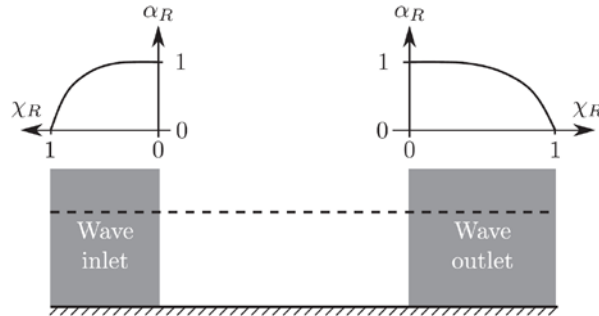


Figure2. A sketch of the variation of χ_R for both inlet and outlet relaxation zones (Jacobsen et al., 2012).

Numerical Examples

The numerical wave tank consists of a submerged reef with a height of 0.618m and a slope of about 1/10 gradient, located 3.8 m from the wave generation zone as shown in Figure 3. The numerical set-up, incident wave parameters and the coordinate system are the same as the experimental conditions that presented in Blenkinsopp and Chaplin (2008) [14], the wave cases that presented in this work is listed in the Table 1. And in the computational domain, several wave gauges were set to measure the wave height at different location.

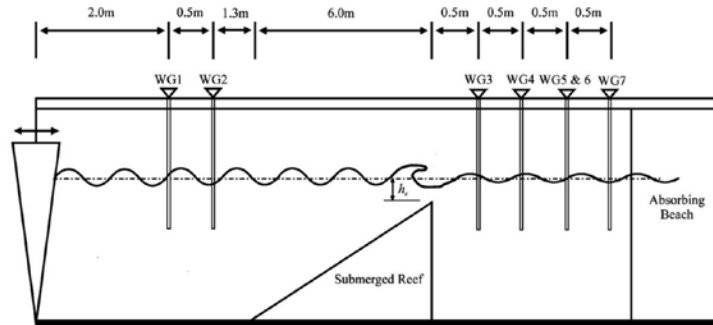
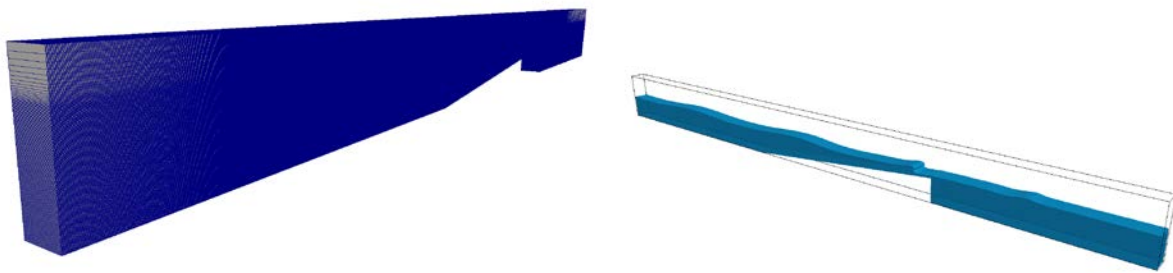


Figure3. Sketch of the computational domain (Blenkinsopp and Chaplin, 2008) [14]

The investigation was set up in a 2D numerical basin. The numerical domain of this work is shown in Figure4 which includes the computational mesh and arrangement of the domain. The total mesh of this work is about 0.82 million, and the mean grid size around the free surface is about 0.5 mm.

Table 1. List of computational cases

Simulation cases	Simulation No.	Wave steepness, H_0/L_0	Reference water depth, d (m)
Based on wave steepness (H_0/L_0)	1	0.02	0.7
	2	0.03	
	3	0.04	
	4	0.05	
	5	0.06	
	6	0.07	


Figure4. Sketch of the mesh and arrangement of the computational domain

The breaking point is assessed in the present study as the point where part of the wave front becomes vertical. Thus, the computed water depth (d_b) and wave height (H_b) at the breaking point are used to calculate the breaker indices. The breaker depth index, γ_b , is the ratio of the breaker height H_b to the water depth at breaking d_b :

$$\gamma_b = \frac{H_b}{d_b} \quad (8)$$

The breaker height index, Ω_b is the ratio of the breaker height H_b to offshore wave height H_0 :

$$\Omega_b = \frac{H_b}{H_0} \quad (9)$$

It is well known that the wave profile becomes asymmetric as it approaches the breaking point and cannot be described by the wave steepness (H/L). Hence, additional parameters are required to describe the asymmetric shape of the wave at breaking. Four additional geometric parameters proposed by Kjeldsen and Myrhaug (1978) are used in this study to describe the asymmetry of the wave profile as shown in Figure 1. And these parameter are described in the following:

$$\varepsilon = \frac{\eta'}{L'}, \delta = \frac{\eta''}{L''}, \lambda = \frac{L''}{L'}, \mu = \frac{\eta'}{H_b} \quad (10)$$

In order to examine the effect of the wave steepness and relative reef submergence it was necessary to define some measurable parameters that could be used to quantify the observed variation in wave breaking intensity. As seen in Figure 5. l_c and w_c are the length and width respectively of the cavity of air enclosed between the overturning jet and the wave face and the x and y axes suitably aligned with the arrows in this figure. From photographs of breaking waves at 23 international surfing breaks, Mead and Black (2001) [19] observed that those on steep reefs broke in a more violent plunging manner, and that there was an inverse linear

relationship between the length to width ratio of the cavity l_c/w_c and the orthogonal reef gradient (for present purposes the same as the actual seabed gradient m), independent of the incoming wave parameters:

$$\frac{l_c}{w_c} = \frac{0.065}{m} + 0.821 \quad (11)$$

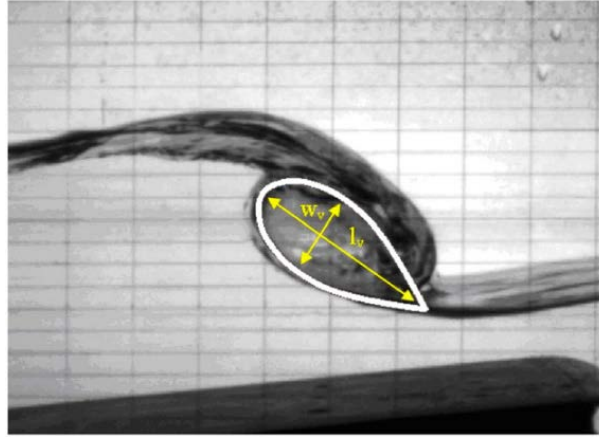


Figure 5. sketch of the l_c and w_c in the cavity (Blenkinsopp and Chaplin, 2008)

Results and Discussions

In the section, wave breaking phenomenon, the asymmetry characteristic and the feature of the cavity in the process of wave over-turning of the wave over submerged reefs will be investigated in detail.

Wave breaking phenomenon over reef

In the practice, wave will be nonlinear in the process that wave propagate from deep sea to the shallow that the asymmetry of the wave will be more clear and when the wave steepness is too large to maintain the waveform of the water particles, the wave breaking will occurs. As shown in Figure 6, which shows the changes in the wave surface profile and the velocity during the breaking process clearly depict that a portion of the wave crest with higher velocity moves forward faster than the rest of the wave. Initially the wave spread to the very shallow position and it is evident that the waveform is very asymmetry which is caused by the accumulation of the water particles from the rear side to the front side and results in the wave height increase at the same time. When the wave crest spread to the breaking position, the wave become to turn, and the velocity of the wave crest become increase and towards the right side which is greater than the wave spread velocity. When the overturned and ejected wave front hits the free surface at the base of the wave, it almost falls over the wave trough of the preceding wave and generates a surface roller. Then water splashes up causing a rise in the water surface with an air cavity inside the water as shown in the Figure 6. An extreme changeover from irrotational flow to rotational flow leads to increased vorticity and turbulence as the wave approaches the beach and eventually violent mixing of air and water occurs. It is worth to notice that the impingement of the rotating plunging vortex is causing a secondary wave with new wave characteristics that propagates shoreward as shown in Figure 6. Moreover, the numerical prediction of the flow pattern and the wave profile changes are very similar to the observation of the flow features of plunging breakers over plane slopes by Basco (1985) [11]. And the return flow phenomenon can be captured clearly in the first figure in the right side of the wave peak which is induced by the shallow water depth and the wave overturning phenomenon.

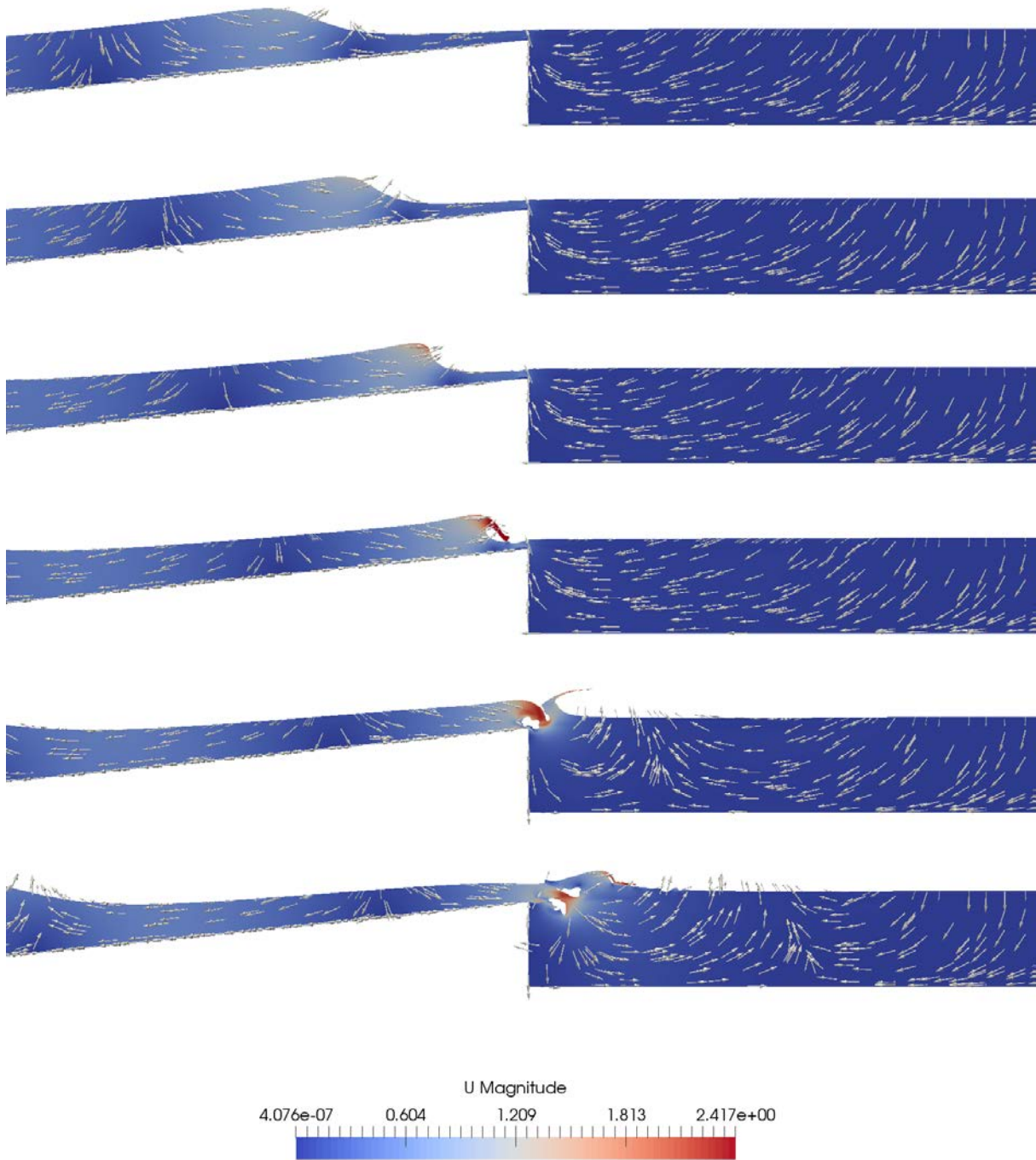


Figure 6. Wave breaking phenomenon over the reef in half wave period colored by velocity.

Study of wave asymmetry parameter

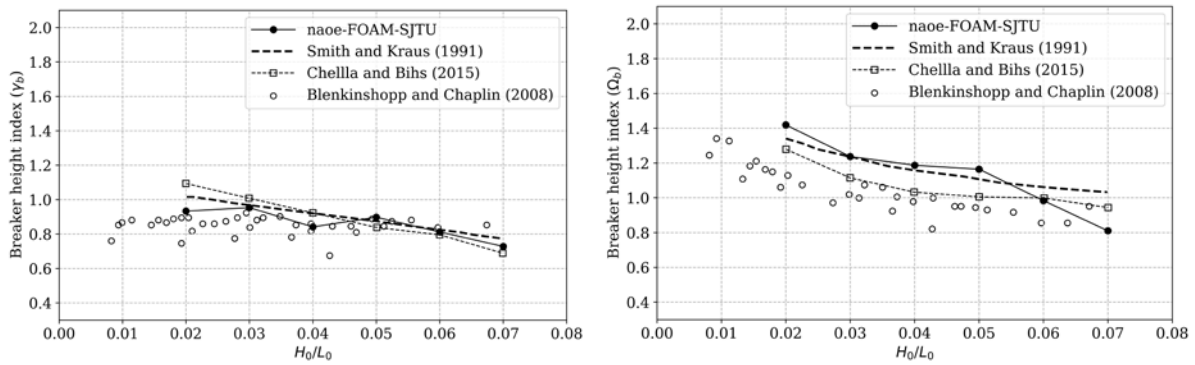


Figure 7. Breaker depth index (left) and breaker height index (right) as a function of offshore wave steepness (H_0/L_0).

Figure 7 presents the comparison of the numerical results and the measured breaker depth index and the breaker height index. It is evident that the γ_b and Ω_b decreases with the increasing of wave steepness. Although the experimental data by Blenkinsopp and Chaplin (2008) [14] do not vary much versus H_0/L_0 . However the computed results are in good agreement with the experimental data. At the same time the present numerical results agree the equation solution that given by Smith and Kraus very well. And this work validates the accuracy of the simulation work in the wave overturning calculation and the solver and computational mesh adopted in this work is reliable. Moreover, it is evident that the breaker height and depth index will decrease with the increasing of the wave steepness.

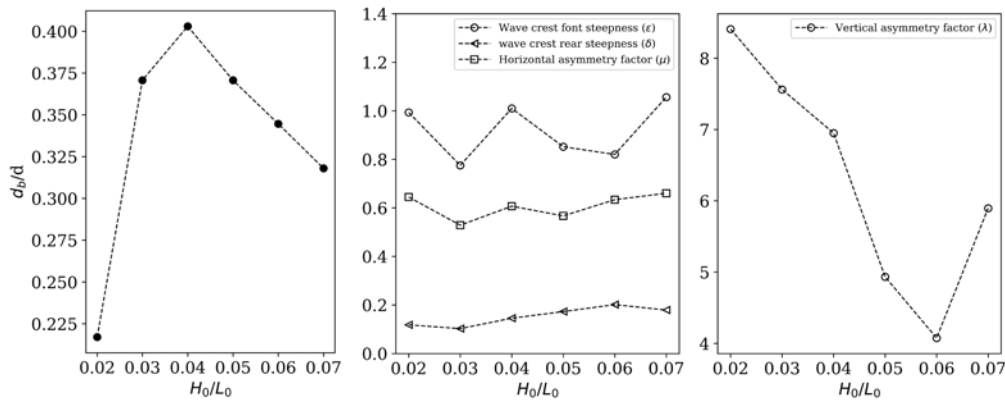


Figure 8. Simulated non-dimensional water depth at breaking (left), wave profile asymmetry parameters as functions of offshore wave steepness (H_0/L_0) (mid and right)

After the validation work, the analysis of the wave asymmetry will be carried out. As shown in the Figure 8, it is evident that, all of these parameters will change with the increasing of the wave steepness. And to analyze the water dimensionless water depth, it is evident that the relative water depth will decrease with the increasing of the wave steepness except the case 1 whose wave steepness is 0.02, and in the author’s opinion, it may be caused by the error in the calculation which is apparently conflicting the other cases and can be removed in the analysis of the trends. In the mid figure, it is evident that all these three parameter do not change very clearly in the change trends, and to do a deep analysis it can be found that the variation tendency of wave crest front steepness, wave crest rear steepness and the horizontal asymmetry factor increase with the increasing of the wave steepness. In the right figure, it is

evident that the wave vertical asymmetry factor will decrease with the increasing of the wave steepness.

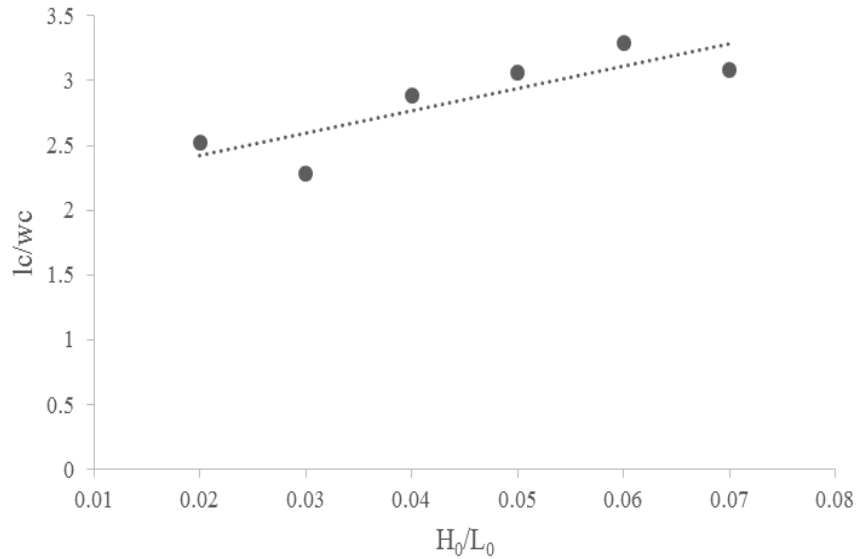


Figure 9. Simulated cavity length to width as a function of wave steepness

In the final section, the characteristic of the cavity in the simulation of the wave overturning process will be analyzed. The parameter of the cavity is related to the wave overturning intensity which is also a significant parameter in the research and quantification of the wave breaking. And as shown in Figure 9, the black point is the calculated results and the dash line is the trend line of the results. From the trend it is obvious that the results of l_c/w_c is greater with the increasing of the wave steepness and so that it means with the increase of the wave steepness, the cavity in the wave overturning process is more and more narrow and long which means that the wave breaking intensity is greater that the velocity of the wave crest if larger. Finally, the vertical asymmetry decrease with the increase of steepness. For a further analysis of the wave breaking intensity, the characteristic of cavity that induced by the wave overturning was investigated and it is found that the cavity will be more narrow and long with the increasing of the wave steepness which shows that the wave breaking intensity is greater.

Conclusions

In this paper, the wave breaking and overturning phenomenon that affected by the wave steepness over a submerged reef was mainly investigated, and a 2D numerical wave basin was set to do the simulation. All these work was carried out by our in-house two-phase flow CFD solver naoe-FOAM-SJTU. Stokes 2nd wave theory was primarily used in the wave generation work. In these simulations, wave overturning phenomenon can be clearly analyzed and some phenomenon in detail such as cavity, return flow, breaking induced vortex can be captured. Moreover, some regularity can be drawn from the analysis of wave asymmetry characteristic. First of all, the numerical results of the breaker depth and height index show great correlation to the experimental data meanwhile, it is apparent that both these two index decrease with the increasing of the wave steepness. Subsequently, the relative breaking depth decrease with the increasing of the steepness, and it can be found that the variation tendency of wave crest front steepness, wave crest rear steepness and the horizontal asymmetry factor increase with the increasing of the wave steepness.

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