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NUMERICAL SIMULATION OF THREE-LAYER-LIQUID SLOSHING BY MULTIPHASE MPS METHOD

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ABSTRACT

In the present study, three-layer-liquid sloshing in a rigid tank is simulated based on the newly developed multiphase MPS method. Firstly, the multiphase MPS method is introduced in detail, including the basic particle interaction models and the special interface treatments employed to extend single phase MPS solver to multiphase flows simulations. The new multiphase MPS method treats the multifluid system as the multi-density and multi-viscosity fluid, thus only a single set of equations needs to be solved for all phases. Besides, extra density smoothing technique, interparticle viscosity model and surface tension model are included in the present method for interface particles. The new multiphase MPS method is then applied to simulate three-layer-liquid sloshing in a rigid tank and verified through comparison with the experiment conducted by Molin et al. [1]. The predicted motion of interfaces by the present method shows a good agreement with the experimental data and other numerical results.

INTRODUCTION

With the increasing demand of oil and gas, offshore oil industry continuously extends to deeper water and many new ideas have been proposed to improve the production/processing efficiency. For example, the wash tanks, alike separators, are manufactured and installed on FPSOs or production platforms, having purpose of initially separating the crude oil from the mixed fluid produced from the oil well. Thus the higher economic effectiveness can be achieved with less transportation costs. However, sloshing phenomena take place insiede the wash tanks under motion of the floating support induced by the complex hydrodynamic environment such as wave force, wind force, currents force, buoyant force anchoring force, gravity force and etc. When the sloshing flows in the wash tanks become violent enough, the motion of the oil-water interfaces is irregular and the separation efficiency of the wash tanks is reduced. Therefore, accurate prediction of the motion of interfaces is necessary to check that no undesirable resonances take place at the design stage.

Most of the studies reported devote to the sloshing flows containing only a single layer fluid. For example, Faltinsen [2] studied the small amplitude liquid sloshing in a rectangular tank under horizontal harmonic excitation and obtained an analytical solution. Chen et al. [3] employed the level-set method to numerically analyze the accuracy of the prediction of sloshing induced impact pressure on the walls and ceiling of tank. Zhang and Wan [4] presented a comparative study of the mesh-less MPS method and level-set method in the simulation of sloshing flows. Yang et al. [5] simulated the liquid sloshing in a rectangular tank under horizontal excitation by the modified 3D MPS method, and the dimension effect has been analyzed through comparison of the results obtained by 2D and 3D MPS method. Zhang and Wan [6] simulated the fluid-structure interaction between the violent sloshing flow and an elastic tank based on the MPS-FEM coupled method.

However, the studies concerning layered sloshing in tanks have rarely been conducted. La Rocca et al. [7] performed a theoretical and experimental investigation on the sloshing of a two-liquid system with both two-phase interface and free surface. Molin et al. [1] reported sloshing experiments with a rectangular tank filled with three fluids of different densities, then an analytical model is proposed based on linearized potential flow theory and compared with a fully nonlinear CFD

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code with VOF tracking of the interfaces. Grotle et al. [8] investigate multiphase sloshing at shallow-liquid depths in a rectangular container by using experimental and level set methods. Choudhary and Bora [9] employed a semi-analytical approach to investigate sloshing flows in an immiscible, incompressible and inviscid two-layer fluid and demonstrated the effects of the parameters such as fluid heights and density ratio on the natural sloshing frequencies. Kim et al. [10] investigated the multiliquid-sloshing simulation by developing a new multifluid-layer MPS method, in which buoyancy correction and surface tension models are included.

The numerical simulation of multiphase flows is challenging since the tracing of the largely deformed multiphase interface. In present paper, a multiphase method is developed based on the moving particle semi-implicit method (MPS). Compared with grid-based methods, the mesh-less MPS method original proposed by Koshizuka [11] is advantageous for the implicit tracing of free surface and interfaces between different phases. In present method, the multiphase system is treated as the multi-density and multi-viscosity fluid. To consider the interaction between particles belonging to different phases, interparticle viscosity defined by the harmonic mean viscosity is firstly adopted. Then the density smoothing technique is employed to reduce pressure discontinuity crossing the interface and obtain the continuous acceleration and velocity fields. To consider the influence of surface tension force on the interface, a contoured continuum surface force (CCSF) model proposed by Duan et al. [12] is utilized in the present method.

The multiphase MPS method is then applied to simulate three-layer-liquid sloshing problems. The chosen immiscible fluids from top to bottom in the tank are respectively cyclohexane, water and dichloromethane and the tank is imposed both harmonic roll and sway motion. The simulation results are compared with the three-layer-liquid sloshing experiment conducted by Molin et al. [1]. The good agreement of the numerical simulation with experimental results proves the applicability and accuracy of the multiphase MPS method.

MULTIPHASE MPS METHOD

Governing Equation

In the present multiphase MPS method, the multifluid system is treated as a single fluid with multi-density and multi-viscosity. Therefore, the governing equation for all fluids is uniform, expressed by continuity and momentum equations. They can be written as:

$$\frac{D\rho}{Dt} = -\rho(\nabla \cdot \mathbf{u}) = 0 \tag{1}$$

$$\rho\frac{D\mathbf{u}}{Dt} = -\nabla P + \nabla(\mu\nabla \cdot \mathbf{u}) + \mathbf{g} + \mathbf{F}_s \tag{2}$$

where ρ and μ are respectively the density and dynamic viscosity, $\mathbf{u}$ is the flow velocity, P is the pressure, $\mathbf{g}$ is a constant gravitational acceleration, $\mathbf{F}_s$ is the interface tension force between different phases. The difference in governing equation for different fluids is mainly reflected in the value of ρ and μ. Compared with grid-based method, the Lagrangian description of governing equation effectively avoids the numerical dispersion induced by convective acceleration term.

Particle interaction models

In MPS method, the governing equation is discretized through a set of disordered particles in space. And particle interaction models based on kernel function are used to replace all terms of differential operators in the right hand of governing equation. In this paper, kernel function is as follow:

$$W(r) = \begin{cases} \dfrac{r_e}{0.85r + 0.15r_e} & (0 \le r < r_e) \\ 0 & (r_e \le r) \end{cases} \tag{3}$$

where r is the distance between two particles and r_e is the radius of the particle interaction.

In simulation of multiphase flows, the mathematical discontinuity of density at two-phase interface causes a discontinuous acceleration field and accordingly numerical instabilities. To deal with this discontinuity, a density smoothing technique is carried out for particles near interface, based on a simple spatial averaging as follow:

$$\rho_i = \frac{\sum_{j\in i}\rho_j W(r_{ij}, r_e)}{\sum_{j\in i} W(r_{ij}, r_e)} \tag{4}$$

The gradient model represents a local weighted average of the gradient vectors between particle i and its neighboring particle j. In the case of pressure gradient, the modified gradient model proposed by Koshizuka et al. [13] is adopted, written as:

$$\langle \nabla P \rangle_i = \frac{d}{n^0}\sum_{j\ne i}\frac{(P_j - P_{i,\min})}{r_{ij}}\mathbf{e}_{ij} W(r_{ij}, r_e) \tag{5}$$

where $\langle\ \rangle$ denotes kernel approximation operator, $P_{i,\min}$ is the minimal pressure among neighboring particles of particle i, d is the number of space dimension, $\mathbf{e}_{ij} = \boldsymbol{r}_{ij}/r_{ij}$ is the unit vector with a direction from particle i to particle j. n^0 is the particle number density at initial arrangement, the particle number density is defined as:

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$$n_i = \sum_{j \neq i} W\left(r_{ij}, r_e\right) \tag{6}$$

The divergence model of velocity is given by:

$$\left\langle \nabla \cdot \mathbf{u} \right\rangle_i = \frac{d}{n^0} \sum_{j \neq i} \frac{\left(\mathbf{u}_j - \mathbf{u}_i\right)}{r_{ij}} \cdot \mathbf{e}_{ij} W\left(r_{ij}, r_e\right) \tag{7}$$

In present multiphase MPS method, viscosity discontinuity cross interface is dealt with by an interparticle viscosity between particles belonging to different phases. According to the numerical test of Shakibaeinia and Jin [14], the multi-viscosity model with harmonic mean interparticle viscosity is adopted:

$$\nabla(\mu \nabla \cdot \mathbf{u}) = \frac{2d}{n^0 \lambda} \sum_{j \neq i} \frac{2\mu_i \mu_j}{\mu_i + \mu_j} (\mathbf{u}_j - \mathbf{u}_i) W(r_{ij}, r_e) \tag{8}$$

where λ is applied to make the increase of variance equal to that of the analytical solution:

$$\lambda = \frac{\sum_{j \neq i} W\left(r_{ij}, r_e\right) r_{ij}^{\ 2}}{\sum_{j \neq i} W\left(r_{ij}, r_e\right)} \tag{9}$$

Interface tension model

For multiphase flow, interface tension is important force acting on the interface. To impose the force to the interface particles, a contoured continuum surface force (CCFS) model developed by Duan et al. [12] has been adopted, where interface tension force is calculated as:

$$\mathbf{F}_s = \sigma \cdot \kappa \cdot \nabla C \tag{10}$$

where σ is the surface tension coefficient which is mainly decided by fluid property, C is a color function defined as:

$$C_i = \begin{cases} 0 & \text{particle } i \text{ belongs to the specified phase} \\ 1 & \text{particle } i \text{ belongs to the other phase} \end{cases} \tag{11}$$

Therefore, the most important parameter of interface tension model is the interface curvature κ. In present CCFS model, the curvature is analytically calculated on the basis of the Taylor series expansion of the local contour of the color function. The calculation process is specially presented here.

Firstly, the smoothed color function f at an arbitrary location near the particle i is obtained through a smoothing process based on the Gaussian kernel function:

$$f\left(x, y\right) = \frac{\sum_j C_j G(r_{ij}, r_s)}{\sum_j G(r_{ij}, r_s)}, \quad G(r_{ij}, r_s) = \frac{9}{\pi r_s^2} \exp\left(-\frac{9 r_{ij}^{\ 2}}{r_s^{\ 2}}\right) \tag{12}$$

where r_s represents the smooth radius. According to the definition of the color function, we can know that the two-phase interface can be approximately represented by the contour of the smoothed color function. So the key issue is how to get the local contour.

Then, f is expanded at point i based on the Taylor series expansion:

$$\begin{aligned} f\left(x, y\right) = f\left(x_i, y_i\right) + f_{x,i}(x - x_i) + f_{y,i}(y - y_i) + \frac{1}{2} f_{xx,i}(x - x_i)^2 \\ + f_{xy,i}(x - x_i)(y - y_i) + \frac{1}{2} f_{yy,i}(y - y_i)^2 + O(r_s^3) \end{aligned} \tag{13}$$

where the subscripts are the first- and the second-order partial derivatives of f with respect to x or y at particle i. $O\left(r_s^3\right)$ is the high order error term which is proven to have no influence on the accuracy of curvature calculation and can be omitted [12].

As we kown, the local contour function of f passing through particle i should satisfy

$$f\left(x, y\right) = f\left(x_i, y_i\right) \tag{14}$$

Thus the local contour at particle i can be obtained by combination of Eq. 13 and Eq. 14, expressed as:

$$\begin{aligned} f_{x,i}(x - x_i) + f_{y,i}(y - y_i) + \frac{1}{2} f_{xx,i}(x - x_i)^2 \\ + f_{xy,i}(x - x_i)(y - y_i) + \frac{1}{2} f_{yy,i}(y - y_i)^2 = 0 \end{aligned} \tag{15}$$

If we regard y as a function of x in Eq. 15, the curvature of particle i can be analytically calculated. The derivation of the above formulation is presented by Duan et al. [12] as follow:

$$\kappa_i = \frac{y''}{\left(1 + y_i'\right)^{3/2}} = \frac{2 f_{x,i} f_{y,i} f_{xy,i} - f_{x,i}^2 f_{yy,i} - f_{y,i}^2 f_{xx,i}}{\left(f_{x,i}^2 + f_{y,i}^2\right)^{3/2}} \tag{16}$$

Model of incompressibility

The compressibility condition in MPS method is presented by a constant particle number density. Each time step consists of two stages, explicit stage and implicit stage. In the explicit stage, temporal velocity of particles is calculated explicitly based on gravity, viscosity and interface tension terms. In the second implicit stage, the temporal velocity is projected into a divergence-free velocity field according to pressure. The

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pressure field is implicitly calculated by solving a Pressure Poisson Equation (PPE). The Pressure Poisson Equation used in this paper is the improved one with mixed source term, written as:

$$\left\langle \nabla^2 P^{k+1} \right\rangle_i = (1-\gamma)\frac{\rho}{\Delta t}\nabla \cdot \mathbf{u}_i^* - \gamma \frac{\rho}{\Delta t^2}\frac{\left\langle n^k \right\rangle_i - n^0}{n^0} \qquad (17)$$

where γ is a relaxation coefficient. According to the numerical tests by Lee et al. [15], $\gamma = 0.01$ is used in this paper to reduce error of the numerical pressure.

Boundary condition

In MPS method, the kinematic and dynamic free surface boundary condition is imposed on the free surface particles. The kinematic boundary condition is automatically satisfied in Lagrangian method, while the dynamic free surface boundary condition is implemented by setting zero pressure on the free surface particles.

To impose the dynamic free surface boundary condition, free surface particles should be detected first. To improve the accuracy, the new detection proposed by Zhang and Wan [16] is employed in this paper. In this method, a function based on the asymmetric arrangement of neighboring particles of the center particle i is defined as:

$$\langle \mathbf{F}_i \rangle = \frac{d}{n}\sum_{j \neq i}\frac{\mathbf{r}_{ij}}{r_{ij}} W\left(r_{ij}, r_e\right) \qquad (18)$$

Particles satisfying

$$\left|\mathbf{F}_i\right| > 0.9\left|\mathbf{F}\right|^0 \qquad (19)$$

are judged as free surface particles, where $\left|\mathbf{F}\right|^0$ is the value of $\left|\mathbf{F}\right|$ for free surface particles at initial arrangement.

As for wall boundary condition, no-slip condition is imposed by introducing moving ghost particles. At each time step, locations and velocities of the ghost particles are rearranged according to corresponding fluid particles near the wall. The locations of ghost particles are symmetrical to corresponding fluid particles about the wall, and their velocities are decided as follow:

$$\mathbf{u}_{dum} \cdot \mathbf{n} = (2\mathbf{u}_w - \mathbf{u}_i) \cdot \mathbf{n}, \quad \mathbf{u}_{dum} \cdot \mathbf{t} = (2\mathbf{u}_w - \mathbf{u}_i) \cdot \mathbf{t}, \qquad (20)$$

where $\mathbf{n}$ and $\mathbf{t}$ are respectively the normal and tangential vectors to the wall. The subscripts dum, i and w represent the ghost, corresponding fluid and wall particles.

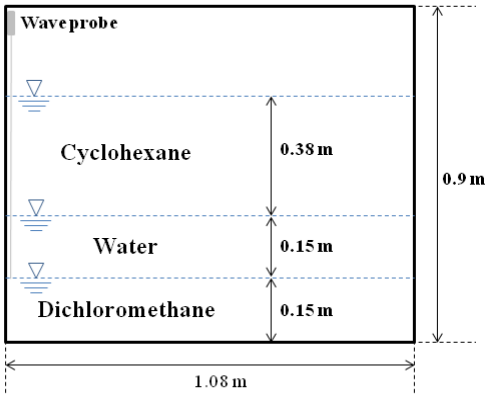

Figure 1 Diagrammatic sketch of three-layer-liquid sloshing in a rectangular tank.

TABLE 1 Density, dynamic viscosity, and surface tension coefficient for the studied three fluids.

	Density	Dynamic viscosity	Surface tension coefficient
Cyclohexane	780kg/m^3	1.014 ×10^{-3} m^2/s	0.00247 N/m
Water	1000kg/m^3	1.0 ×10^{-3} m^2/s	0.0727 N/m
Dichloromethane	1300kg/m^3	3.9 ×10^{-4} m^2/s	0.0278 N/m

NUMERICAL SIMULATIONS

In this study, numerical simulation of three-layer-liquid sloshing is carried out by the above multiphase MPS method. All the simulation conditions are kept identical with the three-layer-liquid sloshing experiment of Molin et al. [1]. In their experiment, the rigid liquid tank with a length of 1.08 m and a width of 0.9 m is secured on a shake table which can excite the tank to move with different freedoms, including roll and sway. The liquid field in the tank is divided into three layers. From top to bottom in the tank, three immiscible fluids, cyclohexane, water and dichloromethane are chosen respectively. The properties of fluids can be found in Table 1, and the height of each layer is shown in Fig. 1.

In numerical simulations, three-layer-liquid sloshing phenomena induced by roll and sway motion of tank are considered respectively. To record the elevations of the free surface and two interfaces (Cyclohexane-Water and Water-Dichloromethane interface) during sloshing, a wave probe is arranged on the left side of the tank, as shown in Fig. 1. The initial particle distance is set to be 0.005 m, and the size of time step is 0.0005s in all cases.

Sloshing induced by roll motion

In this section, the numerical simulation of three-layer-liquid sloshing induced by the harmonic roll motion of the tank is conducted by the above multiphase MPS method. In this case, 1 degree amplitude and 1.83 rad/s frequency is chosen, which is close to the first nature frequency of the middle water layer.

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Figure 2 Snapshots of harmonic roll motion (maximal degree) at 1.83 rad/s. Present method (top), experiment (middle) and simulation (bottom) of Molin et al. (2012). Upper layer-cyclohexane, middle layer-water, lower layer-dichloromethane.

Figure 3 Snapshots of harmonic roll motion (zero degree) at 1.83 rad/s. Present method (top), experiment (middle) and simulation (bottom) of Molin et al. (2012). Upper layer-cyclohexane, middle layer-water, lower layer-dichloromethane.

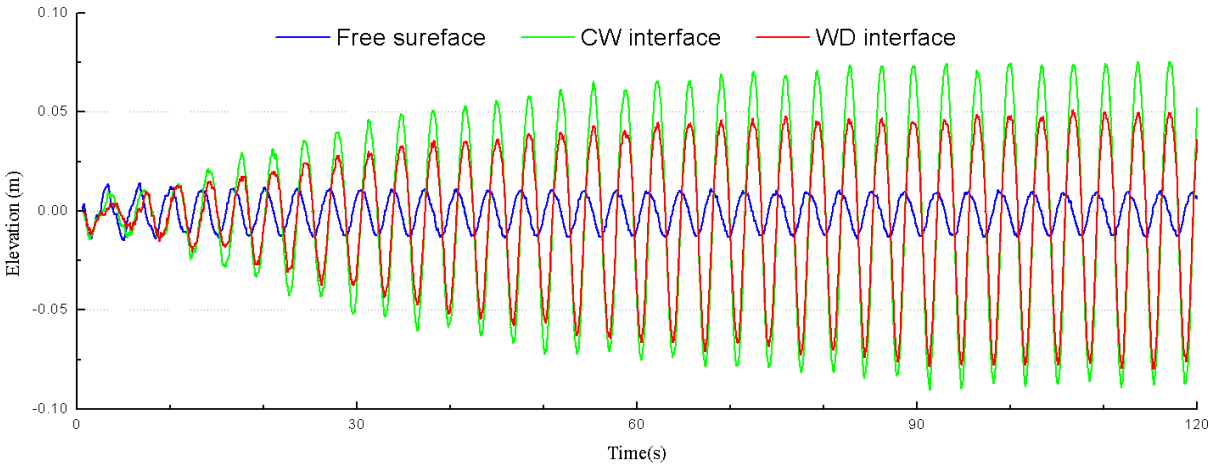

Figure 4 Time history of interfaces elevations at left wall, harmonic roll motion with 1 degree amplitude and 1.83 rad/s frequency.

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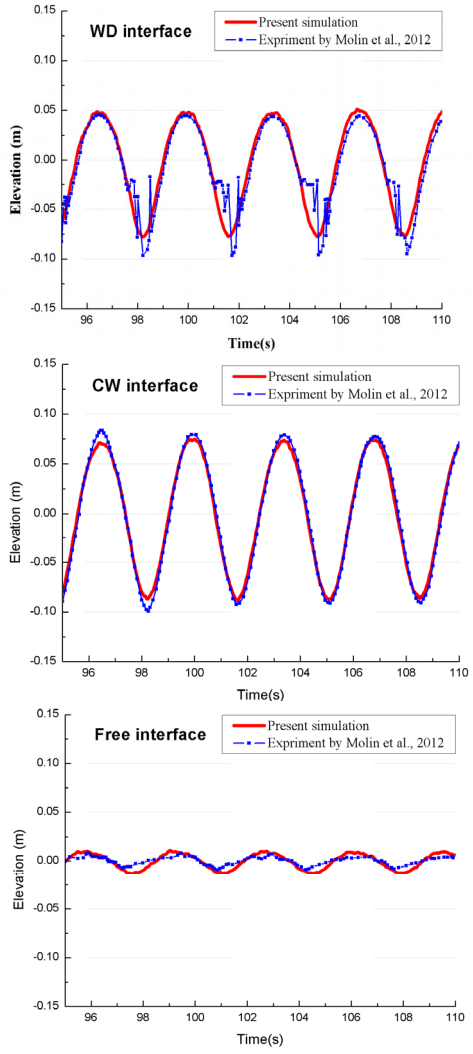

Figure 5 Comparison of interfaces elevations between present method (red) and experiment (blue) of Molin et al. (2012) at left wall, harmonic roll motion with 1 degree amplitude and 1.83 rad/s frequency.

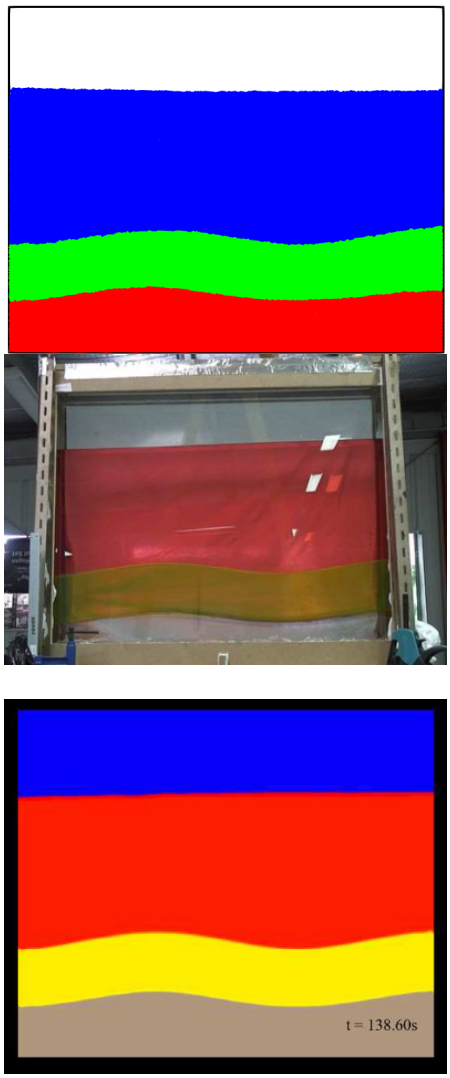

Figure 6 Snapshots of harmonic sway motion at 3.62 rad/s. Present method (top), experiment (middle) and simulation (bottom) of Molin et al. (2012). Upper layer-cyclohexane, middle layer-water, lower layer-dichloromethane.

Fig. 2 and Fig. 3 show the snapshots of liquid layers' motion at two moments respectively corresponding to the largest deformation and smallest of excited interface. The numerical results by the present multiphase MPS method is on the top, and the experiments and simulations by Molin et al. [1] are on the middle and bottom respectively. Good agreement can be observed and the accuracy of present method is validated. In Fig. 2, the lowest modal shape of the middle layer is captured by the multiphase MPS method, which influences the deformations of CW (Cyclohexane-Water) and WD (Water-Dichloromethane) interfaces. In Fig. 3, the slightly excited second mode of the WD interface is observed, because the frequency of 1.83 rad/s in this case is close to the second natural frequency of the dichloromethane layer, which is equal to 1.986 rad/s.

In Fig. 4, the elevations of different interfaces at left wall of the tank are quantitatively compared. The elevation of free surface reaches a steady state soon after the beginning of the roll motion of tank, while the elevations of CW and WD interfaces keep growing until a steady state is reaches at about 90s. Due to the generation of the first modal sloshing of the middle layer, the elevations of CW and WD interface are much larger than that of the free surface.

To quantitatively validate the present multiphase MPS method, the calculated elevations of different interfaces are compared with experimental results of Molin et al. [1] between 95s and 110s, as shown in Fig. 5. It should be noted that the

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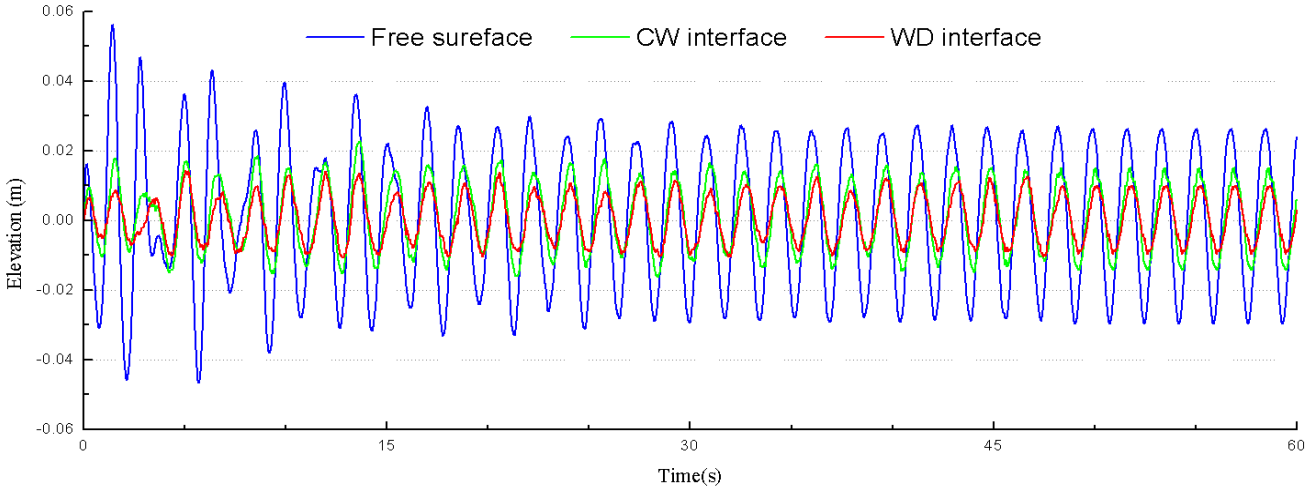

Figure 7 Time history of interfaces elevations at left wall, harmonic roll motion with 1 cm amplitude and 3.62 rad/s frequency.

violent fluctuations in elevation of WD interface in the experiment is induce by the less-effective tracking sensor. For CW and WD interfaces, excellent agreement is achieved. And only a little difference exists in the phase of free surface, while the amplitude agrees well.

Sloshing induced by sway motion

In this section, the numerical simulation of three-layer-liquid sloshing induced by the harmonic sway motion of the tank is conducted by the above multiphase MPS method. In this case, 1 cm amplitude and 3.62 rad/s frequency is chosen, which is close to the third nature frequency of the middle water layer.

Fig. 6 shows the snapshots obtained by different methods, in which the obvious third mode of the CW interface can be observed. Results obtained by present method and experiment of Molin et al. [1] show good agreement. Fig. 7 shows the interface oscillations at left wall. In this case, the elevation of free surface at left wall is largest, while the CW and WD interfaces elevations are very limited due to the third mode of the middle layer.

CONCLUSIONS

In this study, the three-layer-liquid sloshing is investigated by a newly developed multiphase MPS method. The new multiphase MPS method treats the multifluid system as the multi-density and multi-viscosity fluid, thus only a single set of equations needs to be solved for all phases. Besides, extra density smoothing technique, interparticle viscosity model and surface tension model are included in the present method for interface particles. The new method is verified against the three-layer-liquid sloshing experiment by Molin et al. [1] and good agreement is achieved. In case of an excitation frequency of the tank close to the first natural frequency of the middle layer, the lowest modal shape of the middle layer is captured by the multiphase MPS method, which influences the deformations of CW (Cyclohexane-Water) and WD (Water-Dichloromethane) interfaces. In case of an excitation frequency of the tank close to the third natural frequency of the middle layer, obvious third mode of the CW interface is observed.

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