NUMERICAL INVESTIGATIONS ON VORTEX-INDUCED VIBRATION OF A FLEXIBLE CYLINDER EXPERIENCING COMBINED FLOW

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Keywords: *vortex-induced vibration; viv-FOAM-SJTU; combined flow; flow ratio*

1.INTRODUCTION

Vortex-Induced Vibration (VIV) is the main source of risers' fatigue damage. In actual production, offshore floating structures subject to waves, currents or winds may cause the platform to move periodically in the water. Then relatively oscillatory flow is generated between the riser and the water. In recent decades, researches of the sinusoidal motion of a cylinder in viscous fluid have been extensively studied. Fu[3] has taken experimental studies on VIV of a flexible cylinder in oscillatory flow for different KC numbers and raise the VIV development process of build-up, lock-in and die-out. Zhao[2] has carried out numerical simulations of a circular cylinder in combined oscillatory and steady flow. In this paper, VIV of a flexible cylinder experiencing combined uniform and oscillatory flow is investigated numerically based on experiments of Fu[3]. The flow ratio α of the uniform flow in the total combined flow increase from 0 to 1 through an interval of 0.5.

2.METHOD

2.1 Hydrodynamics governing equations

The flow field is supposed to be incompressible, with constant dynamic viscosity μ and constant density ρ. The Reynolds-averaged Navier-Stokes equations are used as the hydrodynamics governing equations as follow:

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial}{\partial t}\left(\rho \overline{u}_i\right) + \frac{\partial}{\partial x_j}\left(\rho \overline{u}_i \overline{u}_j\right) = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(2\mu \overline{S}_{ij} - \rho \overline{u_j' u_i'}\right) \tag{2}$$

where $\overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)$ is the mean rate of strain tensor, $-\rho \overline{u_j' u_i'}$ is referred as Reynolds stress τ_{ij} computed by $\tau_{ij} = -\rho \overline{u_j' u_i'} = 2\mu_t \overline{S_{ij}} - \frac{2}{3}\rho k \delta_{ij}$, where μ_t is the turbulent viscosity and $k = (1/2)\overline{u_i' u_i'}$ is the turbulent energy, computing from the fluctuating velocity field.

2.2 Structural dynamics governing equations

In order to form the relatively oscillatory flow, the supporting frame was forced to oscillate harmonically. The oscillation can be expressed as:

$$x_s = A_m \cdot \sin(\frac{2\pi}{T} t) \tag{3}$$

$$U_s = \frac{2\pi \cdot A_m}{T} \cdot \cos(\frac{2\pi}{T} t) = U_m \cdot \cos(\frac{2\pi}{T} t) \tag{4}$$

$$KC = \frac{2\pi \cdot A_m}{D} = \frac{U_m \cdot T}{D} \tag{5}$$

where A denotes the oscillating amplitude， T is the oscillating period, x_s is the oscillating displacement， U_s is the oscillating velocity ， U_m is the amplitude of the oscillating velocity ， D is the diameter of the cylinder。

Fu[1] uses the support excitation method combined with the Bernoulli–Euler bending beam theory to obtain the structural response of the cylinder. The in-line displacement of the cylinder is the sum of support frame motion and the relative in-line vibration of the cylinder:

$$x_t = x_s + x \tag{6}$$

where x_t is the in-line displacement, x_s is the support motion and x is the relative in-line displacement.
The equilibrium of forces for this system can be written as follow:

$$f_I + f_D + f_S = f_H \tag{7}$$

where f_I, f_D, f_S, f_H are the inertial, the damping, the spring, and the hydrodynamic force, respectively.
Then the equilibrium of forces for the system can be written as:

$$m\ddot{x}_t + c\dot{x} + kx = f_H \tag{8}$$

$$m\ddot{x} + c\dot{x} + kx = f_H - m\ddot{x}_s \tag{9}$$

where m, c, k are the mass, the damping and the stiffness of the system.
In the finite element method(FEM), the equations can be discretized as:

$$\mathbf{M\ddot{x} + C\dot{x} + Kx = F_{Hx} - M\ddot{x}_s} \tag{10}$$

$$\mathbf{M\ddot{y} + C\dot{y} + Ky = F_{Hy}} \tag{11}$$

Where **M**, **C**, **K** are the mass, the damping and the stiffness matrices, while **x**, $\mathbf{x_s}$ and **y** are the relative in-line, the support and the cross-flow nodal displacement vectors. $\mathbf{F_{Hx}}$ and $\mathbf{F_{Hy}}$ are the hydrodynamic force in the in-line and cross-flow direction respectively.

2.3 Strip method

In this paper, numerical investigations are carried out by viv-FOAM-SJTU solver based on the strip method and the pimpDyMFOAM solver attached to the open source code OpenFOAM. The strip method is very appropriate for solving CFD investigations of supramaximal computational domain. It owns high computational efficiency and the computational accuracy is reliable, The reliability of the viv-FOAM-SJTU solver has been testified by Duan[4], in which the benchmark case has been verified in detail.
For a long flexible cylinder, a direct computation of the three dimensional flow field will cost too much resources. Instead of this, we simplify CFD model and obtain the two dimensional flow field on strips distributed equably along the cylinder. The hydrodynamic force is obtained from each strip, which is then applied to the structural field. The structural displacements of all nodes are interpolated to get the boundary motion of dynamic mesh of flow field. The strip theory is shown as figure 1.
During the numerical simulations, the RANS equations and SST *k-ω* turbulence model are adopted to solve the flow field in each strip, while the whole structure filed is solved through Bernoulli–Euler bending beam theory with the finite element method. The fluid-structure interaction is carried out by loose coupling strategy.

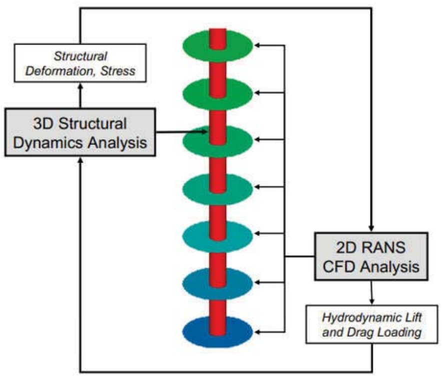


Figure 1: Schematic diagram of strip theory

3. PROBLEM DESCRIPTION

3.1 CFD model

The numerical model follows experiments of Fu[1] and the layout of the experiments is shown in figure 2. Detailed information about parameters of the cylinder is shown in Table 1. 20 strips located equidistantly along the cylinder. Figure 3 shows the distribution of strips. Figure 4 shows the entire computational domain and meshes of strips.

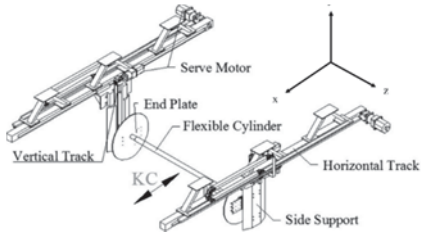


Figure 2: Layout of the experiments of Fu et al

Table 1: Main parameters of the cylinder

	Symbols	Values	Units
Mass ratio	m^*	1.53	−
Diameter	D	0.024	m
Length	L	4	m
Bending stiffness	EI	10.5	$\mathrm{N \cdot m^2}$
Top tension	T_t	500	N
First natural frequency	f_n^1	2.68	Hz
Second natural frequency	f_n^2	5.46	Hz

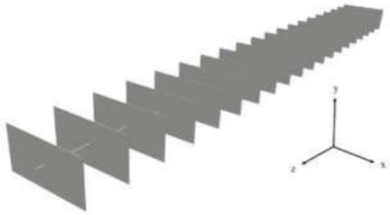


Figure 3: Illustration of multi-strip model

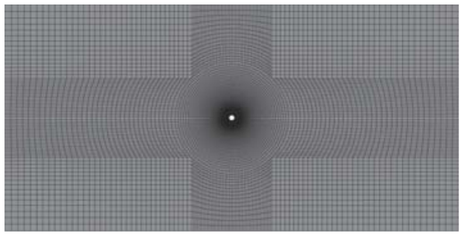


Figure 4: Computational domain and mesh of a strip

3.2 Computational conditions

According to equations (3) and (4), the total velocity and the flow ratio can be written as equations (12) and (13).

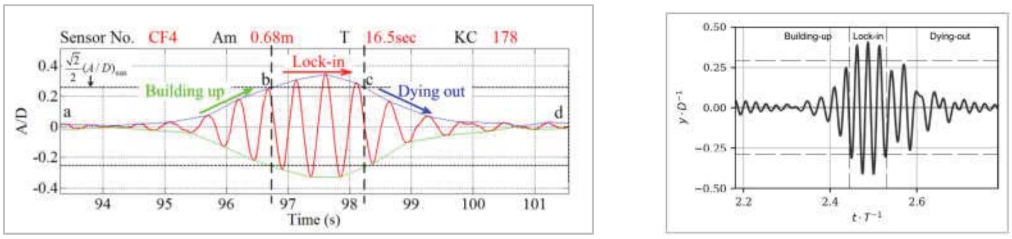
$$U_c\left(t\right) = U_s + U_m \cos(\frac{2\pi}{T}t) = U_s + A_m \frac{2\pi}{T}\cos(\frac{2\pi}{T}t) \tag{12}$$

$$\alpha = \frac{U_s}{U_s + U_m} = 1 - \frac{A_m}{U_c}\cdot\frac{2\pi}{T} \tag{13}$$

where U_s is the uniform flow velocity, U_c is the total velocity, A_m is the amplitude of the oscillation, T is the oscillating period.

4 RESULTS

Figure 5 shows subplots of non-dimensional cross-flow amplitude of the intermediate node of the cylinder between experiment and simulation. From comparison, it can be concluded: (i)the development process of "Building-up—Lock-in—Dying-out" of vortex-induced vibration is observed in both experiment and numerical simulation; (ii)the lock-in region is 17.3% of the half oscillating period in numerical simulation, which is close to the experiment result of 17%; (iii)the non-dimensional cross-flow amplitude is 0.37D in half oscillating period, which is close to the experiment result of 0.36D.



(a)Result of Fu et al. (b)The present simulation

Figure 5: Non-dimensional cross-flow amplitude of the intermediate node in half an oscillating period

Figure 6 shows subplots of non-dimensional cross-flow amplitude of the intermediate node of the cylinder in three flow ratios. It can be known that when flow ratio equals 0, the cylinder is in pure oscillatory flow. And two obvious VIV development processes can be observed in an oscillating period. When flow ratio equals 1, the cylinder is in pure uniform flow. And the obvious VIV phenomenon occurs through the whole computational process. While flow ratio equals 0.5, only one obvious VIV development process can be observed when the oscillatory flow and the uniform flow are in the same direction. Figure 7 shows subplots of Modal weights of each vibration mode of the cylinder for different flow ratios. We can know that the cylinder mainly vibrates in the 1st mode and the effects higher modes of 2nd and 3rd are relatively small. Figure 8 shows subplots of wavelet analysis of the cross-flow displacement. When flow ratio equals 0 and 1, the dominant vibration frequency of the cylinder is close to the first natural frequency of 2.68Hz. While flow ratio equals 0.5, the vibration frequency transition happens in each oscillating period. The dominant vibration frequency is approximately the 1st natural frequency when the oscillatory flow is in the same direction with the uniform flow. And the dominant vibration frequency turns to be approximately 0 when flows are in adverse direction.

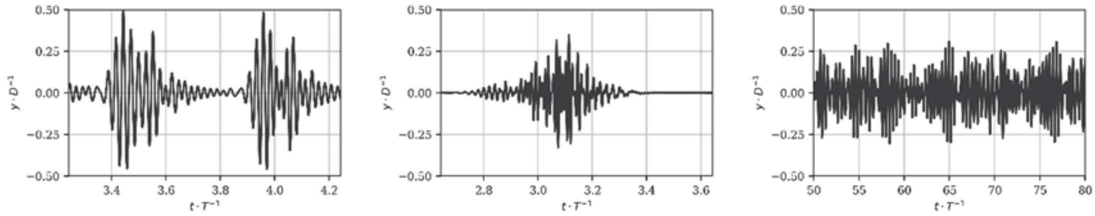
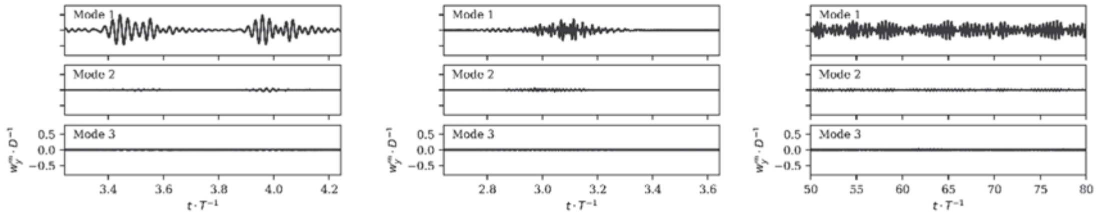

(a)α=0 (b)α=0.5 (c)α=1.0

Figure 6: Non-dimensional cross-flow amplitude of the intermediate node for different flow ratios in an oscillating period



(a)α=0 (b)α=0.5 (c)α=1.0

Figure 7: Modal weights of each vibration mode of the cylinder for different flow ratios

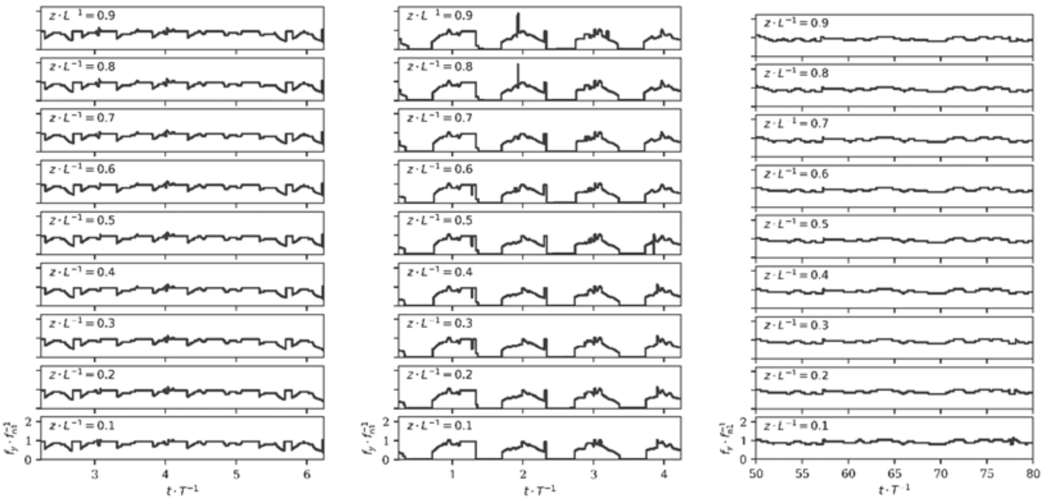

(a)α=0 (b)α=0.5 (c)α=1.0

Figure 8: Cross-flow wavelet analysis of the cylinder for different flow ratios

Acknowledgements

This work is supported by the National Natural Science Foundation of China (51490675, 11432009, 51579145), Chang Jiang Scholars Program (T2014099), Program for Professor of Special Appointment (Eastern Scholar) at Shanghai Institutions of Higher Learning (2013022), Innovative Special Project of Numerical Tank of Ministry of Industry and Information Technology of China (2016-23/09) and Lloyd's Register Foundation for doctoral student, to which the authors are most grateful.

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