Ship Hull Form Design Using a Kriging-based Global Optimization Algorithm

Aiqin Miao*, Decheng Wan‡

State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai 200240, China

*Presenting author: maq046@163.com

‡Corresponding author: dcwan@sjtu.edu.cn

http://dcwan.sjtu.edu.cn

Abstract

The kriging-based global optimization algorithm, Efficient Global Optimization (EGO), is applied to ship hull form design problems. The Kriging-based surrogate model is used to approximate the relationship between the design variables (input) and the objective functions (output) using a stochastic process. The kriging model drastically reduces the computational time required for objective function evaluation in the optimization(optimum searching) process. Expected improvement (EI) is used as a criterion to select additional sample points. This makes it possible not only to improve the accuracy of the surrogate model, but also to explore the global optimum efficiently. The present method is applied to five test functions and a ship hull form design, which makes the optimal hull form with the best resistance performance in calm water. It turns out that the EGO method is a good method for optimization design and can further extend to other ship hull form optimization design problems based on computational fluid dynamics.

Keywords: kriging model; efficient global optimization; uncertainty; ship optimization design; OPTShip-SJTU

Introduction

With the growth in computing power of current computers and the advances in computational techniques, today especially computational fluid dynamics (CFD) has become an invaluable tool for ship hull form optimization design. However, in the process of optimization design, the number of objective function evaluations using high-fidelity numerical analysis solvers, is severely limited by time and cost, even with current supercomputers.

One alternative is to construct a simple surrogate model instead of the complicated numerical analysis. The surrogate model expresses the relationship between the design variables (input) and the objective functions (output) using a stochastic Gaussian process. The model requires very little time to evaluate the objective function. It enables designers to greatly save computational cost. The most widely used surrogate model are the polynomial-based model, the response surface model, the kriging model and so on.

The kriging model (Wu J. W., 2017) has gained popularity for the engineering optimization design, recently. The model predicts the value of the unknown point using stochastic processes. The model has sufficient flexibility to represent the nonlinear and multimodal functions at the expense of computation time. However, the computation time to construct the kriging model is still short compared to that of direct high-fidelity numerical analysis.

However, the traditional optimization algorithms, like genetic algorithm (GA), differential evolution (DE), require many objective function evaluations, which may be impractical if we rely solely on the time-consuming high-fidelity numerical analysis solver. Here, the time-consuming numerical analysis solver in the objective function evaluation process of optimization is replaced with the kriging model. However, that it is possible to miss the global optimum in the searching space if we rely only on the prediction value of the kriging model because the model includes uncertainty at the prediction point. For robust exploration of the global optimum point, both the prediction value and its uncertainty should be considered at the same time. This concept is expressed in the criterion expected improvement (EI). EI includes the probability of a point being optimum in the design space. By the selection of the best EI point as the additional sample point, improvement of the model and robust exploration of the global optimum can be achieved at the same time, which is called Efficient Global Optimization (EGO) (Jones D. R., 1998; Jeong S., 2015).

In this paper, differential evolution (DE) algorithm is adopted as the searching algorithm to find maximum value of the EI function. DE is similar to genetic algorithm. However, compared with other evolutionary algorithms, DE retains the population-based global search strategy, which reduces the complexity of genetic operation by using real number coding, simple mutation based on difference and one-to-one competitive survival strategy. At the same time, DE's unique memory ability makes it possible to dynamically track the current search situation to adjust its search strategy, which has strong global convergence ability and robustness, and does not need to use the characteristic information of the problem. DE is suitable for solving the optimization problem in complex environment that the mathematical programming method may not solve.

In the first half of this paper, the EGO method will be briefly introduced back to the Kriging modelling. In the second of this paper, the method will be applied to the optimization problems of the test functions and ship design of Wigley.

Kriging Model

Kriging model (Simpson et al., 1994, 2004) is developed from mining and geostatistical applications involving spatially and temporally correlated data. This model combines a global model and a local component:

$$y(x) = f(x) + z(x) \tag{1}$$

where $y(x)$ is the unknown function of interest, $f(x)$ is a known approximation function of x, and $z(x)$ is the realization of a stochastic process with mean zero, variance σ^2, and non-zero covariance. With $f(x)$ and $z(x)$, the kriging model can build the surrogate model between the input variables and output variables.
The kriging predictor is given by:

$$\hat{y} = \hat{\beta} + \mathbf{r}^{\mathrm{T}}(x)\mathbf{R}^{-1}\left(\mathbf{y} - \mathbf{f}\hat{\beta}\right) \tag{2}$$

where y is an ns-dimensional vector that contains the sample values of the response; $\mathbf{f}$ is a column vector of length n_s that is filled with ones when f is taken as a constant; $\mathbf{r}^{\mathrm{T}}(x)$ is the correlation vector of length n_s between an untried x and the sampled data points $\left\{x^{(1)}, x^{(2)}, \ldots, x^{(n_s)}\right\}$ and is expressed as:

$$\mathbf{r}^{\mathrm{T}}(x) = \left[R\left(x, x^{(1)}\right), R\left(x, x^{(2)}\right), \ldots, R\left(x, x^{(n_s)}\right),\right]^{\mathrm{T}} \tag{3}$$

Additionally, the Gaussian correlation function is employed in this work:

$$R(x^i, x^j) = \mathbf{exp}\left[-\sum_{k=1}^{n_{dv}} \theta_k \left|x_k^i - x_k^j\right|^2\right] \tag{4}$$

In equation (2), $\hat{\beta}$ is estimated using equation (5):

$$\hat{\beta} = \left(\boldsymbol{f}^{\mathrm{T}}\mathbf{R}^{-1}\boldsymbol{f}\right)^{-1}\boldsymbol{f}^{\mathrm{T}}\mathbf{R}^{-1}\boldsymbol{y} \tag{5}$$

The estimate of the variance $\hat{\sigma}^2$, between the underlying global model $\hat{\beta}$ and y is estimated using equation (6):

$$\hat{\sigma}^2 = \left[\left(y - f\hat{\beta}\right)^T R^{-1}\left(y - f\hat{\beta}\right)\right] \Big/ n_s \tag{6}$$

where $f(x)$ is assumed to be the constant $\hat{\beta}$. The maximum likelihood estimates for the θ_k in equation (4) used to fit a kriging model are obtained by solving equation (7):

$$\max_{\theta_k > 0} \Phi(\theta_k) = -\left[n_s \ln\left(\hat{\sigma}^2\right) + \ln|\mathrm{R}|\right] \Big/ 2 \tag{7}$$

where both $\hat{\sigma}^2$ and $|\mathrm{R}|$ are functions of θ_k. While any value for the θ_k create an interpolative kriging model, the "best" kriging model is found by solving the k-dimensional unconstrained, nonlinear, optimization problem given by equation (7).

The accuracy of the prediction value largely depends on the distance from sample points. Intuitively speaking, the closer point x to the sample point, the more accurate is the prediction $\hat{y}$. This intuition is expressed as

$$s^2(x) = \hat{\sigma}^2\left[1 - r^{'}R^{-1}r + \frac{(1 - 1R^{-1}r)^2}{1^{'}R^{-1}1}\right] \tag{8}$$

where $s^2(x)$ is the mean squared error of the predictor and it indicates the uncertainty at the estimation point. The root mean squared error (RSME) is expressed as $s = \sqrt{s^2(x)}$.

DE algorithm

DE algorithm (Storn R., 1997) is mainly used to solve the global optimization problem of continuous variables. The main working steps are basically the same as other evolutionary algorithms, including Mutation, Crossover and Selection. The basic idea of the algorithm is to start from a randomly generated initial population and use the difference vector of two individuals randomly selected from the population as the random variation source of the third individual, and weight the difference vector and according to certain rules plus the third individual to produce a variation of individuals, which are called mutations. Then, the mutated individuals are mixed with a predetermined target individual to generate the test one. This process is called crossover. If the fitness value of the test individual is superior to the fitness value of the target individual, the test individual replaces the target individual in the next generation, otherwise the target individual is still saved, which is called the selection. In each evolutionary process, each individual vector is taken as the target individual once. The algorithm keeps the good individual and removes the poor individual through the iterative calculation, and the search process is approached to the global optimal solution. Here we use the DE algorithm to maximize the EI value.

A kriging-based global efficient optimization algorithm

Traditionally, once the surrogate model is constructed, the optimum point can be explored using an arbitrary optimizer on the model. However, it is possible to miss the global optimum because the approximation model includes uncertainty at the predicted point.

In Fig. 1, the solid line is the real shape of objective function. Eight points are selected to construct the kriging model, which is shown as red points. The minimum point on the kriging model is located near x=16, whereas, the real global minimum of the objective function is situated near x=17. Searching for the global minimum using the present kriging model will not result in the real global minimum near x=17. For a robust search of the global optimum, both the predicted value by the kriging model and its uncertainty should be considered at the same time.

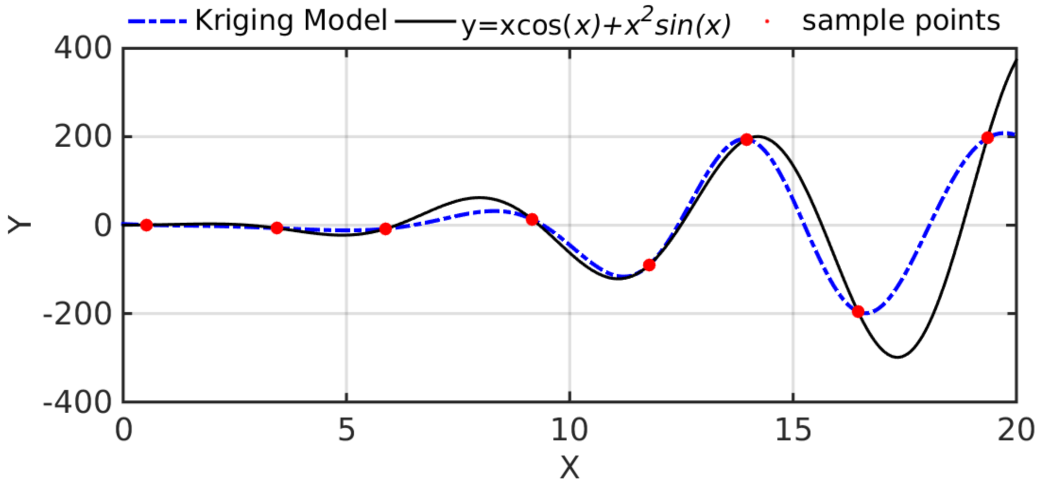

Figure 1. The Kriging model and the real function curve

When Kriging model is built, the mean predicted value and the standard error of the kriging model at any point can be evaluated. Considering the uncertainty of the model, this concept is expressed in the criterion of EI. The EI of minimization problem can be calculated as

$$E[I(x)] = (f_{\min} - \hat{y})\Phi[(f_{\min} - \hat{y})/s] + s\phi[(f_{\min} - \hat{y})/s] \quad (9)$$

Where $f_{\min}$ is the minimum value among n sampled values. Φ and ϕ are the standard distribution and normal density, respectively.
In fact, if we compute the derivative of EI as given in equation (9) with respect to $\hat{y}$ and s, we gets several terms that cancel, resulting in the simple following expressions:

$$\partial EI / \partial \hat{y} < 0, \partial EI / \partial s < 0 \quad (10)$$

It turns out that EI is monotonic in $\hat{y}$ and in s. Thus, we see that the EI is larger the lower is $\hat{y}$ and the higher is s. By selecting the maximum EI point as additional sample point through DE algorithm mentioned above, robust exploration of the global optimum and improvement of the model can be achieved simultaneously.

The overview over the whole efficient global optimization (EGO) procedure mentioned before is shown in Figure 2. First, the initial sample points should be chosen by experiment of design uniformly covering the whole design space. Secondly, an ordinary Kriging model is built and used to predict the objective for each design variable. Thirdly, the expected improvement (EI) balancing between regions of the low mean prediction and of high standard error is constructed to select the next point. The choice of the next sample point is the maximization of the EI value. Next, the objective function in the new point is calculated accurately and used to build a new surrogate model with the initial sample points, thus the next iteration is initiated. Finally, when the EI value is very small after n iterations, i.e., $\max(EI) < \Delta s \cdot (\max(y) - \min(y))$, where Δs is the relative stopping tolerance, or reach the maximization of iteration steps, the loop should be stopped.

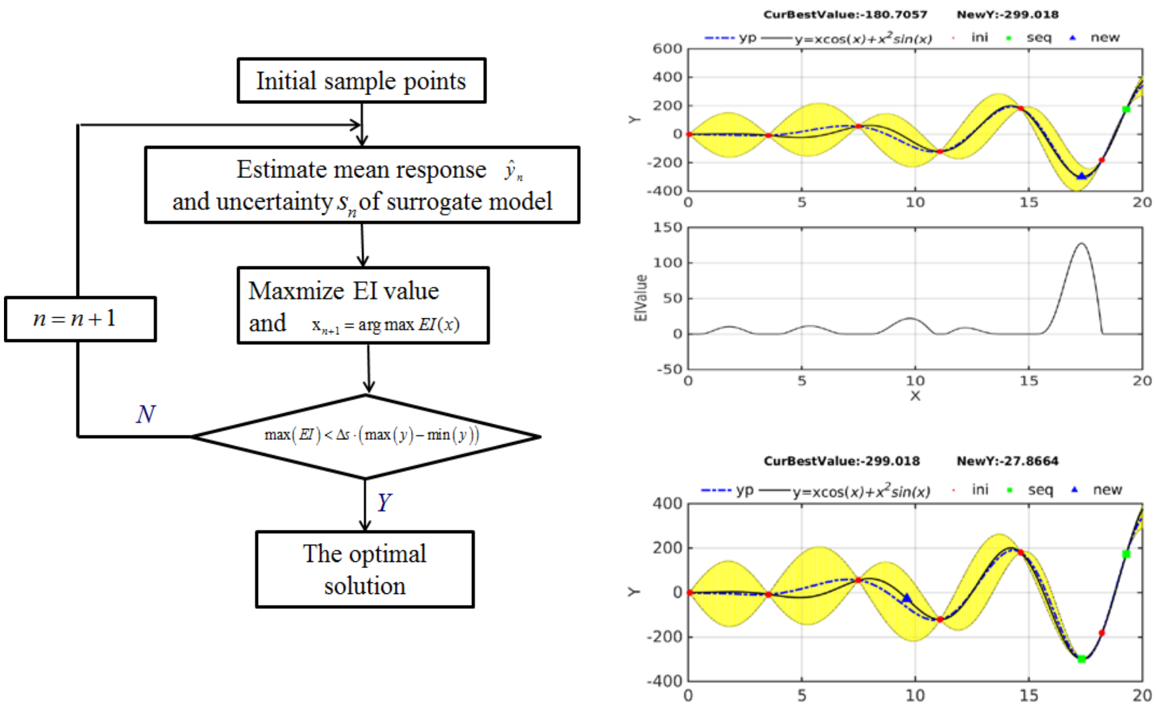

Figure 2. The flow chart of efficient global optimization: on the left, the steps are briefly described; on the right, an example is given (predetermined design points as red dots, the added new points as green squares and the next new point as a blue triangle).

Applications

Application of the test functions

we now apply this method EGO to five test functions, from a literature (Dixon L.C.W. and Szego G. P., 1978): Six-hump camel-back function, the Branin function, the Goldstein-Price function, the Hartman 3 function and the Hartman 6 function. The dimensions of these test functions are 2, 2, 2, 3 and 6, respectively. The results of the test functions by the EGO algorithm is shown in table.

For each function, we report the number of evaluations to meet stopping criterion, and report the actual relative error on convergence.

Table 1. Test function results by the EGO method

Function Name	Dimension	Evaluations to meet stopping criterion	Optimal result by EGO	Real optimal value	Relative error
Six-hump	2	30	-1.0259	-1.03163	-0.56%

Branin	2	30	0.39985	0.398	0.46%
Goldstein-Price	2	31	3.00326	3	0.11%
Hartman 3	3	35	-3.83	-3.86	-0.78%
Hartman 6	6	84	-3.25503	-3.32	-1.96%

As we can see, the results of the former four test functions achieve less than an actual relative error of 1%. For the Hartman 6 function, the actual relative error on convergence is slightly greater. Possibly, it is caused by the unknown but estimated correlation parameters. As a results, the standard error may be a bit too small, causing us to underestimate EI value.

Application of ship optimization design of Wigley

In this study, the EGO method was applied to ship hull form design and the optimization of the resistance performance in calm water.

$$f_{obj} = C_w, Fr = 0.3 \tag{11}$$

The design problem is to minimize the wave-making resistance coefficient of Wigley at the design speed. The Radial Basis Function (RBF) method (De Boer A., 2007) was applied to modify ship hull form. The design variables are parameters closely related to ship hull form modification. The total 7 design variables in table 2 are used to define the geometry of ship hull form. The upper and lower bound of each parameter is determined to avoid unrealistic ship hull geometry.

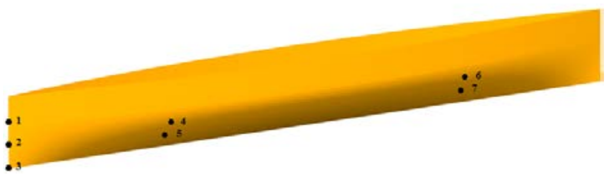

Figure 3. The control points distributed on Wigley hull by RBF method

Table 2. The design variables and their ranges

Design variables	Deformation direction	The bound
1#	x	[-0.0025, 0.0025]
2#	x	[-0.0025, 0.0025]
3#	x	[-0.0025, 0.0025]
4#	y	[-0.0025, 0.0025]
5#	y	[-0.0025, 0.0025]
6#	y	[-0.0025, 0.0025]
7#	y	[-0.0025, 0.0025]

At the early stage of optimization design, 35 sample points are spread over the design space and selected by Optimal Latin Hypercube Sampling (OLHS) to obtain a kriging model (Park

J. S., 1994). In this study, OLHS has the beneficial property of orthogonality and uniformity in the multidimensional design space. The number of sample points is very important to keep the kriging model accurate in the traditional optimization process. however in the present kriging model, additional sample points will be added later in the region where the accuracy is not good enough, based on EI evaluation. The wave-making resistance coefficients of 35 sample hull form and additional sample points are all evaluated using a potential flow theory, Nuemann-Michell method (Noblesse F., 2013; Liu, X. Y., 2016; Wu, J. W., 2016).

After efficient global optimization search, The total number of sample points reached 45, after adding 10 more sample points. The objective function converges to the minimum value, 1.046E-03, a larger reduction of 18.46% than the initial value. the summary of the relative parameters of EGO method is shown in table. And The design values of the optimal hull form is shown in Table 4.

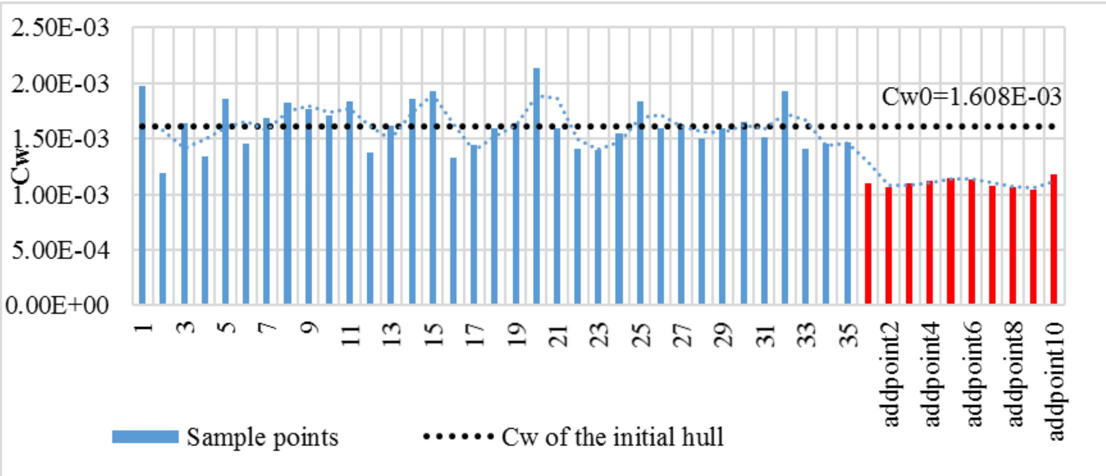

Figure 4. The initial sample hulls and the additional new hulls used in the EGO method

Table 3. The parameters of the EGO method

The initial number of sample points	35	The additional number of sample points	10
The number of iterations	10	Optimization time (h)	1.075h
Node number	1		

Table 4. The values of design variables and the optimal solution of the objective function after optimization procedure

Modification method	Design variables	Convergence value
RBF method	1#	0.002297
	2#	-0.002346
	3#	-0.002450

4#	-0.00250
5#	-0.00250
6#	-0.00250
7#	-0.00250
The initial objective function value	1.618×10^{-3}
The convergent objective function value	1.046×10^{-3}

The following figures depict the comparisons of body lines and shapes of the initial Wigley and the optimal ship. The optimal hull form is thinner than the initial one globally. Another apparently different region is the fore part of the hull. The optimal hull is changed like ‘s’ curve, indent near the bottom and convex close to the top part.

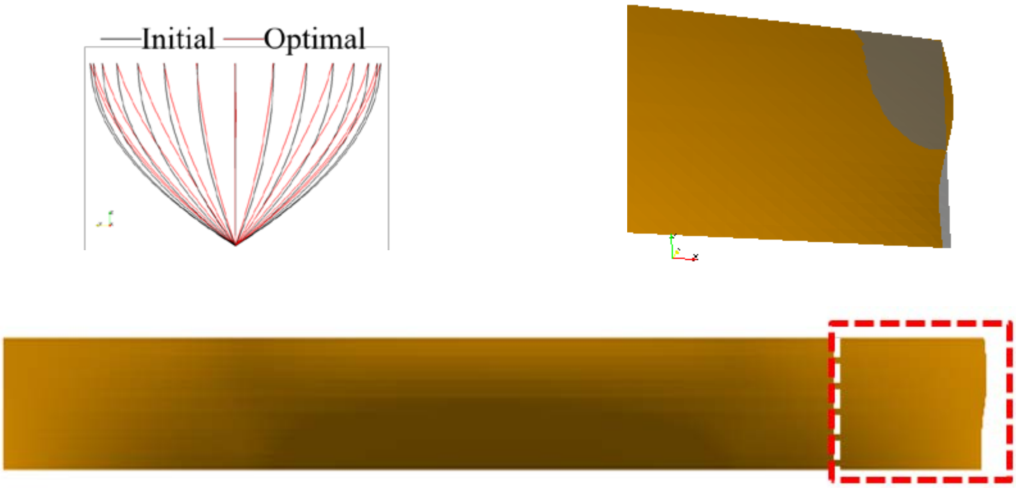

Figure 5. Comparisons of the body lines and shapes between the initial hull and the optimal one

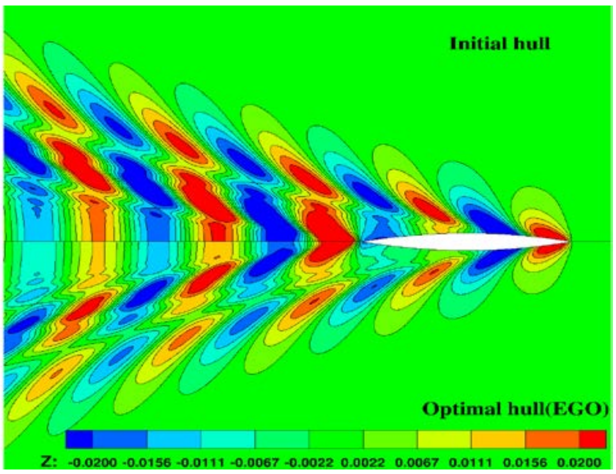

Figure 6. Comparison of free surface elevation between the initial hull and the optimal one

Figure 6 is the wave profile of free surface of the two ships. In contrast, the optimal hull by EGO method generates lower wave than the initial one obviously, thus leads to the reduction of the wave-making resistance of the optimal hull. Also comparing the pressure distribution of the two ships in Fig. 7, the fore part of the optimal hull is with lower pressure than the initial one. Figure 8 shows the wave-making resistance coefficients of the two ships at a series of speeds, which makes it clear that the optimal ship has a lower wave-making resistance coefficient than the initial ship at certain range of speeds. This results validate that our optimization design in this study is robust.

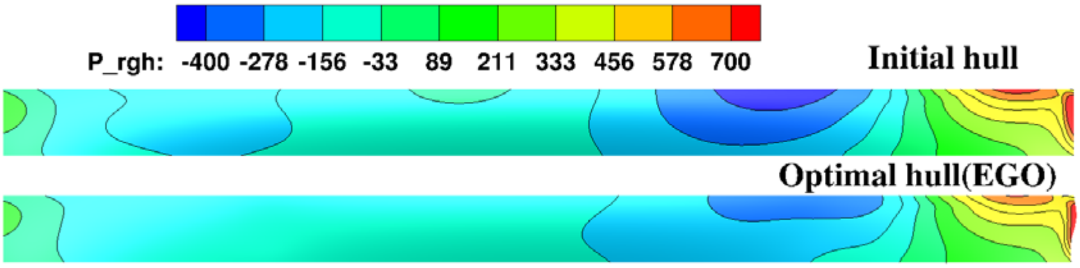

Figure 7. Comparison of pressure distribution between the initial hull and the optimal one

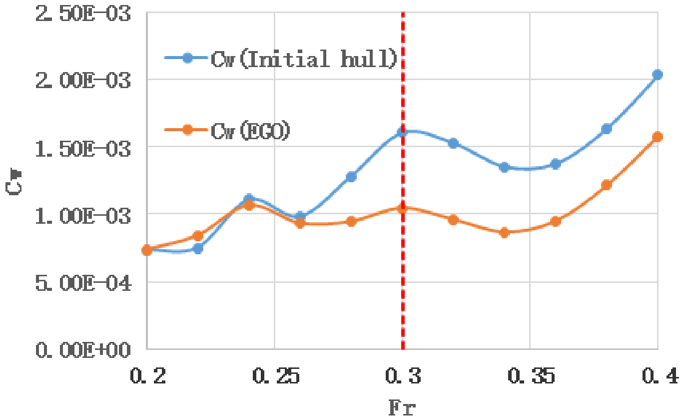

Figure 8. The wave-making resistance coefficients of the best variation at a series of speeds

Conclusions

This paper presents a Kriging-based global optimization method, efficient global optimization (EGO), different from the ordinary optimization method. It combines the surrogate modeling with the optimization algorithm. By this method, not only the accuracy of surrogate model is continuously improved but also the solution of the optimization problem keeps searched in the iterative procedure. The method is successfully applied to the test functions and ship hull form optimization design. The results demonstrate the usability of the method in ship hull form optimization design. In the future, it will be used to the more ship optimization problem, such as the ship hull form design to improve comprehensive hydrodynamic performance, based on entire CFD.

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