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Hull Form Optimization Using Bayesian Optimization Framework

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ABSTRACT

In this paper, a data-driven shape optimization approach is proposed for ship hull form optimization. To avoid the time-consuming evaluation of ships via a viscous flow solver, we developed a Machine-Learning (ML) based model that predicts the hull's hydrodynamic performance. For this purpose, a Bayesian optimization framework is developed and applied to OPTShip-SJTU, an existing ship optimization solver. Among them, the CFD method is used to calculate ship performance, and the Radial Basis Function (RBF) method is adopted for hull surface deformation. To improve the efficiency of hull form optimization, the surrogate model is used to approximate the CFD simulation. Unlike the traditional static approximation models used in the process of hull form optimization, a dynamic approximation model based on expected improvement is proposed. The adaptive balance parameter is taken in the parallel efficient optimization (PEGO) algorithm to make a tradeoff between exploitation and exploration. The Optimal Latin hypercube algorithm is used as the method of design of experiments. The Kriging model is employed as the surrogate model. Wigley ship is used to demonstrate the proposed optimization framework. Lines of the ship are determined and optimization results of the resistance show the effectiveness of the proposed method.

KEY WORDS: OPTShip-SJTU Solver; dynamic approximation model; parallel efficient global optimization algorithm (PEGO); Wigley; resistance

INTRODUCTION

Ship is a significant tool for humans to explore and exploit the ocean. Design of a ship is so complex that multiple performances should be considered, especially hydrodynamic performance which includes maneuverability, rapidity, and seakeeping. In the process of ship design, rapidity is the main concern. Besides, the resistance of ship is an essential aspect that reflects the ship's rapidity performance. Reducing the resistance of ships becomes more and more challenging, which can be achieved by optimizing the ship hull lines. Changes in hull lines can also affect other ship properties, such as seakeeping performance and maneuverability. Noteworthy, how to obtain the best hull form is the main concern in the design stage. With the development of computer technology, Computer Fluid Dynamics (CFD) based on viscous theory has been widely applied to hydrodynamic problems. As a result, simulation-based design (SBD) technology has been widely applied to hull form optimization in the past decades.

Based on SBD technology, scholars at home and abroad have conducted a lot of research and obtained good results. Peri, Rossetti and Campana (2001) modified a tanker ship by using the Bézier patch. Three different algorithms which included Conjugate Gradient (CG), Steepest Descentto (SD) and Sequential Quadratic Programming (SQP) were used to optimize its total resistance and wave amplitude. Peri and Campana (2003) performed a local reconstruction of the sonar cover of DTMB5415 based on the Bezier polynomial surface method and optimized the total drag coefficient when Fr=0.41. The results showed that the total drag coefficient of the optimized ship model was reduced by 6%. Lin, Yang and Guan (2019) used six cross-sectional area curve parameters of Small Waterplane Area Twin Hull (SWATH) as design variables. The optimal Latin hypercube sampling (OLHS) method was employed to obtain 40 sample data. After hydrodynamic evaluation, the Kriging model was constructed, and the optimal ship type was obtained by using Multi-Island Genetic Algorithm. The results showed that the total resistance of the optimal hull was reduced by 28.9% compared with the parent ship. Wang, Chen and Feng (2021) selected 9 parameters of deep-sea aquaculture vessels as design variables and applied 60 sample data which were obtained by uniform sampling method to construct radial basis network surrogate model. The optimization results showed that the total drag coefficient and the non-uniformity of wake flow of the optimal vessel were reduced by 1.67% and 17.12%, under the structural draft and 2.59% and 4.04%, under the ballast draft, respectively. Liu, Zhao and Wan (2022) optimized the resistance and propeller wake distortion of Japan Bulk Carrier (JBC) based on the in-house solver OPTShip-SJTU considering the interaction between hull and propeller. The Free-Form Deformation (FFD) method was applied to modify the stern shape of JBC. 30 sample points were obtained by using the Optimized Latin Hypercube Sampling Method (OLHS). The Kriging model was constructed to reduce the computation cost. The optimal results obtained by using the multi-objective genetic algorithm (NSGA-II) showed that it was necessary to consider the propeller effect in the process of resistance and wake optimization.

As can be inferred from the above introduction, the conventional SBD optimization process usually involved constructing a surrogate model with reliable global prediction accuracy and then using optimization algorithms to search for the optimal design solution. Obviously, this optimization method relied heavily on the global prediction accuracy of the surrogate model. If the prediction accuracy of the surrogate model was low, it will lead to poor or failed optimization results. In addition, even if the global prediction accuracy of the surrogate model was high, it might not be able to predict the true optimal solution in the design space with high accuracy. To tackle this problem, the adaptive sampling method was put forward. In the field of ship and ocean engineering, the adaptive sampling method was relatively little applied. With the strong promotion of adaptive sampling strategies in the aviation field (Viana and Haftka, 2010; Liu, Han and Song, 2012; Forrester and Keane, 2009; Wang, Ni, and Zeng, 2021), it has only been gaining wider attention in the ship and ocean engineering in recent years. Mackman and Allen (2010) developed an adaptive sampling strategy based on the nonlinearity degree in the design space. The strategy took the position with the highest nonlinearity in the initial design space as the position where the next sample point should be added, and so on repeatedly until the highest nonlinearity in the design space was less than the nonlinearity convergence threshold. Rafiee, Haase and Malcolm (2022) proposed a multi-objective Bayesian optimization method for high-speed craft. In order to verify the effectiveness of the proposed framework, the total resistance of a conceptual high-speed catamaran at two speeds was optimized. It's concluded that the proposed method could design hull forms efficiently along the multi-objective Pareto front. Volpi, Diez and Gaul (2015) proposed an adaptive sampling strategy based on the prediction uncertainty of the surrogate model, which selected the position with the largest uncertainty in the prediction of the dynamic surrogate model as the next sample point should be added until the maximum uncertainty in the prediction of the surrogate model was less than the uncertainty convergence threshold.

In this paper, we used a Bayesian optimization method for hull form optimization as it could make a tradeoff between exploration and exploitation with less expensive function evaluations. We proposed an automated framework for hull optimization with a series of objectives and constraints. Finally, we test our framework on optimizing the total resistance of Wigley at Fr=0.3.

METHODOLOGY

In this section, we introduce the Kriging model which is used as a data mining tool to lead the addition of sample points, the Bayesian optimization method, the method for hull surface deformation, and the automated framework for hull form optimization.

Kriging model

The Kriging method, first proposed in 1951, is originally used for mineral reserve estimation. Sacks, Welch and Mitchell (1989) use it to approximate computer calculation, and the Kriging model become more and more popular in optimization problems. Here is a brief introduction to the Kriging model, the concrete derivation can be found in (Sacks, Welch and Mitchell, 1989).

A simple Kriging model is constructed as:

$$y(x) = \mu + \varepsilon(x) \tag{1}$$

where μ is the average value of Gaussian process, $\varepsilon(x)$ is the error term that satisfies the normally distributed $(0,\sigma^2)$, and the covariance is non-zero. The correlation of the deviations can be expressed as follows:

$$Cov\left[\varepsilon\left(x^{(i)}\right),\varepsilon\left(x^{(j)}\right)\right] = \sigma^2 R\left(\left[Corr\left[\varepsilon\left(x^{(i)}\right),\varepsilon\left(x^{(j)}\right)\right]\right]\right) \tag{2}$$

$$Corr\left[\varepsilon\left(x^{(i)}\right),\varepsilon\left(x^{(j)}\right)\right] = \exp\left(-\sum_{k=1}^{m}\theta_k\left|x_k^{(i)} - x_k^{(j)}\right|^{p_k}\right) \tag{3}$$

$$R\left(x_i, x_j\right) = \prod_{k=1}^{n}\exp\left(-\theta_k\left|x_k^i - x_k^j\right|^2\right) \tag{4}$$

where R denotes the matrix of correlation functions between samples, which is presented as Eq.4 , m is the dimension of the design parameters, θ_k and p_k are the parameters to be determined.

In the Kriging model, the predicted values of the $(2m+2)$ parameters: $\mu,\sigma^2,\theta_1,\ldots,\theta_m,p_1,\ldots,p_m$ are obtained by maximizing the likelihood function of the sample points. I denotes the n-dimensional unit vector and the likelihood function is presented as follows:

$$\frac{1}{(2\pi)^{n/2}\left(\sigma^2\right)^{n/2}|R|^{1/2}}\exp\left[-\frac{(y-I\mu)^T R^{-1}(y-I\mu)}{2\sigma^2}\right] \tag{5}$$

The values of μ and σ^2 can be gotten by specifying the correlation parameters θ_k and p_k:

$$\hat{\mu} = \frac{I^T R^{-1} y}{I^T R^{-1} I} \tag{6}$$

$$\sigma^2 = \frac{\left(y - I\hat{\mu}\right)^T R^{-1}\left(y - I\hat{\mu}\right)}{n} \tag{7}$$

For any unknown point x^*, the prediction value as well as the mean squared error are calculated by the Kriging model:

$$\hat{y}\left(x^*\right) = \hat{\mu} + r^T R^{-1}\left(y - I\hat{\mu}\right) \tag{8}$$

$$s^2\left(x^*\right) = \sigma^2\left[1 - r^T R^{-1} r + \frac{\left(1 - r^T R^{-1} r\right)^2}{I^T R^{-1} I}\right] \tag{9}$$

where r is an n-dimensional column vector that represents the correlation between the observation point x^* and the sample points.

Bayesian optimization

Bayesian optimization (Jones, Schonlau and Welch, 1998) is an efficient global optimization algorithm for solving time-consuming optimization problems. It uses the Kriging model as a tool for data mining to update the Kriging model dynamically. The fundamental idea of Bayesian optimization is to construct an initial Kriging model based on the design of experiment, and add sample points iteratively by using infill sampling criteria to finally converge to the optimal solution (Shahriari, Swersky and Wang, 2015). A classical choice for infill sampling criteria is

Expected Improvement (EI) that is the core of the efficient global optimization (EGO) algorithm and is expressed as follows:

$$E[I(x)] \equiv \mathrm{E}[\max(f_{\min} - Y(x), 0)] \quad (10)$$

where the $Y(x)$ is regarded as normal distribution with mean $\hat{y}$ and variance s^2 .The right-hand term of the Eq.10 can be expressed in integral form, and then a lengthy series of derivations leads to the following expression in closed form:

$$E[I(\mathbf{x})] = \left(f_{\min} - \hat{y}\right)\Phi\left(\frac{f_{\min} - \hat{y}}{s}\right) + s\phi\left(\frac{f_{\min} - \hat{y}}{s}\right) \quad (11)$$

where $\phi(x)$ and $\Phi(x)$ is the standard normal density function and the standard normal distribution function.

$$\frac{\partial E(I)}{\partial \hat{y}} = -\Phi\left(\frac{y_{\min} - \hat{y}}{s}\right) < 0 \quad (12)$$

$$\frac{\partial E(I)}{\partial s} = \phi\left(\frac{f_{\min} - \hat{y}}{s}\right) > 0 \quad (13)$$

The highlight of the EGO algorithm is that it can achieve a tradeoff between local search and global search. As can be seen from Eq. 12 and Eq.13, when the prediction value at a point is very small, $\left(y_{\min} - \hat{y}\right)$ is large, leading the search to iterate towards the very small prediction value. When the prediction error at a point is large, it will lead the search to iterate towards the prediction error where it is very small to find the best.

The EGO algorithm is summed as Algorithm 1

Algorithm 1 The framework of the EGO algorithm
1: Initial sample points and their objection function value
2: The optimal solution of current sample set
3: While the stop condition is not met do
4: Constructing a Kriging model based on the initial sample set
5: $x_{new} = \arg\max EI(x)$
6: Calculating the real function value at x_{new}
7: Adding a new sample point to the initial sample point set
8: Getting the optimal solution by using optimization algorithm
9: end While

The parallel efficient global optimization algorithm

The EGO algorithm mentioned above can only obtain one sample point with the highest EI value per iteration to update the Kriging model and sample point set. It can't get the second additional sample point without evaluating the first additional sample point, which constrains its application in hull form optimization. Thus, it's necessary to develop a parallel efficient global optimization algorithm. Scholars have conducted lots of work on parallel efficient global optimization algorithms (Ginsbourger, Le and Carraro, 2010; Sóbester, Leary and Keane, 2004). However, the parallel efficient global optimization algorithm developed by the above scholars is complex and time-consuming. Zhan, Qian and Cheng (2017) propose a parallel efficient global algorithm (PEGO) that is simple and easy to derive. Here will give a brief introduction to parallel efficient global algorithms and the concrete principle can be found in Zhan, Qian and Cheng (2017).

Fig. 1(a) is the initial EI function of the Forrester function (Sobester, Forrester and Keane, 2008). The EI value is highest at x=0.676, therefore, the x = 0.676 is selected as the new sample point to update the Kriging model and EI function which is shown in Fig. 1(b). It can be seen from Fig. 1(b) that the updated EI function decreases sharply around the new sample point and the change is slower far away from the new sample point. That's to say, the new sample point will have affect the EI function, the closer the distance, the greater the effect.

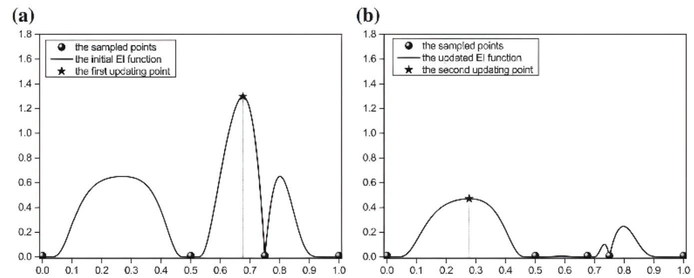


Fig. 1. The initial expected improvement function (a) and the updated expected improvement function (b) (Zhan, Qian and Cheng, 2017)

Thus, in order to approximate the updated EI function to tune the initial EI function, Zhan, Qian and Cheng (2017) propose that an influence function is multiplied by the EI function. By multiplying the influence function, the approximated EI function can be obtained without evaluating the new sample point. The second sample point can be selected according to the highest approximated EI function. In this way, the sample points are obtained continuously. The approximated EI function whose core is the pseudo expected improvement (PEI) can be expressed as follows:

$$PEI(x) = EI(x) \times IF\left(x, x^{(N+i)}\right) \quad (14)$$

According to Zhan, Qian and Cheng (2017), the influence can be any form with follows features:

(1) In the entire design space, the influence function should be continuous.
(2) At the updating sample point, the value of influence function should be zero. At the position far away from the updating point, the value of influence function should be one.
(3) The value of the influence function can only be related to the position of the update point.

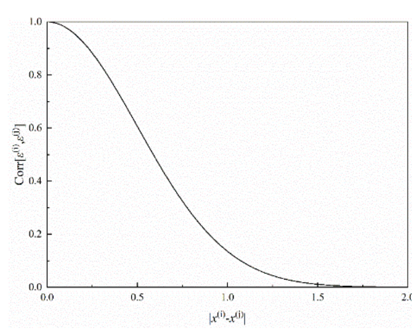


Fig. 2. The correlation function when $\theta = 2$ and $p = 2$

Zhan, Qian and Cheng (2017) propose that the correlation function in the Kriging model can be selected as an influence function. As can be seen in Fig. 2, it's the correlation function that is described in the Kriging model mentioned above between $x^{(i)}$ and $x^{(j)}$ when $\theta = 2$ and $p = 2$. From Fig. 2 we can conclude that the correlation can satisfy the features mentioned above.

Thus, the influence function proposed by Zhan, Qian and Cheng (2017) can be expressed as:

$$IF\left(x, x^{(N+i)}\right) = 1 - Corr[\varepsilon(x), \varepsilon(x^{N+i})] \tag{15}$$

$$IF\left(x, x^{(N+i)}\right) = 1 - \exp\left(-\sum_{k=1}^{n} \theta_k \left|x_k^{(i)} - x_k^{(j)}\right|^{p_k}\right) \tag{16}$$

If we want to select n sample points at one cycle, the PEI function can be presented as follows:

$$\begin{aligned} PEI(x) &= EI(x) \times \prod_{i=1}^{n-1}\left[1 - Corr[\varepsilon(x), \varepsilon(x^{N+i})]\right] \\ &= EI(x) \times \prod_{i=1}^{n-1}\left[1 - \exp\left(-\sum_{k=1}^{m} \theta_k \left|x_k^{(i)} - x_k^{(j)}\right|^{p_k}\right)\right] \end{aligned} \tag{17}$$

Once the n sample points are selected, we can make full use of computing resources and start numerical simulations. The PEGO is summed up in Algorithm 2. In this work, the parallel EGO (Zhan, Qian and Cheng, 2017) is used for Bayesian optimization.

Algorithm 2 The framework of the parallel EGO algorithm
1: Initial sample set (X, Y)
2: $f_{\min}$ of current sample set
3: While the stop condition is not met do
4: Constructing a Kriging model
5: for i=1 to n do
6: $x_{new}^{(m+n)} = \arg\max PEI(x, n-1)$
7: end for
8: Calculating the real function of $\left(x_{new}^{(m+1)}, \ldots, x_{new}^{(m+n)}\right)$
9: Adding n sample point to the initial sample point set
10: Getting the optimal solution by using optimization algorithm
11: end While

Fully automated workflow of hull form optimization

Based on our in-house solver OPTShip-SJTU (Liu, Zhao and Wan, 2021), the parallel efficient global optimization algorithm (PEGO) is added to form the fully automated workflow for hull form optimization. The full optimization workflow is shown in Fig. 3.

As can be seen from Fig. 3, to begin with, the initial sample points are obtained by the optimal Latin hypercube sampling (OLHS) method, and then the value of initial sample points is evaluated to build a Kriging model. Here, the genetic optimization algorithm (Whitley, 1994) is employed to find the new point x_{new}^{m+1} with the highest EI value. In addition, the approximate EI function is computed by multiplying the initial EI function by the influence function instead of evaluating the real value of x_{new}^{m+1} to update the EI function. The n new sample points are selected in a cycle. Next, the n new ships are generated by our in-house solver OPTShip-SJTU and simulated. The hydrodynamic performance of new ships are added to the database to update the Kriging model. In order to guarantee the accuracy of optimization, the convergence conditions are used to improve the efficiency of Bayesian optimization according to the theory of Zhao, Cheng and Ruan (2015).

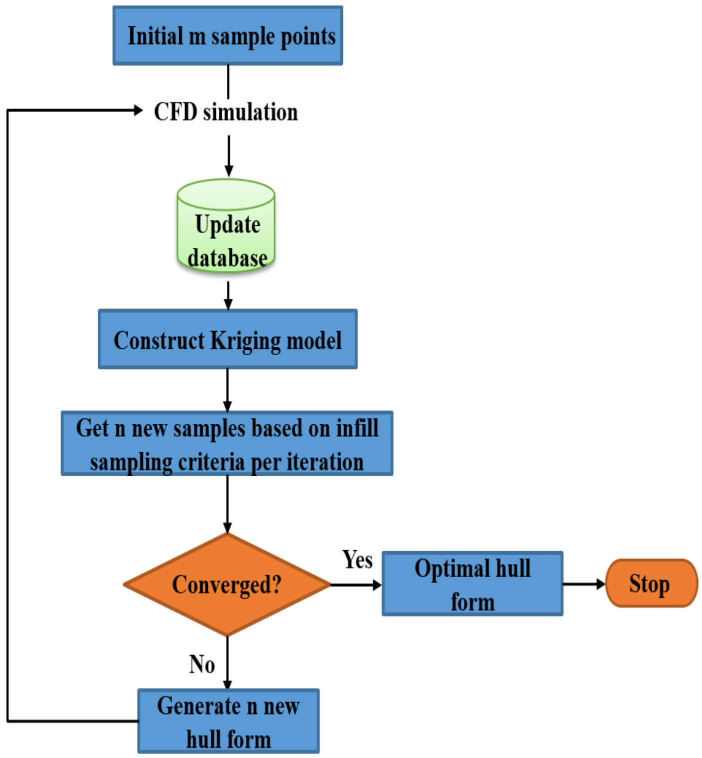


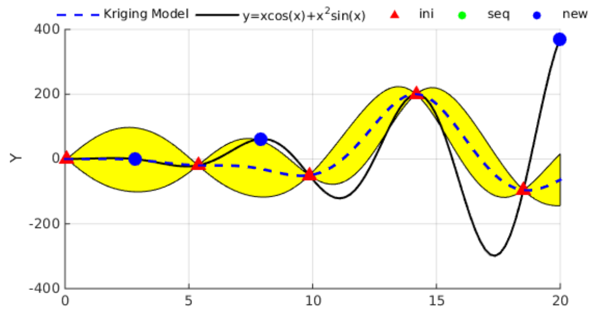
Fig. 3. The automated hull form optimization framework

VALIDATIONS

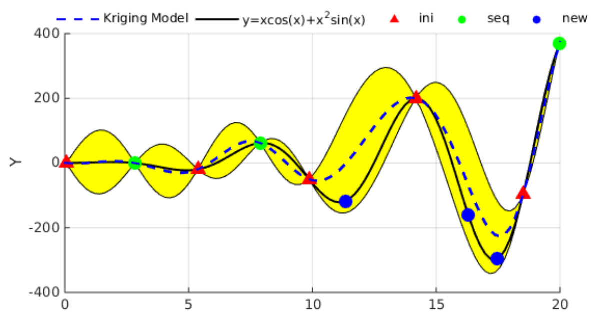
To verify the feasibility of Bayesian optimization in hull form optimization, test functions are employed. Then, three hull form optimization cases are set to show the strength of Bayesian optimization.

Test function

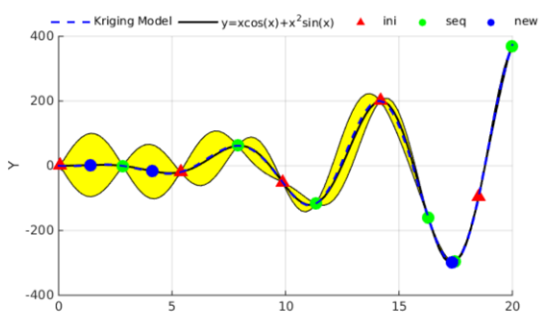
To test the reliability of the Bayesian optimization, test functions are used. Fig. 4 shows that new sample points are added continuously according to the pseudo expected improvement (PEI). The black line is the real function. The dotted line is the Kriging model and the red points are the initial sample points. The green points are the added new sample points and the blue points are the new points that will be added in one cycle. The yellow area is $\hat{y} \pm s$ .The number of initial sample points is five, and three sample points are added in one cycle. From Fig. 4, we can conclude that after 4 iterations, the optimal solution is obtained. The optimal solution by Bayesian optimization is -299.0519 and the true optimal solution is -299.0528, which shows the accuracy of Bayesian optimization is reliable.



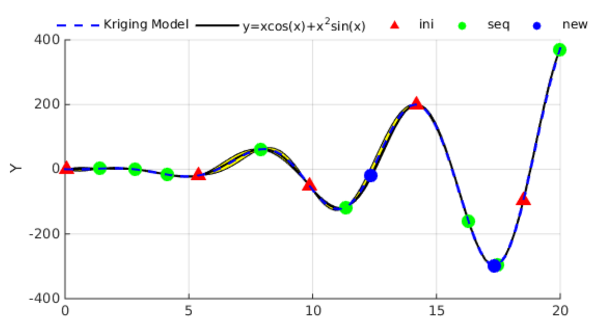
(a) 1st iteration for three additional sample



(b) 2nd iteration for three additional sample



(c) 3rd iteration for three additional sample



(d) 4th iteration for three additional sample

Fig. 4. Optimization for a one-dimensional function based on Bayesian optimization

Hull form optimization

In this part, to evaluate the efficiency and accuracy of Bayesian optimization in hull form optimization. The total resistance of Wigley is optimized at Fr=0.3. The wave-making resistance coefficient C_w is calculated by our in-house solver NMShip-SJTU whose accuracy is fully proved (Liu, Zhao and Wan), and the fractional resistance coefficient C_f is calculated by ITTC 1957 formula which is presented as:

$$C_f = \frac{0.075}{(\lg \mathrm{Re} - 2)^2} \quad (18)$$

In this work, the Radial Basis Function (RBF) method (Liu, Zhao and Wan, 2021) is applied to deforming the hull. Firstly, a bulbous bow is generated using the RBF method, and then the bow is further deformed on this basis, which is shown in Fig. 5. The main dimensions of Wigley are shown in Table 1 and the design variables are shown in Table 2.

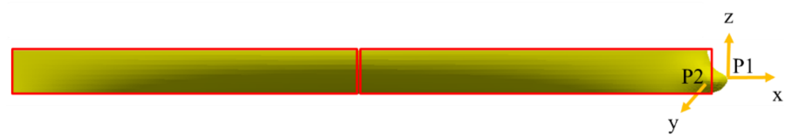


Fig. 5. The control points for Wigley deformation

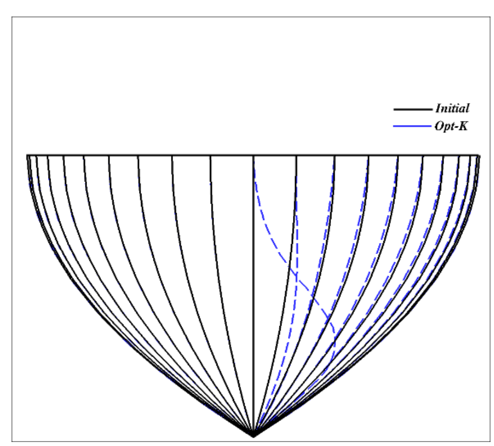
Table 1. The main dimensions of Wigley

Ship	Length (m)	Width (m)	Height (m)
Wigley	4	0.4	0.25

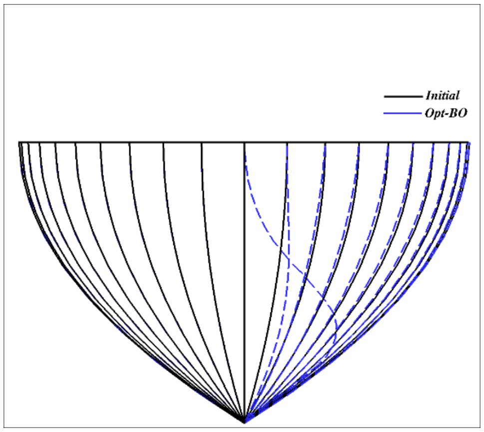
Table 2. Design variables

	Deformation direction	Range
P1	x	[0.515,0.555]
	z	[-0.042,-0.0348]
P2	y	[0.0045,0.024]

A comparison is made with traditional simulation-based on design (SBD) optimization techniques. Based on the SBD method, 36 sample points are selected by OLHS. Then, the hydrodynamic performance is simulated by NMShip-SJTU. Based on the Bayesian optimization method, 15 initial sample points are obtained by OLHS, and 3 new sample points are added based on Bayesian optimization in one cycle. Herein, the optimal hull "Opt-K" is obtained based on the static Kriging model which belongs to the SBD method, while the optimal hull "Opt-BO" is obtained based on Bayesian optimization. A comparison of transverse hull lines between the two optimal and the initial hulls is shown in Fig. 6. It can be seen that for optimal hulls "Opt-K" and "Opt-BO", a relatively large bulbous bow is generated. The bulbous bow of "Opt-BO" has a wide range of variations along the ship length.



(a) Opt-K



(b) Opt-BO

Fig. 6. The transverse hull lines comparison between the initial and the optimal hulls

Table 3 shows that the Bayesian optimization method has a better result with a drag of 18.087N than the SBD method with a drag of 18.499N. Besides, only 24 sample points are needed to obtain a more optimal result, and the efficiency is improved by more than 33.33% compared with the conventional SBD optimization method.

Table 3. Design variables

	Number of Sample points	Rt (N)	Reduction ratio
Initial	-	20.512	-
Opt-K	36	18.499	10.35%
Opt-BO	24	18.087	11.82%

Fig. 7 shows the comparison of the free surface wave evaluation of the initial hull and optimal hull which is obtained by the Bayesian optimization method. On the whole, the amplitudes of the peaks and troughs decline obviously with the generation of the bulbous bow.

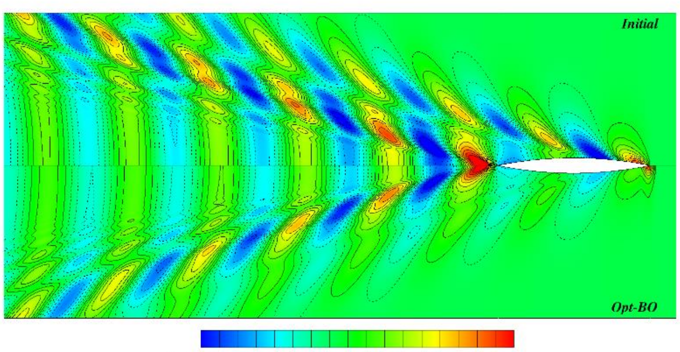


Fig. 7. Comparison of the free surface wave evaluation between initial hull and optimal hull

CONCLUSIONS

For the hull form optimization based on SBD, it's noted that the static surrogate models are popular while the dynamic surrogate models are paid less attention to. The sample points and structure of static surrogate models are fixed in the process of hull optimization. For a complex hull form optimization problem, more sample points are required, which can't guarantee the efficiency. On the contrary, the dynamic surrogate models update the sample points with proper infill sampling criteria is preferable in terms of both accuracy and efficiency.

In this paper, the Bayesian optimization method is applied to hull form optimization to update the Kriging model during each iteration. A fully automated hull form optimization framework is proposed based on Bayesian optimization. The accuracy and efficiency of the Bayesian optimization method are verified based on test functions firstly. The results show that the framework proposed can get the optimal solution with fewer iterations.

In order to verify the accuracy and efficiency of the proposed framework, the total resistance of Wigley at Fr=0.3 is optimized. The wave-making resistance coefficient is simulated by NMShip-SJTU and the fractional resistance coefficient is calculated by ITTC 1957 formula. The initial samples are 15, and 3 new samples are added in one cycle. Finally, the number of total samples is 24. Compared to the SBD method, the efficiency of hull optimization is improved by more than 33.33% with a better resistance performance.

This paper only studies the principle of Bayesian optimization and its simple application in hull form optimization and does not pay attention to the application in more complex optimization problems. In the future, the Bayesian optimization will be improved and applied to multi-objective hull form optimization to further prove its accuracy and efficiency. Besides, how many sample points to add in each iteration is needed to be further discussed.

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