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Effect of density stratification on a self-propelled generic submarine near the free surface

*Linjie Li[1], Yangjun Wang[2], Liushuai Cao[1], Decheng Wan[1]**

[1] Computational Marine Hydrodynamics Lab (CMHL), School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai, China
[2] College of Advanced Interdisciplinary Studies, National University of Defense Technology, Nanjing, China

*Corresponding Author

ABSTRACT

The self-propelled Joubert BB2 submarine near the free surface is studied in homogeneous, strongly stratified, and linearly stratified fluids respectively. The differential effects of the flow field are analyzed under varying submergence conditions across these three kinds of fluids. The findings indicate that when the submarine operates very close to the free surface, the flow field results in different fluids are nearly identical. However, as the dive depth increases, the differences in flow field perturbations between strongly and linearly stratified fluids become more pronounced compared to a homogeneous density fluid, under these conditions, accounting for density stratification becomes essential.

KEY WORDS: Submarine; self-propulsion; stratified Fluid; wakes; free Surface.

INTRODUCTION

The successful completion of underwater operation tasks by new marine equipment such as "Jiaolong" and "Dream" marks the unprecedented development and attention to China's underwater vehicles, creating opportunities for China's ambition to advance into the deep sea. As a common underwater model, SUBOFF and Joubert BB2 submarines have therefore received the attention and research of many scholars.

It is well known that submarines traveling under the free surface are subject to greater drag as well as lift and pitching moments relative to deep-water environments. Amiri et al. (2018) investigated how the free surface affects the hydrodynamics of a shallowly submerged submarines, and Sudharsun et al. (2022) investigated Bernoulli's hump for submarines moving underwater. Dong et al. (2022) investigated the hydrodynamic performance of a submarine navigating near the free surface with long-crested waves, analyzing the effects of irregular waves and underwater depth. However, the effect of propeller rotation on the flow field was not considered in these studies.

A number of self-propulsion research efforts have been conducted on the DARPA Suboff geometry. Liefvendahl and Tröeng (2011) used LES to study the periodic variation of propeller blade loads on a submarine. Chase and Carrica (2013) computed the application of submarine propeller to self-propulsion, and showed that the same method applied to surface ships can be used to calculate the self-propulsion of a submarine. The results showed that the same method applied to surface ships can be used to calculate self-propulsion in submarines. Sezen et al. (2018) used the RANS method to study self-propulsion of Suboff and found that the body force method estimated lower propulsive efficiency and higher transferred power, while the hull was more efficient. Lungu (2022) studied the numerical simulation of self-propelled Suboff working in deep water and near free surface based on DES, revealing the free surface working regimes.

Unlike the slender hull structure of the Suboff submarine, the Joubert BB2, a generic submarine, is shorter and fatter. Li et al. (2015a) first put forward the effects of the free surface on the performance of the Joubert BB2 submarine with a propeller, showing that operating closer to the water surface resulted in higher propeller loads and lower efficiency. Carrica et al. (2019) comparatively analyzed submarine self-propulsion in calm water and waves, found that hull/free surface interactions lead to wake fluctuations. Li et al. (2021) evaluated the effect of free surface on the stern propeller performance and showed that the free surface increases the submarine self-propulsion point.

The effect of density stratification on submarine self-propulsion has not been considered in the above studies, whereas the density of seawater varies along the depth direction in the actual ocean due to temperature, salinity, and other factors. There have been some studies considering the effect of density stratification on the submarine hydrodynamics wake. Ma et al. (2019) studied the internal and free surface waves induced by the Suboff in a strongly stratified fluid. Huang et al. (2022) compared and analyzed the difference between the wake and free surface characteristics of the Joubert BB2 submarine in a homogeneous and linearly stratified fluid. Cao et al. (2023) numerically simulated the hydrodynamic performance and wake of Suboff in continuously stratified fluid. However, these studies still represent the fixed speed of a submarine by the incoming flow method and do not involve the rotation of the propeller. This paper devoted to analyzing the effect of density stratification on a self-propelled generic submarine near the free surface.

NUMERICAL METHODS

Governing equations

In this paper, in addition to the continuity and momentum equations under the Boussinesq approximation, the heat equation is also solved in the numerical simulation of the linear density stratified fluid (Cao et al., 2021), and the governing equations are shown in Eqs. (1) to (3) below.

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\boldsymbol{u} + \beta \boldsymbol{g} T' + \boldsymbol{f} \tag{2}$$

$$\frac{\partial T'}{\partial t} + (\boldsymbol{u}\cdot\nabla)T' + \gamma u_z = \alpha\nabla^2 T' \tag{3}$$

where $\boldsymbol{u}$, ρ, P, υ, α, β, $\boldsymbol{g}$, T', and $\boldsymbol{f}$ respectively denote velocity, density, pressure, kinematic viscosity, thermal diffusivity, volume expansion coefficient, gravitational acceleration, temperature fluctuation, and large-scale forces.

The thermocline model

In this paper, the density is set as a polynomial function of temperature, and the linear stratification of the density is achieved by setting the temperature to vary with the depth direction, and the expressions for the temperature T and the density are shown as Eqs. (4) and (5).

$$T(z) = T_0 + c_1 \cdot z \tag{4}$$

$$\rho(z) = \rho_0 - c_2 \cdot z \tag{5}$$

Linear stratified fluids can be realized by coupling equations (1) to (5). For homogeneous and strongly stratified fluids, c_1 and c_2 are set to 0 and the temperature is treated as a constant.

Turbulence modeling

In order to keep the equations closed, the SST k-ω turbulence model is chosen for the solution, and the transport equations for k and ω are as Eqs. (6) and (7).

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j}\left(\Gamma_k \frac{\partial k}{\partial x_j}\right) + G_k - Y_k + S_k \tag{6}$$

$$\frac{\partial}{\partial t}(\rho \omega) + \frac{\partial}{\partial x_i}(\rho \omega u_i) = \frac{\partial}{\partial x_j}\left(\Gamma_\omega \frac{\partial \omega}{\partial x_j}\right) + G_\omega - Y_\omega + S_\omega \tag{7}$$

where Γ_k, Γ_ω represent the effective diffusivity; G_k, G_ω represent the turbulent kinetic energy, Y_k, Y_ω represent the energy dissipation, S_k, S_ω. represent the source item (Menter, 1994).

The volume of fluid method

The VOF method is used in present numerical simulation to capture the fluid interfaces that are not compatible with each other. The VOF method deals with the interface problem by calculating the volume fraction occupied by different fluids in each grid.

$$\frac{\partial r}{\partial t} + \vec{u} \cdot \nabla r = 0 \tag{8}$$

$$r^{(k)} = V^{(k)} / \sum_{k=1}^{n} V^{(k)} \tag{9}$$

The interface between the two fluids is defined in the range $0 < r < 1$. Depending on the different values of the volume fraction r, we can distinguish the position of the interface.

PI-controller

The rotational speed of the submarine propeller at self-propulsion point is determined by the PI controller method, which continuously adjusts the rotational speed according to the difference between the target speed and current speed of the submarine during numerical iterative solving process. The expression of the PI controller is Eq. (10).

$$R(t) = R_0\left(1 + K_p \cdot error + K_i \cdot \sum error\right) \tag{10}$$

Where R(t) is the rotational speed at different times, R_0 is the initial value of the rotational speed, error is the target speed of the submarine minus the current speed, K_p is the gain of the proportional component, and K_i is the gain of the integral component.

COMPUTATIONAL SETUP AND VALIDATIONS

The Joubert BB2 submarine model

The geometric object studied in this paper is the Joubert BB2, a generic submarine designed by Joubert and adapted by MARIN in the Netherlands. The geometry is shown in the Figure 1, the main parameters are shown in the Table 1.

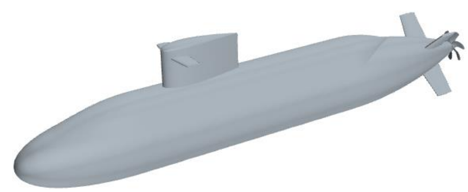


Fig. 1 The Joubert BB2 geometry

Table 1. Main parameters of the Joubert BB2 (model scale 1:18.348)

Parameters	Symbol	Value (m)
Overall length	L	3.826
Beam	B	0.523
Draft to Duck	D_d	0.578
Draft to Sail top	D_s	0.883
Diameter of propeller	D_p	0.273

Computational domain and discretized mesh

This study adopts a computational domain that follows the motion of the submarine, significantly reducing the length of the computational domain compared to traditional methods. The total length of the computational domain is 3L, with 1L in front of the submarine, 2L behind it, and 2L on each side. The top boundary of the computational domain is set as a pressure outlet, while all other boundaries are configured as velocity inlets, ss shown in the Figure 2.

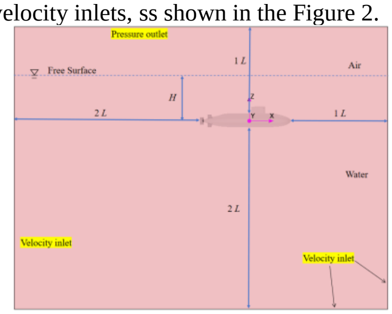
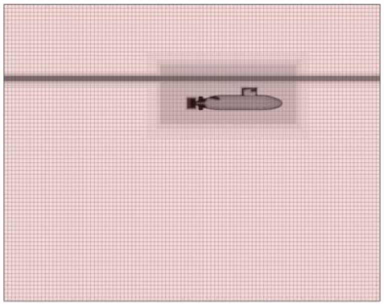
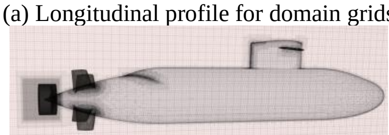


Fig. 2 The computational domain and boundary conditions

The trimmed cell mesh was used to discretize the computational domain, with mesh refinement applied near the free surface region and around the propeller. Boundary layer meshes were added on the submarine surface to capture near-wall flow phenomena. The final mesh consists of approximately 16 million cells. The mesh results are shown in the Figure 3.



(a) Longitudinal profile for domain grids

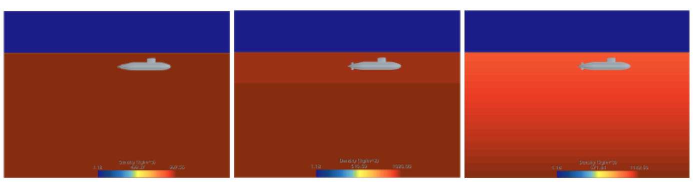


(b) Grids near the submarine

Fig. 3 Mesh details for the domain

Initial density fields for the stratification simulation

In this paper, the density at the center of mass of the submarine is uniformly set to 997.56 kg/m^3, and the position of the free surface H determines the interface between air and water. When the fluid is strongly stratified, 0.32 L below the center of mass of the submarine is set to be heavy-density fluid with the density of 1020 kg/m^3. The coefficients c_1, c_2(in Eqs. 4 and 5) are 1.5 and 18.3 respectively, when the fluid is continuously stratified. The density fields of the three kinds of fluid at the initial moment are shown in the Figure 4.



(a) Homogeneous (b) strongly stratified (c) linearly stratified

Fig. 4 Initial density fields for the three kinds of fluids

Numerical validation

To verify the reliability of the submarine's self-propulsion results in the homogeneous fluid, the experimental results under deep-water conditions from Overpelt (2015) were used for comparison. The results are shown in the Table 1.

Table 1. Results of present and experiments

Parameters	Present	Experiment	Error
Velocity (m/s)	1.2	1.19	——
Rotating speed (rpm)	272	268	1.49%
Thrust (N)	24.92	26.6	-6.32%

Table 1 presents the comparison of propeller rotational speed and thrust between the current numerical results with the experimental results of Overpelt (2015). The error in propeller rotational speed is only 1.49%, and the error in thrust is still within 7%. Therefore, the numerical method used in this study is reliable for submarine self-propulsion simulations.

The results of surface waves induced by the motion of a submarine with a constant speed in a strongly stratified fluid have been compared with the results of liu et al (2021) in previous work (li et al. 2023), and the result is shown in Figure 5.

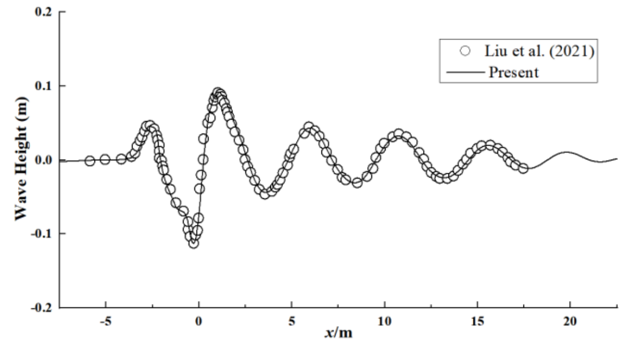


Fig. 5 Verification of surface wave height results in the strongly stratified fluid

As for the results of surface waves induced by the motion of submerged structures in a linearly stratified fluid have been published in previous articles of our research team (Cao et al., 2021; Huang et al., 2022; Cao et al., 2023), and more details can be found in references.

RESULTS AND DISCUSSIONS

Hydrodynamic results of Self-propulsion

The rotational speeds and thrusts corresponding to the self-propelled points of the Joubert BB2 submarine for the nine conditions considered in this paper are shown in the Table2. The speed of the model scale submarine is 1.2m/s, which corresponds to 10kn in the full scale, and the submerged depths include 0.17L, 0.21L, and 0.26L. The density stratification of the fluid is divided into 3 categories, which are homogeneous density fluid, strongly stratified fluid, and linearly continuous stratified fluid.

Table 2. Rotational speed and thrust results for 9 cases

Case	Depth/ L	Fluid	Rotating speed (rpm)	Thrust (N)
1	0.17	homogeneous	320.7	39.23
2		strongly stratified	315.2	39.39
3		linearly stratified	316.7	39.35
4	0.21	homogeneous	288.4	29.05
5		strongly stratified	281.4	29.89
6		linearly stratified	283.9	29.78
7	0.26	homogeneous	280.8	26.64
8		strongly stratified	277.6	26.67
9		linearly stratified	278.3	26.98

From the Table 2, it can be seen that the rotational speed and thrust are smaller as the dive depth increases, which is due to the fact that the farther away from the free surface, the smaller the rising wave resistance. And at the same dive depth, relative to the homogeneous density fluid, the changes of rotational speed and thrust in the two kinds of density stratification are within 3%, which indicates that when the submarine is self-propelled in near-surface conditions, the effect of density stratification on rotational speed and thrust is very small.

Free surface waves

Similar to a ship navigating on the water surface, a submarine operating near the free surface under self-propulsion generates a Kelvin-like wave system on the free surface. The surface wave patterns for 9 cases are shown in Figure 6, while the wave heights along the submarine's longitudinal profile are presented in Figure 7.

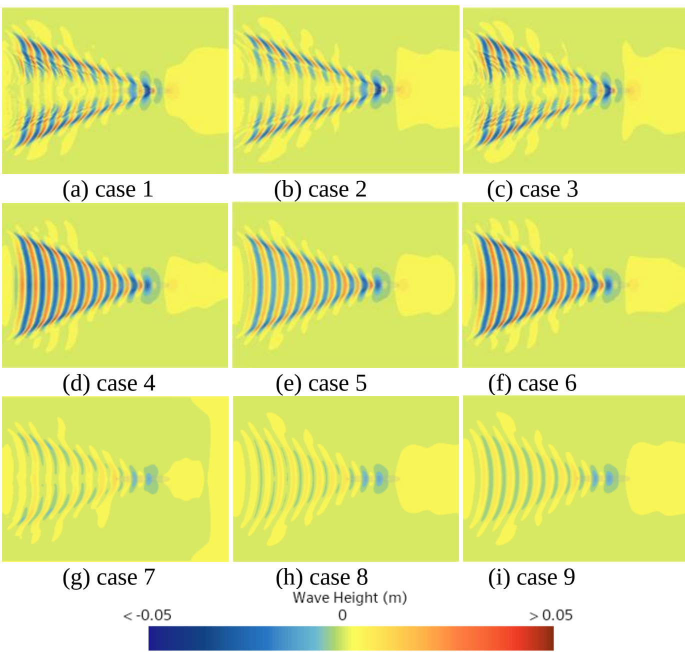


Fig. 6 Free surface wavee shapes for 9 cases

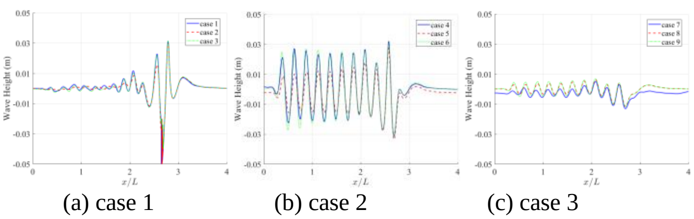


Fig. 7 Surface wave heights for 9 cases

From the Figures 6 and 7, it can be observed that when the submarine is at a depth of 0.17 *L* from the free surface, the surface wave patterns and wave height results in the three types of fluids are very similar, indicating that density stratification has no effect on the surface waves generated by the self-propulsion of the submarine under these conditions. When the depth increases to 0.21 *L*, the surface wave patterns remain almost identical, but the wave height in the strongly stratified fluid is slightly lower compared to the other two fluids. At a depth of 0.26 *L*, the differences in surface wave patterns are most pronounced. The surface waves generated in the uniform-density fluid are less complete than those in the other two cases, as evidenced by a significantly lower wave height along the submarine's centerline.

Internal waves

When a submarine operates under self-propulsion in a density-stratified fluid, it generates not only surface waves but also internal waves. In strongly stratified fluids, the internal waves are located at the interface between the two density layers. For comparative analysis, the internal waves in continuously stratified fluids are represented by the isosurface of a density of 1020. The internal wave patterns for six density stratification conditions are shown in Figure 8, while the wave height results along the submarine's longitudinal profile are presented in Figure 9.

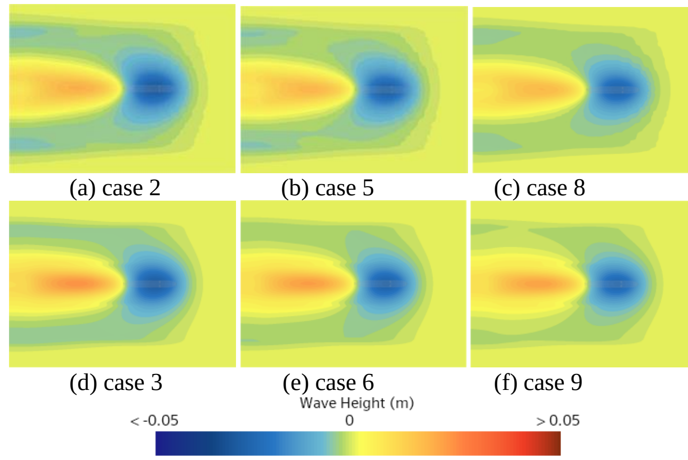


Fig. 8 Internal wave shapes for 6 cases

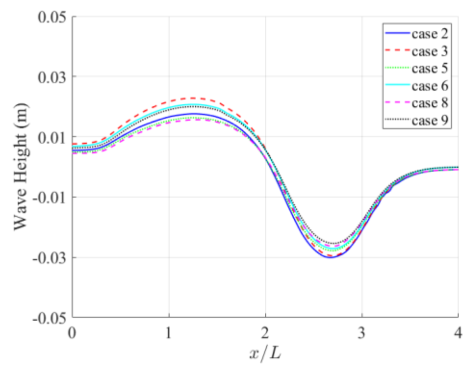


Fig. 9 Internal wave heights for 6 cases

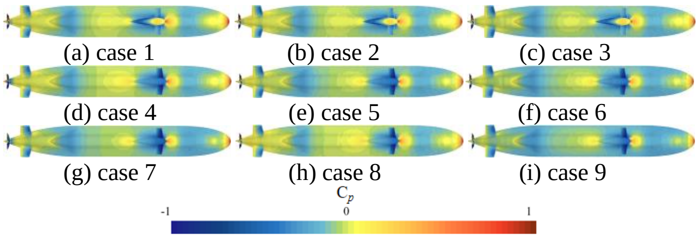
From Figure 8, it can be observed that for the Joubert BB2 submarine model moving at a speed of 1.2 m/s above the internal interface, the internal wave patterns consistently show a trough directly beneath the submarine and a crest forming behind it. Figure 9 reveals that when the distance between the submarine and the internal interface remains constant, the closer the submarine is to the free surface, the greater the numerical values of the internal wave crest and trough. Under the strongly stratified fluid condition, the trough values are slightly larger than those under the continuously stratified condition, while the opposite trend is observed for the crest values.

Pressure coefficients

Figures 10 and 12 show the pressure coefficient results on the upper and lower surfaces of the self-propelled submarine under 9 cases. Figures 11 and 13 present the pressure coefficient distributions along the centerline of the upper and lower surfaces, respectively. The expression for the pressure coefficient is Eq. (11). In the homogeneous and strongly stratified fluids, ρ_h is 997.56 kg/m³. In the linearly stratified fluid, ρ_h varies with depth.

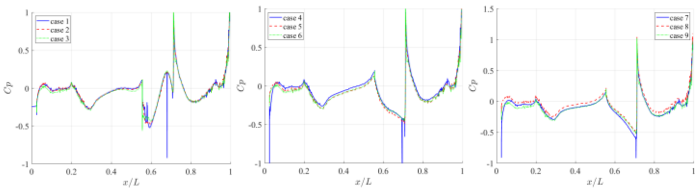
$$C_p = (P - P_h) / (\frac{1}{2} \rho_h U^2) \tag{11}$$

From the Figures 10~13, it can be observed that under the same depth condition, the effect of strongly stratified density fluid on the pressure coefficient of the submarine's upper and lower surfaces is minimal. For the linearly stratified fluid, the effect on the upper surface is also small, but it has a significant impact on the lower surface, as indicated by the pressure coefficient being higher than that in the uniform density fluid.



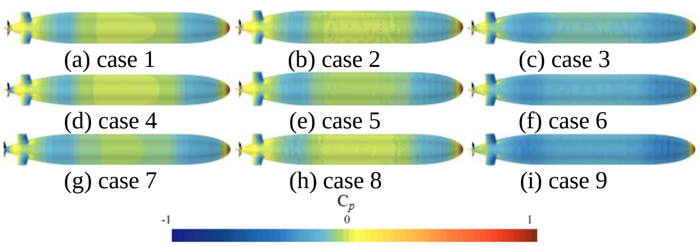
(a) case 1 (b) case 2 (c) case 3
(d) case 4 (e) case 5 (f) case 6
(g) case 7 (h) case 8 (i) case 9

Fig. 10 Pressure distribution on the upper surface of the submarine



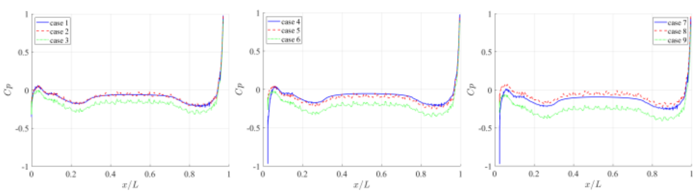
(a) H = 0.17 L (b) H = 0.21 L (c) H = 0.26 L

Fig. 11 The pressure coefficient along the longitudinal section of the submarine's upper surface



(a) case 1 (b) case 2 (c) case 3
(d) case 4 (e) case 5 (f) case 6
(g) case 7 (h) case 8 (i) case 9

Fig. 12 Pressure distribution on the lower surface of the submarine



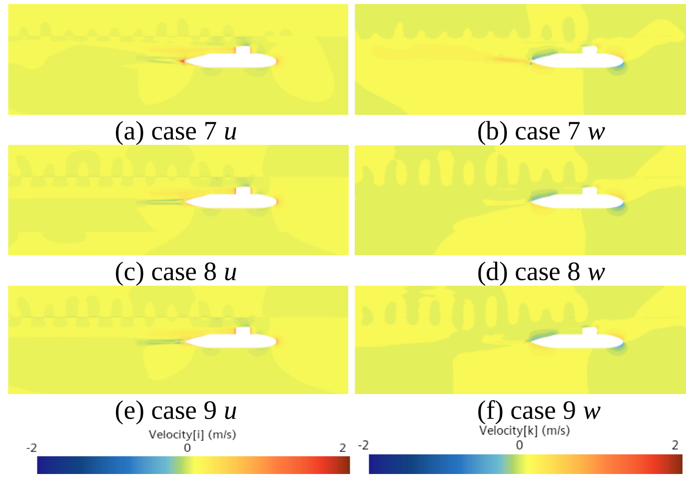
(a) H = 0.17 L (b) H = 0.21 L (c) H = 0.26 L

Fig. 13 The pressure coefficient along the longitudinal section of the submarine's lower surface

Velocity field and density fluctuation

To investigate the reasons behind the significant differences in surface wave results generated by the submarine's self-propulsion at a depth of 0.26L in the three types of fluids, an analysis of the velocity field for this submergence condition was conducted. The horizontal and vertical velocity field results along the submarine's longitudinal profile are shown in Figure 14.

From Figure 14, it can be observed that when the submarine is in a uniform fluid, the horizontal and vertical velocity fields behind the propeller exhibit an upward motion toward the water surface. However, this upward trend is absent in the strongly stratified and linearly stratified fluids, which explains the differences in the surface wave results shown in Figure 6(g)~ (i). This is consistent with More and Ardekani (2023), in density stratified fluids, the density gradient inhibits fluid motion in the vertical direction, compared to a homogeneous fluid where the wake is more likely to spread upstream.



(a) case 7 u (b) case 7 w
(c) case 8 u (d) case 8 w
(e) case 9 u (f) case 9 w

Fig. 14 Velocity field at the submergence depth of 0.26 L

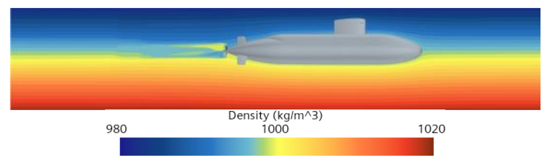


Fig. 15 Density field in the wake region of the linear stratified fluid

In addition, due to the suction effect of the propeller, mixing and oscillation of fluids with different densities are observed in the wake region within the linearly stratified fluid in Figure15. Low-density fluid rises, while high-density fluid sinks, causing the horizontal velocity in the wake to converge toward the propeller axis over a certain distance, as illustrated in the Figure 16.

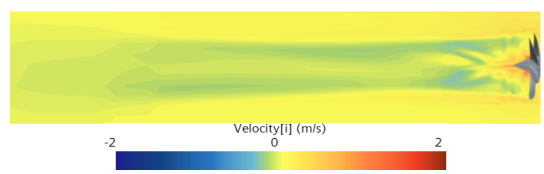


Fig. 16 The horizontal velocity field in the wake region of the linearly stratified fluid

CONCLUSIONS

This study conducted numerical simulations of the BB2 submarine model operating near the free surface under self-propulsion at a speed of 1.2 m/s in three different fluid environments: homogeneous fluid, strongly stratified fluid, and linearly stratified fluid. Simulations were performed for three different depths, resulting in a total of nine cases, to analyze the effects of density stratification on the flow field of a near-surface self-propelled submarine. The main findings are as follows:

(1) When the submarine operates near the free surface under self-propulsion, both the propeller rotational speed and thrust are minimally affected by strong stratification or linear stratification compared to the homogeneous fluid environment.

(2) If the submarine operates at a very shallow depth, such as the 0.17L cases calculated in this study, the surface wave results generated by self-propulsion are almost identical across the three kinds of fluids. However, as the depth increases, the differences become more pronounced. At a depth of 0.26L, the wave shape in the homogeneous fluid is less complete than in stratified environments and the wave

height along the submarine's longitudinal profile is significantly lower.

(3) Regarding the pressure coefficient on the submarine's surface, the values on the submarine's lower surface are higher in the linearly stratified fluid compared to those in the homogeneous fluid and strongly stratified fluid.

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