Numerical Study of Vortex-Induced Vibration of a Flexible Riser under Offshore Platform Horizontal Motion

Yuqi Zhang¹, Decheng Wan*¹, Changhong Hu²

¹Computational Marine Hydrodynamics Lab (CMHL), State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai, China
²Research Institute for Applied Mechanics, Kyushu University, Fukuoka, Japan
*Corresponding author

ABSTRACT

In this paper, numerical simulation of a flexible riser is carried out at first. Results are compared with the model test to verify the validity of the solver. Then, cases with different KC number with the same motion amplitudes are simulated to explain the effect of KC number in the VIV of the riser in the oscillatory flow. Typical characteristics of the vibration are observed including the modal transition and the “build-up-lock-in-die-out” process. Finally, through wavelet analysis, it was found that the main control modes at different positions were in three states when KC number was 84 and 168.

KEY WORDS: VIV; viv-FOAM-SJTU solver; CFD; platform horizontal motion; flexible riser.

INTRODUCTION

The development of deep-sea oil and gas resources relies on large-scale oil drilling platforms. The riser connects the seabed mineral deposits and the offshore operation platform. On the one hand, the riser needs to bear its own gravity and top tension, on the other hand, the vortex induced vibration (VIV) of the riser generated under the action of the shaking of the top platform and the current. When the excitation frequency is close to the natural frequency of the riser, the lock-in phenomenon will occur and the response amplitude will increase obviously. Under the influence of these factors, riser becomes the weakest link in offshore platform structure.

Many previous work have been done for the VIV problem. The research work mainly focuses on experimental research, semi empirical semi numerical simulation and CFD method. The results of the experimental study are the most reliable, but it is difficult to carry out the model test, because the test needs to invest a lot of manpower, material and financial resources. Semi empirical and semi numerical simulation methods simplify the original complex phenomena through a series of assumptions, which are widely used in industry, but the accuracy is poor. CFD method can show the details of motion that can not be captured by experimental research, and it is easy to analyze the formation mechanism of the phenomenon, but it has requirements for much computing resources.

Vandiver & Marcollo (2003) discussed the role of the additional mass. They thought that in the locking range, the additional mass decreased sharply with the increase of the reduced velocity, which resulted in the increase of the natural frequency of the cylinder. It is suggested that the locking region is widened in shear flow, because the change of the added mass to the natural frequency causes it to follow the frequency of the vortex shedding, so that the locking exists continuously.

De Wilde & Huijmsans (2004) tested the riser model with slenderness ratio of 767. The riser was placed horizontally and was under tension. When the tension is 1kN, the basic frequency of riser is about 1.5Hz. The Reynolds number is in subcritical region. They found that when the riser was locked in the VIV, the resistance increased and the resistance coefficient was between 1.5 and 2.7.

Chaplin (2005a) carried out the experimental study of the riser in stepped flow. The structure is shown in figure 1. The mass ratio of riser is 3, and the slenderness ratio is 467. The riser is placed in the stepped flow field, 55% of the top length is in the still water, the rest 45% is in the uniform flow, and the flow velocity can reach up to 1 m/s. It is observed that the maximum transverse vibration mode of riser can reach 8 orders, and the standard deviation of transverse vibration displacement is more than 50% of riser diameter. The resistance
coefficient of the riser measured in the test is about 120% of that of the static cylinder at the same Reynolds number.

Fig. 1: Layout of the experiment of Chaplin (2005)

Iwan & Blevins (1974, 1981) deduced the coupling equation of wake vibrator and structure motion according to the principle of conservation of momentum. The physical meaning of the model is clear, which reflects the hydrodynamic characteristics of the vortex induced vibration problem, and it is widely used in engineering. The empirical parameters in the equation are determined by the test results of forced vibration, so the model of wake vibrator depends on the selection of empirical coefficient, while the empirical parameters selected by different models of wake vibrator are quite different, so the prediction results of the same research object are also quite different.

Facchinetti (2004a) improved the dynamic characteristics of the wake vibrator model, and considered the coupling effects of displacement, velocity and acceleration on the wake vibrator. By comparing the prediction results of different coupling forms with the test results, it is found that the coupling of acceleration and vortex induced lift can quantitatively reflect the vortex induced vibration characteristics of rigid cylinder to a certain extent.

Srinil & Zanganah (2012) use double Duffing-Van der Pol oscillators to predict the two-way coupled VIV response. The model can predict the amplitude response of two-way VIV successfully.

Lucor (2001) further carried out a series of further research on vortex induced vibration by DNS method, and carried out numerical simulation on the vortex induced vibration of flexible riser with slenderness ratio greater than 500 in shear flow. The vibration response characteristics of risers in shear flow with linear and exponential changes are studied. The numerical results show that the lateral vibration excited in the linearly varying shear flow is locked in the third order, while the multi-modal vibration is found in the exponentially varying shear flow neutral tube, with the vibration modes up to 12-14.

Schulz & Meling (2004) combined the RANS method with the dynamic response of the finite element structure, established a multi slice method, and analyzed the fluid structure coupling of the flexible riser under the shear flow. By using this method, the numerical simulation and load analysis of vortex induced vibration of axially tensioned risers are carried out.

Duan M.Y. (2016) verified the standard problem of vortex induced vibration of slender flexible risers based on the RANS method. The numerical simulation accurately predicted the modes and amplitudes of risers in the transverse direction and flow direction.

Kamble & Chen (2016) simulated the vortex induced vibration of flexible riser with slenderness ratio of 1400 and 4200 respectively, and analyzed the fatigue damage of riser.

Constantinides & Oakley (2008) later used the three-dimensional model to analyze the riser with large slenderness ratio. The slenderness ratio is as high as 4200, which is in good agreement with the results of the test Deepstar-MIT Gulf Stream. The three-dimensional model can simulate the harmonic components observed in the experiment.

Wang et al (2014) built a fluid structure coupling solver based on the ANSYS MFX module. The vortex induced vibration of the riser with slenderness ratio of 943 in two kinds of uniform flow fields with flow velocity of 0.1m/s and 0.5m/s was simulated. When the flow velocity increased to 0.5m/s, the flow displacement was 20 times of 0.1m/s, and the lateral displacement was 6 times of 0.1m/s. The larger the velocity is, the weaker the axial correlation of vorticity field is.

**NUMERICAL METHOD**

In this paper, numerical simulations are conducted by the viv-FOAM-SJTU solver, which is developed based on the open source CFD software OpenFOAM.

**Computation of Fluid Fields**

For incompressible viscous fluid flow, the energy conversion caused by temperature change can be neglected. That is to say, the energy conservation equation can be ignored for incompressible viscous flow, but the continuity equation and momentum equation must be satisfied.

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0 \quad (1)
\]

\[
\frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} (\rho u_j u_i) = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} (2 \mu \varepsilon_{ij}) \quad (2)
\]

Where \(\rho\) refers to the fluid density, \(\mu\) refers to dynamic viscosity coefficient.

A direct computation of the three-dimensional flow field is a typical large-scale and high-strength problem, which will cost too many resources. A comparative approach is to select several strips along the vertical axis of the cylinder and apply the CFD method to solve the hydrodynamic force at each strip. It is considered that the fluid force of the cylinder can be calculated according to the length of the cylinder. The strip method is shown as figure 2.

**RANS with turbulence model** is the main method to solve the problem of viscous flow in engineering, because it can save lots of computing resources. The Reynolds-Averaged Navier-Stokes equations are used as follow:

\[
\frac{\partial \overline{u_i}}{\partial x_i} = 0 \quad (3)
\]
Where \( S_{ij} \) is the mean rate of strain tensor, and \(-\rho \overline{u'_i u'_j}\) represents turbulence effects which is often expressed by \( \tau_y \). \( \tau_y \) is a new unknown number and needs to introduce turbulence model to close equations. Based on Boussinesq hypothesis, we have the expression of \( \tau_y \) as follows:

\[
\tau_y = -\rho \overline{u'_i u'_j} = 2\mu \bar{S}_{ij} - \frac{2}{3} \rho k \delta_{ij} \tag{5}
\]

Where \( k = (1/2) u'_i u'_i \) is the kinetic energy of turbulence. The unknown parameter \( \mu \) can be obtained through SST \( k-\omega \) condition.

**Computation of Structure Field**

Infinitesimal beam element is shown in figure 3. Considering the balance of force and moment, the transverse motion of beam can be expressed as a fourth order differential equation.

\[
EI \frac{\partial^4 u(z,t)}{\partial z^4} - \frac{\partial}{\partial z} \left(T(z) \frac{\partial u(z,t)}{\partial z}\right) + m \frac{\partial^2 u(z,t)}{\partial t^2} = f(z,t) \tag{6}
\]

In the structural dynamics calculation module, the riser is regarded as the Euler-Bernoulli bending beam with both ends set as pinned, regardless of the change of top tension with time. Therefore, the structural motion governing equations of two risers can be expressed as a set of second-order ordinary differential equations as follows:

\[
\begin{align*}
[M][\ddot{x}] + [C][x] + [K][x] &= [F_x] \\
[M][\ddot{y}] + [C][y] + [K][y] &= [F_y]
\end{align*} \tag{7}
\]

\[
[M] = \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}, \quad [C] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad [K] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad [F_x] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad [F_y] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]

Where \( M \) refers to the mass of the riser, \( K \) refers to the stiffness matrix, \( C \) refers to the damping, \( F_x \) and \( F_y \) are the load vector of inline direction and cross flow direction respectively. The control equation is discretized to each element for solution, and the element mass matrix

\[
M^e = \frac{ml}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{9}
\]

Structural damping is introduced into the vibration system, and the damping model selected here is widely used in the engineering field. In the Reyleigh model, the control equation is as follows:

\[
M\ddot{u} + C\dot{u} + Ku = F \tag{10}
\]

**Fluid-solid Interaction**

At the beginning of each time step, the fluid force obtained from the solution of the fluid field is mapped to the nodes of the structural model, and then the movement of the riser is calculated. After the movement of the riser is obtained in the structural field, the grid deformation operation can be carried out in the fluid field and a new fluid field can be obtained. The process can be seen in figure 4.

**Problem Description**

To validate the viv-FOAM-SJTU solver, the numerical simulation of the model follows the Chaplin’s (2005 a) experiment, in which the riser is placed in the stepped flow field. 55% of the top length is in the still water, the rest 45% is in the uniform flow. The main parameters are shown in Table 1. The slice model is shown in figure 5 and figure 6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>D</td>
<td>0.028</td>
<td>m</td>
</tr>
<tr>
<td>Length</td>
<td>L</td>
<td>13.12</td>
<td>m</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>L/D</td>
<td>469</td>
<td>-</td>
</tr>
<tr>
<td>Length in fluid</td>
<td>L</td>
<td>5.94</td>
<td>m</td>
</tr>
<tr>
<td>Bending stiffness</td>
<td>EI</td>
<td>29.88</td>
<td>Nm²</td>
</tr>
<tr>
<td>Top tension</td>
<td>T</td>
<td>1610</td>
<td>N</td>
</tr>
<tr>
<td>Velocity</td>
<td>U</td>
<td>0.605</td>
<td>m/s</td>
</tr>
<tr>
<td>Mass Ratio</td>
<td>m*</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>Re</td>
<td>16940</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 4: viv-FOAM-SJTU diagram of fluid-structure interaction

![Diagram of fluid-structure interaction](image)

![Meshing of single slice](image)

![Three dimensional sketch of the slice](image)
Under the action of the flow, the flow displacement of each node of the riser increases gradually from 0. After a period of time, it will reach its equilibrium position. By connecting the corresponding equilibrium positions of each node, the equilibrium state of flow direction can be obtained, and the riser makes instantaneous micro amplitude vibration around the equilibrium state. After stabilization, connect the flow direction displacement of all nodes to the time average to get the equilibrium state as shown in Figure 7. The red line is the numerical calculation result, and the blue line is the experimental result.

![Fig.7: The mean in-line displacement](image)

After balancing, the maximum value of flow displacement and the corresponding position of the maximum value are compared with the test value. The position corresponding to the maximum value of the flow displacement is $Z/L = 0.367$. Compared with the experimental results, we accurately predict the position corresponding to the maximum value of the flow displacement, with an error of only 1.1%. The maximum displacement of flow direction is 3.072 D, which is close to the experimental value, with an error of 6.7%. So we can make sure that the viv-FOAM-SJTU solver is reliable.

RESULTS

VIV of the riser under single direction motion of platform

After completion of the verification work, the study is performed on the effect of KC number on the vortex-induced vibration of a flexible cylinder excited at the top end. The Keulegan–Carpenter (KC) number can be given by:

$$KC = \frac{\mu_{\text{max}} T_w}{D} = \frac{2\pi A}{D}$$  \hspace{1cm} (11)

In this paper, we compute three cases that the midpoint of the cylinder with KC number of 42, 84 and 168 respectively. Only the surge motion of the platform is considered. In each case, we all observe the “build-up-lock-in-die-out” process and find the dying-out process is obviously longer than the building-up process. The process is shown in Figure 8.

![Fig.8: The “build-up-lock-in-die-out” process](image)

The analysis of the riser is mainly from two aspects of cross flow direction and inline flow direction. According to the expression of KC number, we can infer that the bigger the KC number is, the larger the amplitude of the riser is. The amplification of amplitude is mainly shown by the displacement of riser. We can confirm this conclusion from the cross-flow amplitude of the intermediate node of the riser in Figure 9.

![Fig. 8: The “build-up-lock-in-die-out” process](image)

![Fig. 9: Non-dimensional cross-flow amplitude of the intermediate node of the riser in three cases](image)

From the contour of the crossflow power spectral density of the three cases shown in Figure 10, it shows the first-order mode when KC number is 42. The second-order mode appears when KC number is 84 and it is clearly that the first-order mode is the dominant mode. When KC number is 168, it shows third-order mode and the second-order mode is the dominant mode. The reason for this phenomenon is that the randomness and instability of turbulence may cause the instability of modal amplitude, which leads to the occurrence of multimode.

![Fig. 10: The contour of the crossflow power spectral density of the three cases](image)
vibration. We also observe second-order mode when KC number is 84 and first order mode when KC number is 42. These phenomenon verify the previous conclusion for Figure 10.

Fig. 11: The outline of the crossflow of three cases

Figure 12 shows subplots of the cross-flow wavelet analysis of the riser for different cases. When KC number is 84, we find dominant frequency has three states. Because the midpoint of the cylinder is the stationary point of the second-order mode, the phenomenon that second-order natural frequency dominates the vibration doesn’t appear. As the distance from the riser center increases, the dominant second-order frequency gradually increases. Since the bottom of the riser is not shaken, it is dominated by the low frequency at very few moments. As gradually away from the bottom, the low-frequency vibration increases, and the low-frequency gradually control the vibration process.

Fig. 12: The cross-flow wavelet analysis of the riser

The standard deviations of inline displacements and curvatures of the riser of three cases are shown in Figure 13. We can observe that the fluctuation of standard deviation of displacement curvature near the top of riser is obviously larger than that at the bottom. It is obvious that as long as the top of the vertical riser shakes, the relative shear flow is formed, and the upper velocity is large, so the curvature fluctuation is much larger.

Fig. 13: The standard deviations of inline displacements and curvatures of the riser of three cases

VIV of the riser in two directions of the platform motion

We choose Cartesian reference system to model the riser. The origin is at the bottom of the vertical riser, and the x-axis is parallel to the incoming flow direction, the z-axis is along the axis of the vertical riser, and the y-axis is perpendicular to both of them. The Cartesian reference system is shown in Figure 14.

In the numerical simulation analysis of vortex induced vibration of riser in two directions of the platform motion, the main parameters of riser are shown in Table 2, and the test conditions are shown in table 3. We guarantee that the KC number in x-axis direction is 84 and remains unchanged, and the KC number in y-direction is 0, 21 and 42 respectively.

Fig. 14: The Cartesian reference system

<p>| Table 2: Main parameter in numerical simulation |</p>
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ration</td>
<td>( m^* )</td>
<td>1.53</td>
<td>-</td>
</tr>
<tr>
<td>Diameter</td>
<td>( D )</td>
<td>0.024</td>
<td>m</td>
</tr>
<tr>
<td>Length</td>
<td>( L )</td>
<td>12</td>
<td>m</td>
</tr>
<tr>
<td>Bending stiffness</td>
<td>( EI )</td>
<td>10.5</td>
<td>( Nm^2 )</td>
</tr>
<tr>
<td>Top tension</td>
<td>( T )</td>
<td>500</td>
<td>N</td>
</tr>
<tr>
<td>First-order natural frequency</td>
<td>( f_{n1} )</td>
<td>1.08</td>
<td>Hz</td>
</tr>
<tr>
<td>Second-order natural frequency</td>
<td>( f_{n2} )</td>
<td>2.16</td>
<td>Hz</td>
</tr>
<tr>
<td>Third-order natural frequency</td>
<td>( f_{n3} )</td>
<td>3.25</td>
<td>Hz</td>
</tr>
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</table>
Table 3: Parameter for three cases

<table>
<thead>
<tr>
<th>Case</th>
<th>$KC_x$</th>
<th>$KC_y$</th>
<th>$U_{\text{max}}$</th>
<th>$V_{\text{max}}$</th>
<th>$\Phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>84</td>
<td>0</td>
<td>12</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>84</td>
<td>21</td>
<td>12</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>84</td>
<td>42</td>
<td>12</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 15 and Figure 16 show the outline of inline flow and the mean value of the inline displacement respectively. Generally, for the outline of inline flow, we will give its envelope to judge its vibration mode, and use the displacement mean value diagram to support the judgment of the envelope. It can be seen from the Figure 15 that when $KC_y = 42$ and $KC_y = 21$, the mode shape in the inline flow direction is of order 2, and when $KC_y = 0$, the mode shape in the inline flow direction is of order 1.

Figure 16 shows the position of the stagnation point well. We find that the larger the value of $KC_y$, the larger the value of the mean value of the displacement is, the more obvious the deviation from the equilibrium position is. Therefore, it can be inferred that if the KC number in one direction is fixed, the larger the KC number in the other direction is, the more obvious the vibration amplitude is.

Figure 17 and Figure 18 show the weighted power spectral density and relative displacement modal weight of the inline flow direction of the riser. We observe that when $KC_y = 42$ and $KC_y = 21$, the weighted power spectral density of the riser reflects that the second-order vibration mode is the main control mode, while the third-order and fourth-order vibration are almost absent. When $KC_y = 0$, the second-order vibration mode is the main control mode, accompanied by weak third-order and fourth-order vibration modes.

In Figure 18, it can be seen that when $KC_y = 42$ and $KC_y = 21$, the main control frequency is the second-order frequency, and when $KC_y = 0$, the main control frequency is the second-order frequency. By comparing three cases, we find that when the KC number in y direction is not zero, the vibration in y direction does not affect the main control frequency and mode.

Figure 19 shows the riser’s actual trajectories of three cases. We respectively intercept the positions from the bottom and make the actual trajectories line. When the KC number is not zero, the movement trajectories of riser presents "∞" shape, while when the KC number is zero, the movement trajectories is almost a straight line.

In all three cases, we find that the size of the trajectories contour near the bottom of the riser is significantly smaller than that at the top, because the velocity at the top of the riser is larger, the vibration
amplitude of the riser is more intense, and the maximum displacement at $3/4$ of the riser is about 3 times of the maximum displacement at $1/4$ of the riser. We have known that $KC_x$ is 84. It is found that the closer $KC_y$ is to $KC_x$, the fuller the two circles of "$\infty$" shape are. It is also found that the KC number is doubled from 21 to 42, and the maximum displacement at the corresponding position is increased by about 1.8 times, while $KC_x$ is increased from 0 to 21, and the maximum displacement at the corresponding position is increased by about 1.1 times, so the riser vibration amplitude and KC number are not simply linear relationship.

In Figure 19, we also find that the vibration trajectories curve of the riser is not very smooth, and there is a part of slight fluctuation, which is caused by low-frequency vibration. When the KC number is 0, it can be clearly observed that when approaching the origin, the contour width of the trajectories line is significantly larger than that of the two ends of the shaking direction of the riser. This is because there is a falling vortex at the tail of the riser when it passes through the origin. When the riser passes through the origin again, it meets the vortex. The vibration frequency of the two is close and the instability of the vortex causes the contour width of the trace line to be significantly enlarged. As it takes a long time for the riser to pass through the two ends again, and the vortex has dissipated in this time, the contour of the trajectory line is relatively regular.

![Fig. 19: Riser’s actual trajectories of three cases](image)

In the analysis for VIV of the riser under single direction motion of platform, we can clearly observe the “build-up-lock-in-die-out” process and find the dying-out process is obviously longer than the building-up process from the cross flow displacements diagram. We also find that the larger the KC number is, the larger the vibration amplitude is. We observe that the fluctuation of standard deviation of displacement curvature near the top of riser is obviously larger than that at the bottom. Through wavelet analysis, we found that when the KC number is 84 and 168, we observed that the main control modes of different positions of the riser are in three states.

In the analysis for VIV of the riser in two directions of the platform motion, we find that the larger the value of $KC_y$ is, the larger the value of the mean value of the displacement is and the more obvious the deviation from the equilibrium position is. Through the analysis of the riser’s actual trajectories, we observe when the KC number is not zero, the movement trajectories of riser presents "$\infty$" shape, while when the KC number is zero, the movement trajectories is almost a straight line. We also find that there is a part of slight fluctuation in the vibration trajectories curve of the riser, which is caused by low-frequency vibration.

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