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Nonlinear Seakeeping Solution near the Critical Frequency

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Applications of Nonlinear Wave Hydrodynamics

Calculation of wave induced loads of ships and offshore structures is a classical problem.

- For most cases, linear theory is valid under small wave steepness assumption.
- Nonlinearity is important for bodies with large-amplitude motions or in steep waves or solutions obtained by linear theory is trivial or singular.
- For instance, a body moving with forward speed U and oscillate with angular velocity ω is often approximated as source points with linearized velocity potential (Haskind 1954):

$$\phi(x, z) \sim \frac{\epsilon}{\sqrt{1 - 4\tau}}$$

Here ϵ = wave steepness;

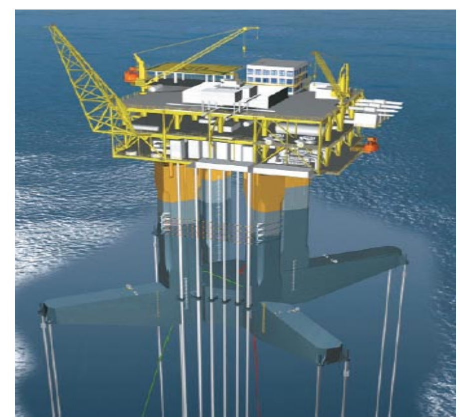
$\tau \equiv \frac{U\omega}{g}$: when $\tau \to \frac{1}{4}$, the solution becomes infinity, which is known as the **critical frequency**.

Applications of Nonlinear Wave Hydrodynamics

- Stability and global performance analysis of ship with forward speed:
  - The combination of ship velocity U and the encounter wave frequency ω_e satisfies $\tau = \frac{U\omega_e}{g} \approx \frac{1}{4}$.

- Hydrodynamic loads on offshore structures (e.g. TLP, FPSO, LNG) due to current-wave induced motions:
  - The average current speed in Gulf of Mexico is $U_{current} = 0.3{\sim}1.2\ m/sec$ (Data from NODC);

    the natural period of TLP in heave mode is $T_{heave} = 2{\sim}3\ sec$ (Johannessen, 2006), leading to $\tau \equiv \frac{U\omega}{g} = \frac{2\pi U_{current}}{gT_{heave}} \approx \frac{1}{4}$



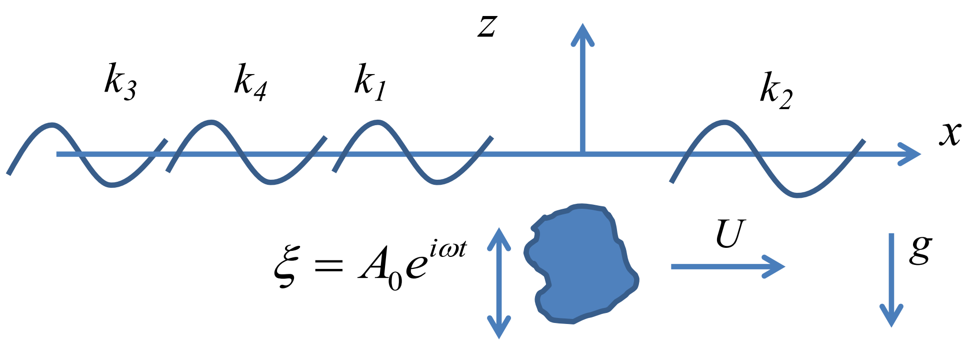
Research Purposes

- Understand the role of body geometry and nonlinearity in the seakeeping solution near the critical frequency with $\tau \equiv \frac{U\omega}{g} = \frac{1}{4}$
- Develop theoretical analysis in the frequency domain to understand the effect of nonlinearity on the seekeeping solutions for a two- and three-dimensional bodies
- Apply independent time-domain numerical nonlinear simulations to validate the theoretical analysis

1. LIU, Y. & YUE, D. K. P. 1993 On the solution near the critical frequency for an oscillating and translating body in or near free surface. J. Fluid Mech. **254**, pp 251–266.
2. Liu, Y. & Yue, D. K. P. 1996 On the time dependence of the wave resistance of a body accelerating from rest. J. Fluid Mech. **310**, pp 331-363
3. LI, C. & LIU, Y. 2018 On the weakly nonlinear seakeeping solution near the critical frequency. J. Fluid Mech. **846**, pp 999-1022.

Background

- Wave generation by an oscillating body with a forward speed:



- $\boldsymbol{\tau < \frac{1}{4}}$,
  $\boldsymbol{k_1}$, $\boldsymbol{k_3}$, $\boldsymbol{k_4}$ = downstream waves;
  $\boldsymbol{k_2}$ = upstream wave

- $\boldsymbol{\tau > \frac{1}{4}}$,
  $\boldsymbol{k_3}$, $\boldsymbol{k_4}$ = downstream wave

- At Critical Frequency $\boldsymbol{\tau = \frac{1}{4}}$,
  $\boldsymbol{k_3}$, $\boldsymbol{k_4}$ = downstream wave;
  $\boldsymbol{k_1}$, $\boldsymbol{k_2}$ merge into a single wave with group velocity $V_g = U$. Wave energy cannot radiate away from the body.
  Linear solution at $\tau = \frac{1}{4}$ may be singular.

- Non-Dimensional Frequency:

$$\tau \equiv \frac{U\omega}{g}$$

Existing Knowledge

(A) Linear Solution for a Single Point Source (Haskind, 1954):

$$\phi(x,z) \sim \frac{\epsilon}{\sqrt{1-4\tau}} = \frac{\epsilon}{\delta}$$

- As $\delta = \sqrt{1-4\tau} \to 0$, the velocity potential becomes singular.

(B) Nonlinear Solution for a Single Point Source (Dagan & Miloh, 1982):

$$\phi(x,z) \sim \frac{\epsilon}{\sqrt{\delta^2 + \sqrt{\delta^4 + \epsilon^2}}}$$

- As $\delta = \sqrt{1-4\tau} \to 0$, the velocity potential remains finite, and is proportional to *O(*$\epsilon^{1/2}$*).*

(C) Linear Solution for Actual Bodies (e.g. Grue & Palm 1984)

- Frequency-domain numerical solution for a submerged circular cylinder indicates that the linear solution is O(ϵ) and finite. No theoretical proof was provided.

Q: (1) Proof of a finite linear solution for a real body?
(2) What is the effect of ***nonlinearity*** in the case for real bodies?

Linearized Boundary-Value Problem

$$\Phi^{*}(x, z, t) = \bar{\phi}(x, z) + \Phi(x, z, t) = \bar{\phi}(x, z) + \mathrm{Re}\{\phi(x, z)e^{i\omega t}\}$$

Field equation: $\phi_{xx} + \phi_{zz} = 0$

Linearized free-surface boundary condition:

$$(i\omega - U\frac{\partial}{\partial x})^2\phi + g\frac{\partial \phi}{\partial z} = 0 \qquad \text{on} \quad z = 0$$

Body boundary condition:

$$\frac{\partial \phi}{\partial n} = f(x, z) \qquad \text{on} \quad S_B$$

Deepwater condition:

$$\nabla\phi \longrightarrow 0 \quad as \quad z \longrightarrow -\infty$$

Green Function

Haskind (1954):

$$G(x,z;x',z') = G_0 + G_1 + G_2 + G_3 + G_4,$$

where

$$G_0 = \tfrac{1}{2}\{\ln[(x-x')^2 + (z-z')^2] - \ln[(x-x')^2 + (z+z')^2]\}$$

$$G_1 = \frac{\mathrm{i}\pi}{(1-4\tau)^{\frac{1}{2}}}\mathrm{e}^{k_1[-\mathrm{i}(x-x')+(z+z')]} + \frac{1}{(1-4\tau)^{\frac{1}{2}}}\int_0^\infty \frac{1}{m-k_1}\mathrm{e}^{m[-\mathrm{i}(x-x')+(z+z')]}\mathrm{d}m\,,$$

$$G_2 = \frac{\mathrm{i}\pi}{(1-4\tau)^{\frac{1}{2}}}\mathrm{e}^{k_2[-\mathrm{i}(x-x')+(z+z')]} - \frac{1}{(1-4\tau)^{\frac{1}{2}}}\int_0^\infty \frac{1}{m-k_2}\mathrm{e}^{m[-\mathrm{i}(x-x')+(z+z')]}\mathrm{d}m\,,$$

$$G_3 = \frac{-\mathrm{i}\pi}{(1+4\tau)^{\frac{1}{2}}}\mathrm{e}^{k_3[\mathrm{i}(x-x')+(z+z')]} + \frac{1}{(1+4\tau)^{\frac{1}{2}}}\int_0^\infty \frac{1}{m-k_3}\mathrm{e}^{m[\mathrm{i}(x-x')+(z+z')]}\mathrm{d}m\,,$$

$$G_4 = \frac{\mathrm{i}\pi}{(1+4\tau)^{\frac{1}{2}}}\mathrm{e}^{k_4[\mathrm{i}(x-x')+(z+z')]} - \frac{1}{(1+4\tau)^{\frac{1}{2}}}\int_0^\infty \frac{1}{m-k_4}\mathrm{e}^{m[\mathrm{i}(x-x')+(z+z')]}\mathrm{d}m\,,$$

$$k_{1,2} = \frac{\kappa}{8\tau^2}(1 - 2\tau \pm (1-4\tau)^{\frac{1}{2}})\ ; \qquad k_{3,4} = \frac{\kappa}{8\tau^2}(1 + 2\tau \pm (1+4\tau)^{\frac{1}{2}})\ ;$$

$$\tau \equiv U\omega/g \qquad\qquad \kappa \equiv 4\omega^2/g$$

Asymptotic Expansion of Green Function near $\tau = 1/4$

For convenience, we define $\delta^2 \equiv |1 - 4\tau|$. For $\delta^2 \ll 1$, we have

$$k_{1,2} = \kappa[1 + O(\delta)] , \qquad \delta^2 \ll 1$$

$$G_1 + G_2 = \frac{2\pi \mathrm{i}}{\delta} \mathrm{e}^{\kappa[-\mathrm{i}(x-x')+(z+z')]} + G' + O(\delta) , \qquad \delta^2 \ll 1 .$$

$$\frac{1}{4}G' + 1 = \kappa[-\mathrm{i}(x - x') + (z + z')]\mathrm{e}^{\kappa[-\mathrm{i}(x-x')+(z+z')]} \,\fint_{-\kappa}^{\infty} \frac{1}{m} \mathrm{e}^{m[-\mathrm{i}(x-x')+(z+z')]}\mathrm{d}m$$

G_3 and G_4 are regular and O(1)

Solution for a Submerged Body near $\tau = 1/4$

Using source formulation:

$$\phi(x,z) = \int_{S_B} \sigma(x',z')G(x,z;x',z')\mathrm{d}s'$$

Imposing the body boundary condition to find unknown source distribution σ:

$$\pi\sigma(x,z) + \fint_{S_B} \sigma(x',z')\frac{\partial}{\partial n}G(x,z;x',z')\mathrm{d}s' = f(x,z)\ , \qquad (x,z) \in S_B$$

In the neighborhood of $\tau = \frac{1}{4}$:

$$\pi\sigma(x,z) + \frac{2\pi\kappa}{\delta}(n_x + \mathrm{i}n_z)\mathrm{e}^{\kappa(-\mathrm{i}x+z)} \int_{S_B} \sigma(x',z')\mathrm{e}^{\kappa(\mathrm{i}x'+z')}\mathrm{d}s'$$
$$+ \fint_{S_B} \sigma(x',z')\tilde{G}_n(x,z;x',z')\mathrm{d}s' = f(x,z) + O(\delta)\ , \qquad \delta^2 \ll 1$$

We now define the Kochin function

$$\alpha \equiv \int_{S_B} \sigma(x,z)\mathrm{e}^{\kappa(\mathrm{i}x+z)}\mathrm{d}s$$

Then, $$\sigma(x,z) = -\frac{2\kappa\alpha}{\delta}(n_x + \mathrm{i}n_z)\mathrm{e}^{\kappa(-\mathrm{i}x+z)}$$
$$-\frac{1}{\pi}\fint_{S_B} \sigma(x',z')\tilde{G}_n(x,z;x',z')\mathrm{d}s' + \frac{f(x,z)}{\pi} + O(\delta)\ , \qquad \delta^2 \ll 1$$

$$\alpha = \frac{\delta}{\pi(\delta + 2\mathrm{i}\kappa\Gamma)}[\mathscr{F} - \int_{S_B} \sigma(x', z')P(x', z')\mathrm{d}s'] + O(\delta^2) ,$$

where the kernel P is given by

$$P(x', z') = \fint_{S_B} \mathrm{e}^{\kappa(\mathrm{i}x+z)} \frac{\partial}{\partial n}(G' + G_0)\mathrm{d}s ,$$

and the constants $\mathscr{F}$ and Γ are given by

$$\mathscr{F} = \int_{S_B} f(x, z)\mathrm{e}^{\kappa(\mathrm{i}x+z)}\mathrm{d}s, \qquad \Gamma = \int_{S_B} (-\mathrm{i}n_x + n_z)\mathrm{e}^{2\kappa z}\mathrm{d}s$$

For $\boldsymbol{\Gamma \neq 0}$,

$$\pi\sigma(x, z) - \frac{(n_x + \mathrm{i}n_z)}{\delta/2\kappa + \mathrm{i}\Gamma}\mathrm{e}^{\kappa(-\mathrm{i}x+z)} \int_{S_B} \sigma(x', z')P(x', z')\mathrm{d}s'$$
$$+ \fint_{S_B} \sigma(x', z')\tilde{G}_n(x, z; x', z')\mathrm{d}s' = F(x, z) + O(\delta) ,$$

where

$$F(x, z) = f(x, z) - \mathscr{F}\frac{(n_x + \mathrm{i}n_z)}{\delta/2\kappa + \mathrm{i}\Gamma}\mathrm{e}^{\kappa(-\mathrm{i}x+z)} = O(1) .$$

$$\boldsymbol{\sigma = O(1), \qquad \alpha = O(\delta)}$$

$$\phi(x, z) = \frac{2\pi\mathrm{i}\alpha}{\delta}\mathrm{e}^{\kappa(-\mathrm{i}x+z)} + \int_{S_B} \sigma(x', z')\tilde{G}(x, z; x', z')\mathrm{d}s' + O(\delta) = \mathrm{O}(1)$$

For $\boldsymbol{\Gamma = 0,\ \alpha = O(1),\ \sigma = O\left(\delta^{-1}\right),\ \phi =}$ unbounded

Body Geometry Parameter

$$\Gamma \equiv \int_{S_B} (-\mathrm{i}n_x + n_z)\mathrm{e}^{2\kappa z}\mathrm{d}s$$

With the use of the divergence theorem, we obtain immediately

$$\Gamma = 2\kappa \iint_B \mathrm{e}^{2\kappa z}\mathrm{d}S \ ,$$

where B is the (mean) body section. Since the integrand in (3.14) is positive definite, $\Gamma \neq 0$ if and only if the (submerged) body has non-zero cross-section area. The known singular solution for a point source turns out to be a special case of $\Gamma = 0$.

Comparison of Theoretical Solution with Direct Numerical Computation

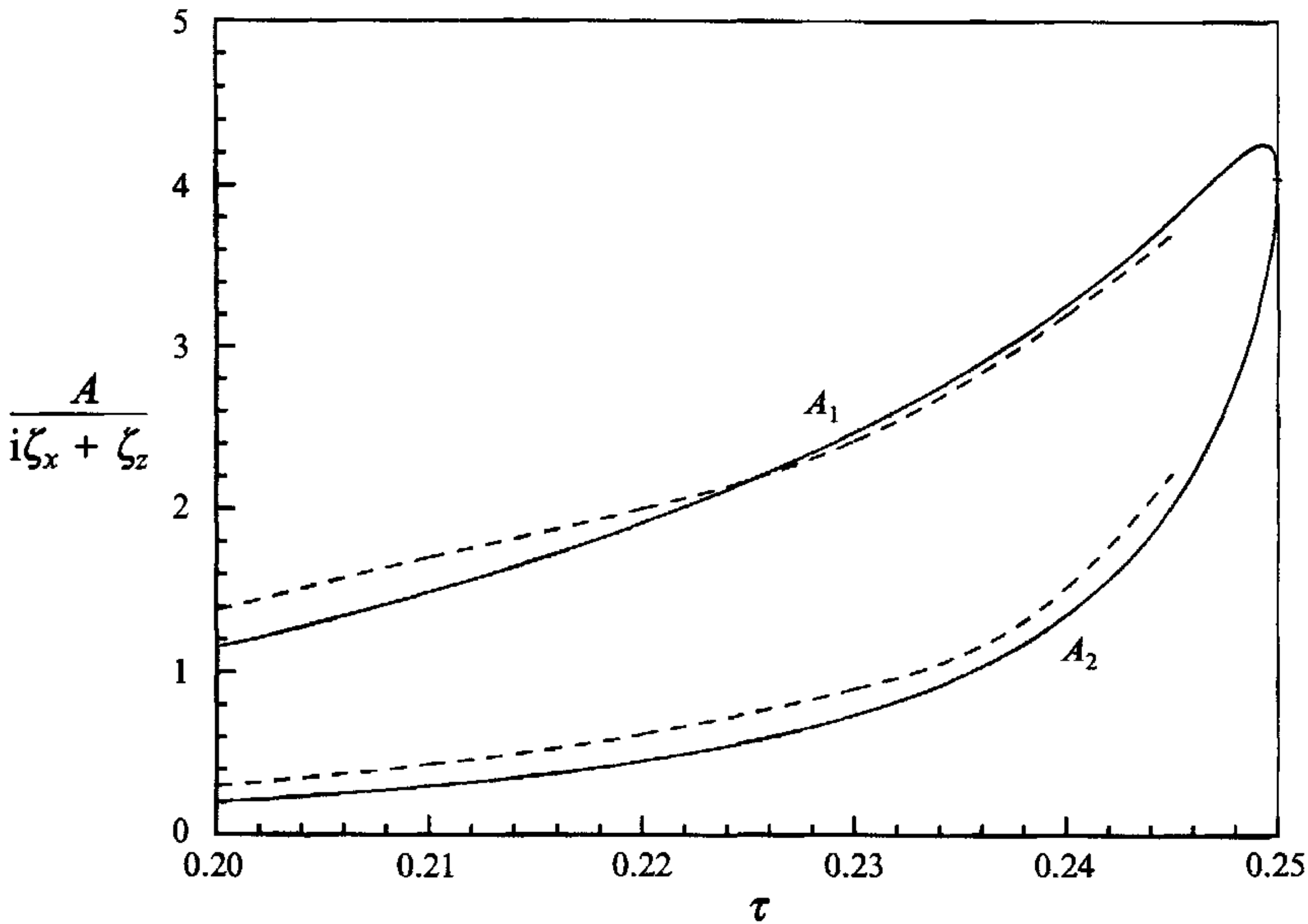


FIGURE 3. Amplitudes of the k_1 (upper branch) and k_2 (lower branch) waves radiated by the heave and sway oscillations of a submerged circular cylinder as a function of $\tau \equiv U\omega/g$. Asymptotic solution (5.13) (——); direct numerical calculations (Grue & Palm 1985) (- - -). ($F_r = U/(gR)^{\frac{1}{2}}$=0.4, h/R=2).

Theoretical Analysis of Nonlinear Effects in Frequency Domain

Step 1: Potential Flow Theory
(Governing Equations)

↓

Step 2: Perturbation Method
(Nonlinear FSBC by adding 3^{rd}-order Nonlinear Terms)

↓

Step 3: Combine the Nonlinear FSBC and
Body Boundary Condition (BBC)
to obtain the Boundary Integral Equation (BIE)

↓

Step 4: Solve BIE for
Nonlinear Potential Solution $\phi(x,z)$

Step 2—Nonlinear Free-Surface Boundary Condition

- Following Dagan & Miloh (1982), for a single source, assumed perturbation for potential: $\phi = \phi_1 + \phi_2 + \phi_3 + \ldots.$

The analysis showed: $\phi_1 \sim \frac{\epsilon}{\delta}$ $\phi_2 \sim \frac{\epsilon^2}{\delta}$ $\phi_3 \sim \frac{\epsilon^3}{\delta^5}$

When $\delta = \sqrt{1 - 4\tau} \to 0$: $\frac{\phi_3}{\phi_1} \sim \frac{\epsilon^2}{\delta^4}$ (Non-uniform Convergence)

Perturbation valid only when: $\delta^2 \sim |\epsilon|^\alpha,\ \alpha < 1$

- Add the **cubic term** to the free surface boundary condition (FSBC) :

$$\underbrace{\frac{\partial^2 \phi_1}{\partial t^2} - 2\frac{\partial^2 \phi_1}{\partial t \partial x} + \frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial \phi_1}{\partial z}}_{\text{Linear Terms}} + \underbrace{\frac{1}{2} \nabla \phi_1 \nabla (\nabla \phi_1 \nabla \phi_1)}_{\text{Cubic Term}} = 0 \qquad z = 0$$

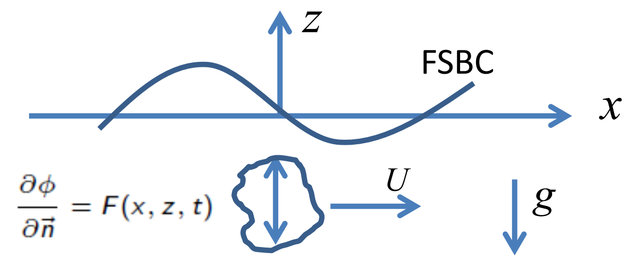
when $\delta = \sqrt{1 - 4\tau} \to 0$, the magnitude of the cubic term would be comparable with the leading order terms.

$\frac{\phi_3}{\phi_1} \sim \epsilon^2$ (Uniform Convergence)

Step 3—Apply Body Boundary Conditions

- Construct the potential solution for a submerged body into two parts:

$$\phi(x,z,t) = \underbrace{\int_{S_B} \sigma(s' t) ln(r/r_1) ds'}_{\substack{\psi(x,z,t) \\ \text{Source distribution} \\ \text{on body surface}}} + \underbrace{H(x,z,t)}_{\substack{\text{Surface wave effect} \\ \text{(free \& evanescent waves)}}}$$



Nonlinear FSBC: $H_{tt} - 2H_{tx} + H_{xx} + H_z + \frac{1}{2} \nabla H \nabla (\nabla H \nabla H) = -\psi_z \qquad z = 0$

BBC: $\frac{\partial \psi}{\partial n}(x,z,t) + \frac{\partial H}{\partial n}(x,z,t) = F(x,z,t) \quad on\ \bar{S}_B$

Here body forcing *F(x,z,t)* is related to the body velocity and is known.

- Consider steady harmonic oscillation problem: *i.e.* $F(x,z,t) = \Re\{f(x,z)e^{i\omega t}\}$

$H(x,z,t) = \Re\{h(x,z)e^{i\omega t}\} \quad \psi_z(x,z,t) = \Re\{P(x,z)e^{i\omega t}\} \quad \psi(x,z,t) = \Re\{X(x,z)e^{i\omega t}\}$

$$\begin{cases} \text{Laplace Eq.:} & h_{xx} + h_{zz} = 0 \\ \text{FSBC:} & -\omega^2 h - 2i\omega h_x + h_{xx} + h_z + \frac{1}{2}\nabla h \nabla(\nabla h \nabla h) = -P(x,0) \quad z = 0 \\ \text{BBC:} & X_n(x,z) + h_n(x,z) = f(x,z) \quad (x,z) \in S_B \\ \text{Bottom BC:} & \nabla h \to 0 \quad z \to -\infty \end{cases}$$

Step 4—Nonlinear Solution

- Velocity potential could be obtained:

$$h_{NL}(x,z) = \frac{i2\mathcal{F}(k_c)}{\sqrt{\delta^2+4d}+i2k_c\Gamma} e^{-ik_{1g,2g}x+|k_{1g,2g}|z}$$

Body volume effect: $\Gamma = \int_{S_B} (-in_x + n_z) e^{2k_c z'} ds' \sim O(1);$

Body forcing : $\mathcal{F}(k_c) = \int_{\bar{S}_B} [f(x,z) - \bar{\mathcal{X}}_n(s)] e^{ik_c x + k_c z} \, ds. \sim O(\epsilon)$

1. $\Gamma = 0$: $\phi \sim \frac{\tilde{f}}{\sqrt{|\tilde{f}|}} \propto O(\epsilon^{1/2})$ at $\delta = 0$ (*i.e.* $\tau = ¼$), consistent with Dagan and Miloh (1982);
2. $\Gamma \neq 0$: total nonlinear solution remain finite, $\phi \propto O(\epsilon)$ at $\delta = 0$ with

$$k_{1g} = \frac{1}{2}(1-2\tau+\sqrt{\delta^2+4d}); \quad k_{2g} = \frac{1}{2}(1-2\tau-\sqrt{\delta^2+4d})$$

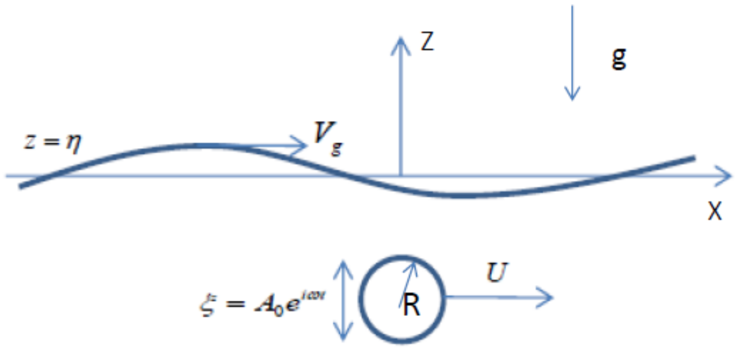
$$d_{1,2} = -\frac{1}{2}\left(\frac{\delta^2}{4}+k_c^2\Gamma^2\right) \pm \frac{1}{2}\left[\left(\frac{\delta^2}{4}+k_c^2\Gamma^2\right)^2 + 4k_c^4|\mathcal{F}(k_c)|^2\right]^{\frac{1}{2}}$$

- Total nonlinear corrections $\propto O(\epsilon)$:

$$h_{COR2}(x,z) = |h_{NL}| - |h_L| \sim \frac{2\mathcal{F}(k_c) e^{k_c z}(e^{\pm \nu x} - 1)}{\delta^2 + 4k_c^2\Gamma^2} \qquad \nu = \mathrm{Im}(\delta^2 + 4d_2)^{1/2}/2.$$

- ν represents the ***nonlinear correction***: spatial damping coefficient to the resonance waves A_1 , A_2.
- **Nonlinear Correction due to cubic interactions is of the same order of magnitude as the leading-order linear solution.**

Analytical Nonlinear Solution for Submerged Circular Cylinder



- Approximation method: treat the potential of steady flow past the circular cylinder as that around a dipole (e.g. Grue & Palm 1984).

$$\begin{cases} \tilde{f} = \pi k_c R^2 e^{-k_c H}(\omega + k_c U)(-\xi_x + i\xi_z) \\ \Gamma = 2\pi R e^{-2k_c H} I_1(2k_c R) \end{cases}$$

- The d term will take the form as:

$$d_2 = -\frac{1}{2}\left[\frac{\delta^2}{4} + 4\pi^2 k_c^2 R^2 e^{-4k_c H} I_1^2(2k_c R)\right] - \frac{1}{2}\left\{\left[\frac{\delta^2}{4} + 4\pi^2 k_c^2 R^2 e^{-4k_c H} I_1^2(2k_c R)\right]^2 + 16 k_c^8 \pi^2 R^4 e^{-2k_c H}(\xi_x^2 + \xi_z^2)\right\}^{\frac{1}{2}}.$$

- The resonant waves $A_{1,2}(x) \sim e^{\nu_{1,2} x}$ with $\nu_{1,2} = \pm\frac{1}{2}\Im(\delta^2 + 4d_2)^{\frac{1}{2}}$

Verification of Analytical Solution for Submerged Circular Cylinder by Comparison with Independent Time-Domain Nonlinear Simulation

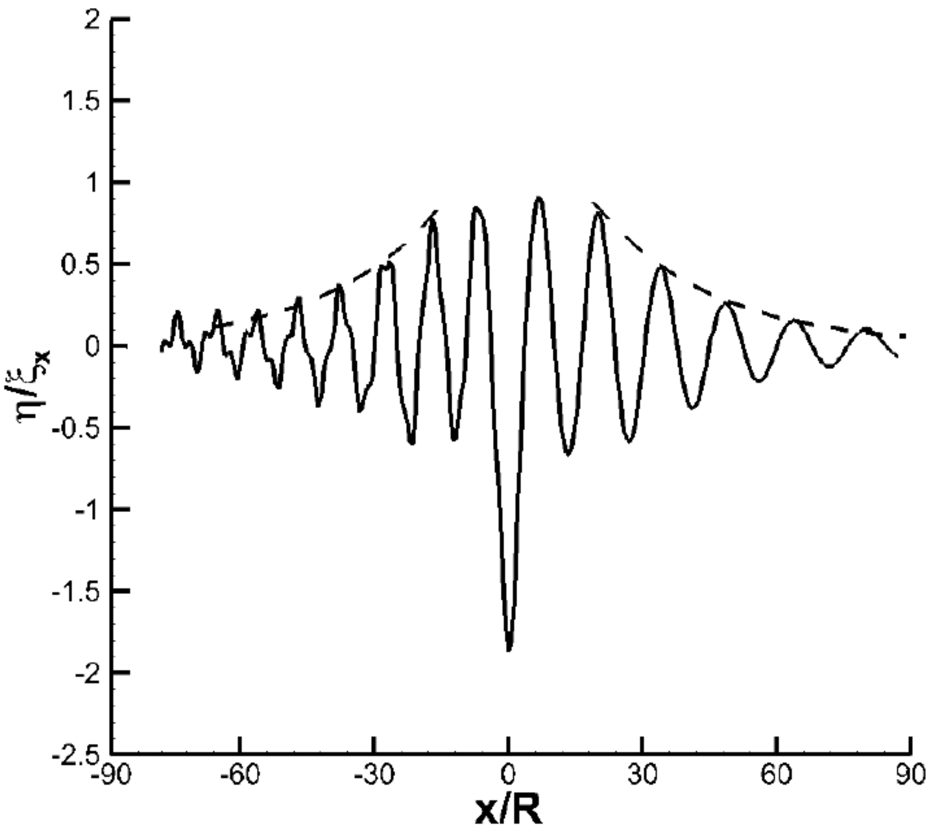


FIGURE 2. Representative instantaneous free-surface shape of steady-state wave profile above the submerged circular cylinder (with its center at x=0) obtained by the time-domain nonlinear simulation with M=3 (——). Analytical solution of the envelop of decaying wave amplitude of resonant waves at downstream ($x < 0$) and upstream ($x > 0$) of the body $A_{1,2}(x) \sim e^{\pm 0.022x}$ from equation (4.6) (- - -) is also plotted for comparison. (H/R=6, F_r=0.75, τ=0.25, ξ_x/R=0.05).

Verification of Analytical Solution for Submerged Circular Cylinder by Comparison with Independent Time-Domain Nonlinear Simulation

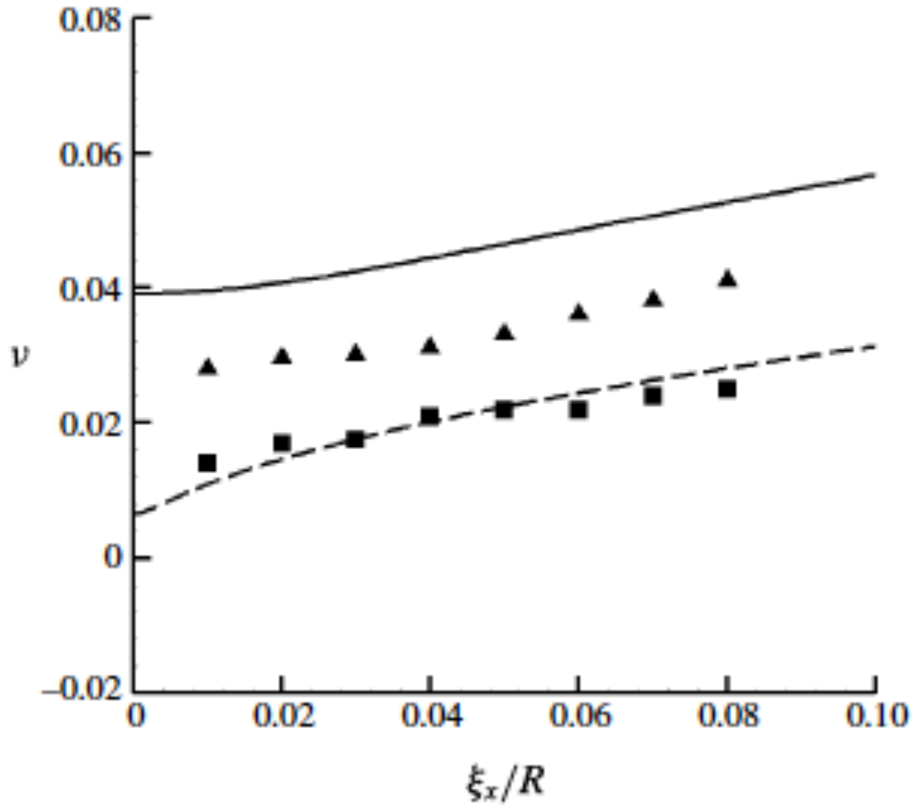


FIGURE 3. Comparison of the damping factor ν_2 of the resonant wave upstream of the body ($A_2(x)$) between the analytic solution from (4.6) with $H/R=4$ (——) and 6 (– – –), and nonlinear simulations ($M=3$) with $H/R=4$ (▲) and 6 (■) as a function of the surge motion amplitude ξ_x/R. (Here, $\tau=0.25$ and $F_r=0.75$.)

Verification of Analytical Solution for Submerged Circular Cylinder by Comparison with Independent Time-Domain Nonlinear Simulation

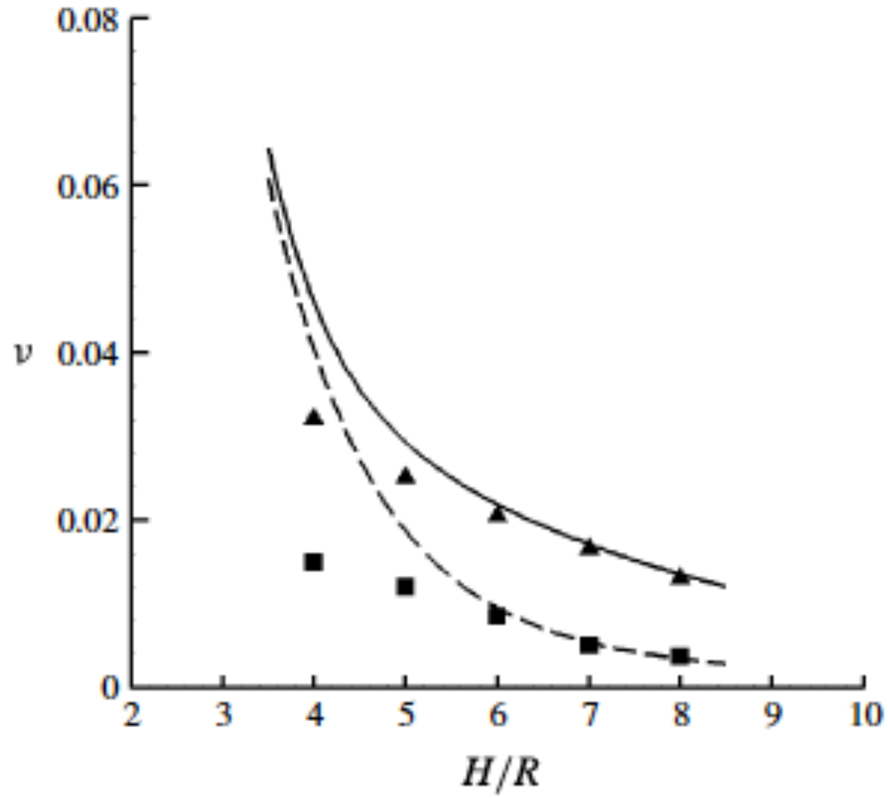


FIGURE 4. Comparison of the damping factor ν_2 of the resonant wave upstream of the body ($A_2(x)$) between the analytic solution from (4.6) at $\tau = 0.245$ (– – –) and 0.25 (——), and nonlinear simulations ($M = 3$) at $\tau = 0.245$ (■) and 0.25 (▲) as a function of body submergence H/R. (Here, $\xi_x/R = 0.05$ and $F_r = 0.75$.)

Time-Domain Nonlinear Simulation Confirms that Cubic Interactions Gives Leading-Order Contribution at the Critical Frequency

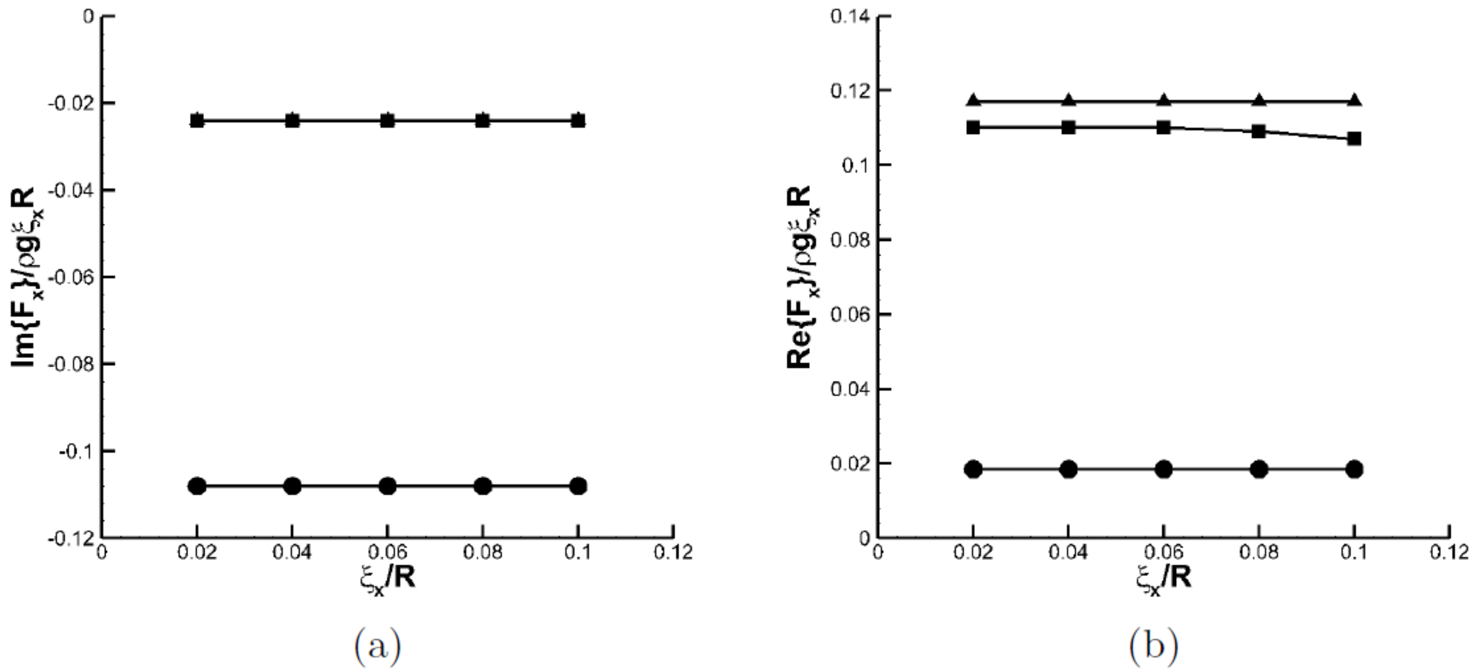


FIGURE 6. The imaginary part (a) and real part (b) of the radiation force in the horizontal direction on the cylinder at τ=0.25 as a function of the surge motion amplitude (ξ_x/R) obtained by nonlinear simulations with M=1 (●), 2 (▲), and 3 (■). The results with M=2 and 3 overlap each other graphically. (H/R=6 and F_r=0.75).

Added Mass and Damping Coefficient of S60 Ship at the Critical Frequency by Nonlinear Time-Domain Simulation

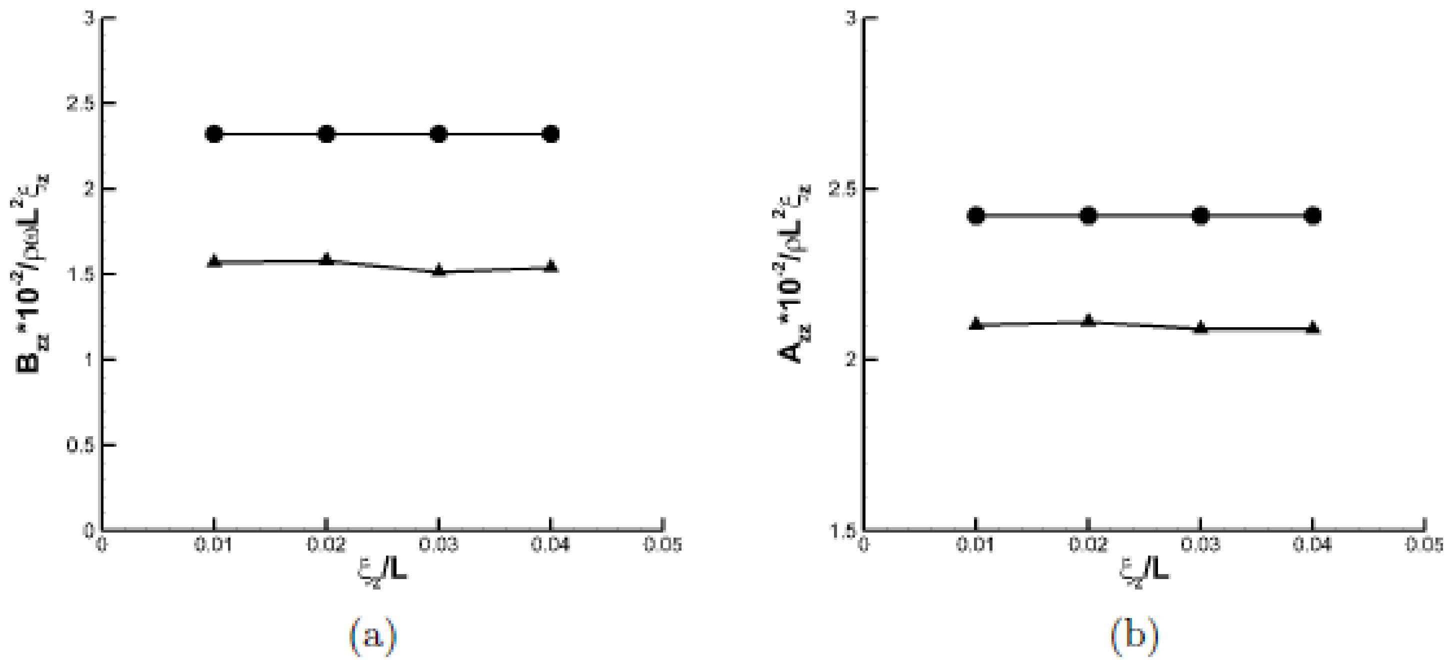


FIGURE 9. Heave damping coefficient (a) and added mass coefficient (b) of a Series 60 ship hull at the critical frequency τ=0.25 as a function of the heave motion amplitude ξ_z/L, obtained by linear (●) and fully-nonlinear (▲) numerical simulations (F_r=0.2).

Time Dependence of Wave Resistance of a Body Accelerating from Rest

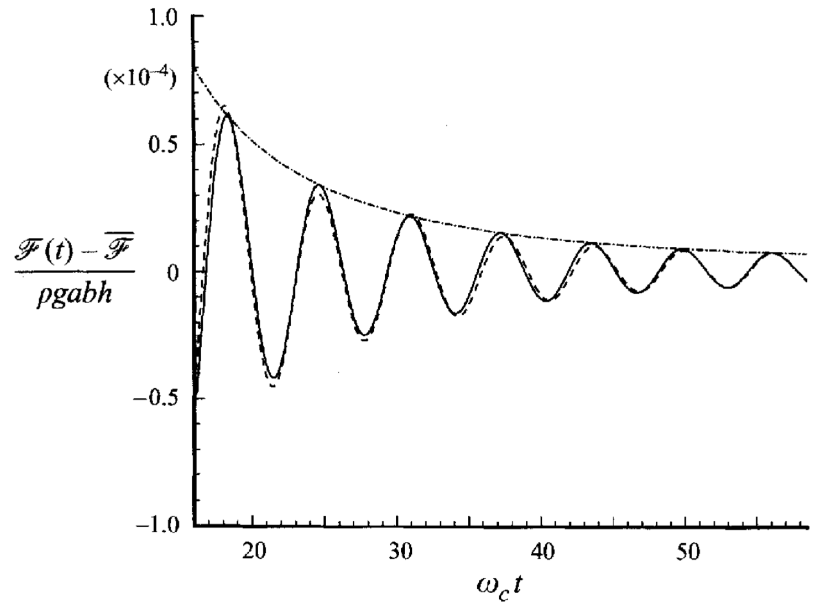
- For a 2D body:

$$\frac{\mathscr{F}(t)-\overline{\mathscr{F}}}{\rho g a^2}=\frac{a_1}{\omega_c t}+\frac{a_2}{(\omega_c t)^{3/2}}\cos(\omega_c t+a_3)$$ for t>> 1

- For a 3D body:

$$\frac{\mathscr{F}(t)-\overline{\mathscr{F}}}{\rho gabh}=\frac{a_1}{(\omega_c t)^2}+\frac{a_2}{(\omega_c t)^2}\cos(\omega_c t+a_3)$$

for t>> 1



Comparison between the asymptotic prediction (—) and direct time-domain simulation result (- - -) for the unsteady wave resistance on a Wigley hull.

Conclusion

For the general 2D and 3D seakeeping problems:

- when $\Gamma \neq 0$, the nonlinear correction due to self cubic interactions of resonant waves near the critical frequency is proportional to $O(\epsilon)$, which is in the same order as the leading-order (linear) solution.
- In the prediction of seakeeping solution near the critical frequency, the nonlinear cubic terms in the free-surface boundary conditions should be included since they will provide the leading-order contribution.

THANK YOU!

QUESTIONS??

Added Mass and Damping Coefficient of S60 Ship near the Critical Frequency by Nonlinear Time-Domain Simulation

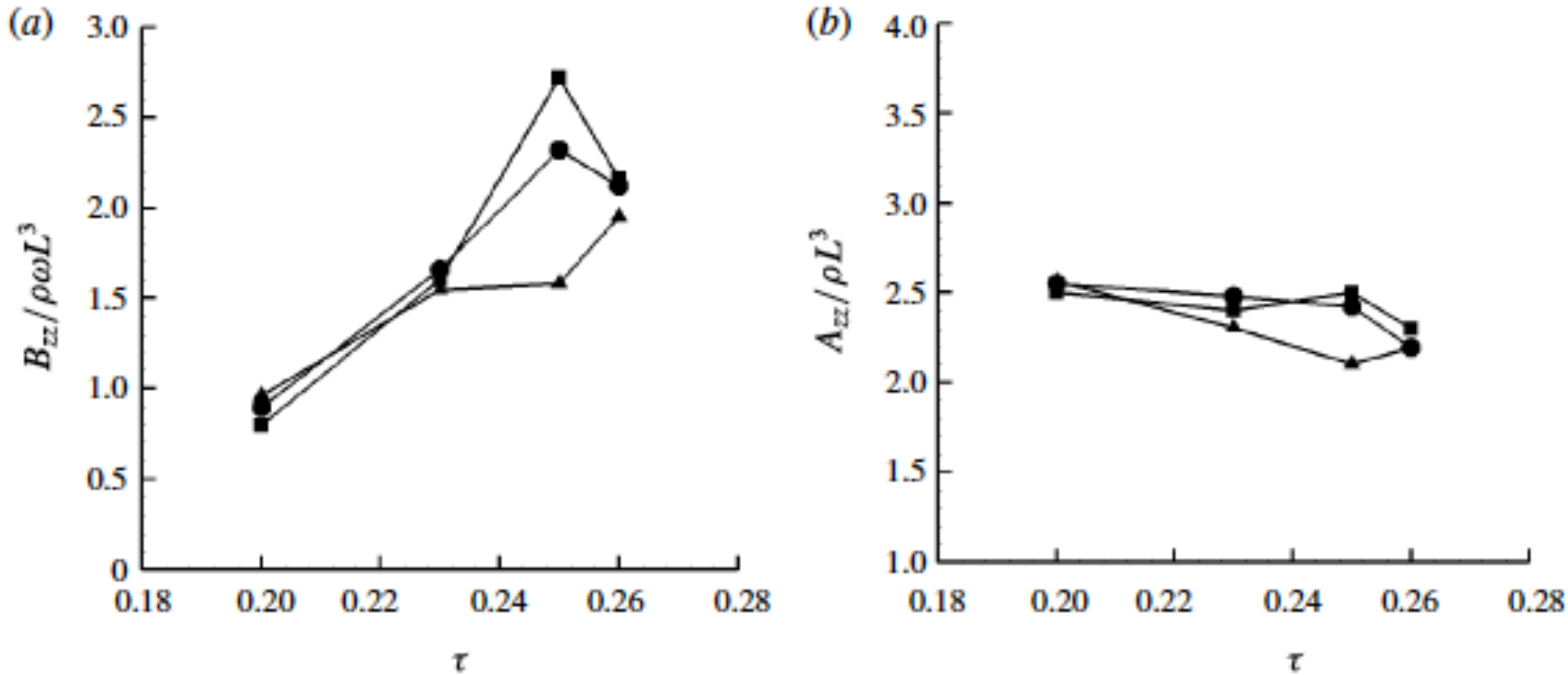


FIGURE 10. The heave damping coefficient (a) and added mass coefficient (b) of a S60 ship hull in the neighbourhood of the critical frequency $\tau = 0.25$ obtained by linear (●) and fully nonlinear (▲) numerical simulations. The linear numerical solution by Bingham (1994) (■) is shown for comparison. (Here, $F_r = 0.2$ and $\xi_z/L = 0.01$.)