Benchmark Computations on Motion Responses and Bow Waves of the Ship in Regular Waves

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ABSTRACT

The bow wave depending on the bow shape can be determined by relatively simple geometric parameters due to little influence of the downstream wave. Accurate prediction of the bow wave is essential for ship resistance and motion responses. In this paper, the benchmark ship model, the Blunt modified Wigley, is used for all the simulations. The bow wave generated by the Blunt modified Wigley hull at a constant speed in incident waves is considered. The bow wave and the ship hydrodynamics are investigated by using Reynolds-averaged Navier-Stokes (RANS) method with Volume of Fluid (VOF) method to capture the free surface. In-house computational fluid dynamics solver, naoe-FOAM-SJTU, is applied to predict the hydrodynamics. The added resistance and motion responses of the Blunt modified Wigley in incident waves are studied to help understand the effect of the wave length on motion responses. The 1ˢᵗ harmonic amplitude and corresponding phase for heave and pitch motions are analyzed in detail. Furthermore, the wave patterns are visualized to discuss the evolution of the bow wave.

KEY WORDS: Bow wave; ship motions; hydrodynamics; Blunt modified Wigley; naoe-FOAM-SJTU solver.

INTRODUCTION

The regulations of ship Energy Efficiency Design Index (EEDI) (MEPC, 2008) was proposed by the International Maritime Organization (IMO) (Minchev, 2013). The energy efficiency of newly built vessels must be assessed by means of the EEDI and meets its requirements. The bow shape can significantly affect the pattern and parameters of bow waves, which has an important impact on the hydrodynamic performance of the ship. Therefore, it is vital to accurately predict the bow wave profile and study the relationship between the bow wave and ship resistance. Compared with the ship in calm water, there is a certain increase of ship resistance when ship advances in waves. In order to maintain sufficient propulsion power for the ship in waves, the issue that must be solved is to reliably predict the ship resistance during the ship design.

The predictions of ship's wave force and motions are mainly implemented by Experimental Fluid Dynamics (EFD), Potential Flow and Computational Fluid Dynamics (CFD). The potential flow methods mainly include two-dimensional slice theory, two-dimensional semi-theory, and three-dimensional panel theory. These methods have relatively high calculation efficiency and accuracy in linear problems, but they cannot accurately predict the strong nonlinear phenomena such as wave breaking, large-scale motions and short-wave conditions. With the development of the computer technology, CFD method becomes an important tool to predict the hydrodynamic performance of ships. Especially in the nonlinear problems, such as the bow wave prediction, the CFD method presents its unique advantages solving Navier Stokes equations.

Preliminary simulation researches of the ship added resistance are investigated and published by Havelock (1937) and Maruo (1957) which were based on analytical considerations combined with strip theory methods. Fujii and Takahashi (1975) investigated the added resistance due to the bow reflection in regular oblique waves. Based on Maruo’s formula, the added resistance is divided into two components, one is diffraction resistance due to ship motions and another one is radiation resistance due to bow reflection. Kashiwagi (2013) adopted unsteady wave analysis to investigate the effects of non-linear ship-generated unsteady waves with the Blunt and Slender modified Wigley. Simonsen (2013) used the URANS method to study the variation of pitch, heave and added resistance with the KCS (KRISO 3600 TEU Container Ship) model at different speeds in regular waves of different wave lengths. Shen (2013) calculated added resistance and ship motions of DTMB 5512 in the waves with various steepness by naoe-FOAM-SJTU. Sadathosseini (2013) used CFDShip-Iowa to verify and validate the characteristics of KVLCC2 in short and long head waves. The results of its added resistance and ship motions are well in agreement with the experimental results. Olivieri (2007) studied the scars and vortices induced by bow and shoulder wave breaking of DTMB 5415. Ren (2018) analyzed the speed effects on the phenomena of ship wave breaking, four different speeds in calm water

In this paper, the benchmark ship model, the Blunt modified Wigley, advances at a constant speed (Fr = 0.2) in incident waves. The in-house computational fluid dynamics solver, naoe-FOAM-SJTU, is used to...
predict the added wave resistance and motion response of Blunt modified Wigley in the first-order Stokes waves in deep water with ten different wave lengths ($\lambda/L=0.3$–2.0). The coupled equations of velocity and pressure are solved by pressure-implicit split-operator (PISO) algorithm and the computational domain is discretized by Finite Volume Method (FVM) (1997) with Volume of Fluid (VOF) method to capture the free surface. The added resistance and motion responses of the Blunt modified Wigley in incident waves are studied to help understand the effect of the wave length on motion responses. The 1st harmonic amplitude and corresponding phase for heave and pitch motions are analyzed. Furthermore, the wave pattern are visualized to discuss the evolution of the bow wave system.

NUMERICAL METHOD

Governing Equations

The incompressible Navier-Stokes equations (Eq. 1 ~ 2) are adopted as governing equations to solve the viscous flow.

$$\nabla \cdot \mathbf{U} = 0$$  
$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) = -\nabla p + \nabla \cdot (\mu \nabla \mathbf{U})$$ (1)

$\mathbf{U}$ is velocity field. $\mathbf{U}_f$ means the velocity of mesh nodes. $p$ means pressure and $p_d$ is dynamic pressure. $g$ and $p$ are the acceleration vector of gravity and the mixture density of the two phases separately considered.

The shear stress transport (SST) $k$-$\omega$ model which is proposed by Menter (1981) is applied for the closure of the numerical model. It has been validated that this turbulence model can accurately predict the hydrodynamics and 6DOF motions of ships.

VOF Method

A Volume of Fluid (VOF) method with the artificial bounded compression technique proposed by Hirt and Nichols (1994) is used to capture the free surface. The free surface is considered as a mixture of water and air. The VOF transport equation is expressed in Eq. 3.

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\rho \mathbf{U} \alpha) + \nabla \cdot (\mu \nabla \alpha) = 0$$ (3)

where $\mathbf{U}_g$ is the velocity of grid nodes and $\mathbf{U}_r$ is the relative field used to compress the interface. The volume fraction $\alpha$ is defined as the relative volume proportion of water in a cell. $\alpha$ is expressed as Eq. 4.

$$\begin{cases} \alpha = 0 & \text{air} \\ 0 < \alpha < 1 & \text{interface} \\ \alpha = 1 & \text{water} \end{cases}$$ (4)

Wave generation and damping

The first-order Stokes regular wave is generated and eliminated by damping zone at the outlet by naoe-FOAM-SJTU. Assuming that the wave propagates along the negative $x$-axis direction, the wave elevation $\zeta$, the encounter frequency $\omega_e$ and the incident wave potential $\Phi$ are defined as

$$\begin{align*} \zeta &= A \cos(kt + \phi) \\ \omega_e &= \omega - kU \\ \Phi &= \frac{gA \cosh(\frac{kz + H}{2\omega})}{2\omega \sinh(\frac{kH}{2\omega})} (kt + \phi) \end{align*}$$ (5)

The velocity component of the water particle is defined as

$$\begin{align*} u(x, y, z, t) &= U + A \omega e^{\omega t} \cos(kt + \phi) \\ v(x, y, z, t) &= 0 \\ w(x, y, z, t) &= A \omega e^{\omega t} \sin(kt + \phi) \end{align*}$$ (6)

where $A$ is the wave amplitude, $\omega$ is the wave natural frequency, $k$ is the wave number, $H$ is the water depth and $U$ is the ship speed.

The length of the damping zone is set 1$\lambda$ for long waves ($\lambda/L \leq 0.6$) and 2$\lambda$ for short waves ($0.6 < \lambda/L \leq 1.3$) and medium waves ($\lambda/L \geq 1.3$). The reason why the lengths damping zone for long, medium and short waves are different is the differentiation of the lengths of long, medium and short waves. Whether the lengths of waves are long or short or not, the length of the damping zone should be guaranteed a certain length to ensure the waves decay. The vertical velocity $w$ of a particle is forced to attenuate and the horizontal velocity $u$ is not changed to ensure the continuity of the flow. The forced attenuation equation is

$$w(x, y, z, t) = w(x, y, z, t) \mu(x, z)$$ (7)

$$\mu(x, z) = \beta \left( \frac{x - x_s}{x_e - x_s} \right)^2 \frac{z_e - z}{z_e - z_s}$$ (8)

where $w(x, y, z, t)$ is the vertical velocity after decay and $\mu(x, z)$ is the decay term. The range of $x$ is $x_s < x < x_e$, $z_s < z < z_e$, and the subscripts of $s$ and $e$ represent the begin and the end of the damping zone along $x$-axis. $\beta$ is a parameter of attenuation.

Dynamic Mesh Deformation

The dynamic mesh deformation is applied in all cases to solve ship motions. The topology between the meshes is constant, but the shape of the mesh changes as the nodes stretch or compress. The velocity of grid nodes is determined by Laplace’s equation with fixed or mutative diffusivity.

$$\nabla \cdot (\nabla \mathbf{U}_g) = 0$$ (9)

where, $\gamma = \frac{1}{r^2}$ is the quadratic inverse distance of cell center to the nearest moving wall boundary. $\mathbf{U}_g$ is the grid velocity.

The naoe-FOAM-SJTU Solver

Numerical simulations are performed by using the in-house computational fluid dynamics solver naoe-FOAM-SJTU (Shen, Cao, 2013), which is developed on the open source platform OpenFOAM. Governing equations are RANS equations with $k$-$\omega$ shear stress transport (SST) model. VOF method is applied to treat the free surface. The solver mainly includes the six Degree of Freedom (6DOF) module, dynamic grid module and the numerical wave tank module.

The 6DOF motion module is developed for predicting the ship motion. The earth-fixed and ship-fixed coordinate systems are adopted to solve 6DOF equations.

The Finite Volume Method (FVM) is used to discretize the flow field of the Blunt modified Wigley under different wave length waves. Rigid body (Blunt modified Wigley) motions are solved by 6DOF equations. Then, the solution of the governing equations is achieved by using the
The pressure-implicit split-operator (PISO) algorithm is applied for temporal discretization. The Van Leer scheme is applied to discretize VOF equation discretization. A second-order total variation diminishing (TVD) scheme and a second-order central difference scheme are separately used to discretize the convection term and diffusion term.

GEOMETRY MODEL AND GRID GENERATION

Geometry Model

The benchmark ship model of Blunt modified Wigley is used in all simulations. The Blunt modified Wigley is a mathematical hull form defined only under the mean waterline and the mathematical form is defined as Eq. 10. Therefore, hull shape for above mean waterline should be vertical wall sided.

\[
\begin{align*}
\bar{y} &= (1 - \bar{x}^2)(1 - \bar{z}^2) \left[ 1 + 0.6(\bar{x}^2 + \bar{z}^2) + \bar{z}^2 \left( 1 - \bar{x}^2 \right) \left( 1 - \bar{z}^2 \right) \right] \times \\
\bar{x} &= x \frac{2}{L}, \quad \bar{y} = y \frac{2}{B}, \quad \bar{z} = z \frac{d}{d}
\end{align*}
\]

(10)

The three views and principle particulars of the Blunt modified Wigley are separately given in Table. 1 and Fig. 1.

Table. 1. Principal particulars of the ship model

<table>
<thead>
<tr>
<th>Principal particulars</th>
<th>Ship model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars</td>
<td>L (m)</td>
</tr>
<tr>
<td>Waterline breadth</td>
<td>B (m)</td>
</tr>
<tr>
<td>Draft</td>
<td>d (m)</td>
</tr>
<tr>
<td>Displacement</td>
<td>V (m³)</td>
</tr>
<tr>
<td>Wetted surface area</td>
<td>S (m²)</td>
</tr>
<tr>
<td>Longitudinal center of gravity</td>
<td>LCG (m)</td>
</tr>
<tr>
<td>Vertical center of gravity</td>
<td>KG (m)</td>
</tr>
</tbody>
</table>

Grid Generation

The fluid flow around the hull is considered to be symmetric which means that the computational domain can be considered only for half of hull to reduce the computational cost. The computational domain extends from \(-1Lpp < x < 4Lpp, 0 < y < 1.5Lpp, -1.5Lpp < z < 0.5Lpp\). A symmetry boundary condition is used according to the symmetric problem. The ship axis is aligned with \(x\) with the mid-ship at \(x = 0\). The \(y\)-axis is positive to starboard with \(z\) pointing upward. The free surface at rest lies at \(z = 0\). For the calm water case, there is no damping zone. For the regular head wave cases, the damping zone with a length of \(2\lambda\) is set at the outlet in order to avoid the wave reflection.

Unstructured hexahedral grids are generated by HEXPRESS. Six refinement regions are used to refine the area around hull and free surface. Seven-layer boundary layer grids are set on the hull surface with the \(y\) plus of 30. The mesh near the bow and free surface is shown in Fig. 2.

Table. 2. Simulation conditions

<table>
<thead>
<tr>
<th>Froude number (Fr)</th>
<th>Wave length (\lambda/L)</th>
<th>Wave height (H/L)</th>
<th>Encounter frequency (\omega_e)</th>
<th>Wave period (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.013</td>
<td>15.850</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.4</td>
<td>0.013</td>
<td>12.848</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>0.5</td>
<td>0.013</td>
<td>10.955</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>0.6</td>
<td>0.013</td>
<td>9.639</td>
</tr>
</tbody>
</table>
Verification is the process of evaluating the numerical error/uncertainty. The numerical error consists of the grid error, the time step error and the iterative error. The grid convergence should be considered primarily. Grid convergence study is useful for deciding the level of discretization error existing in the CFD solution.

Three sets of different grids amounts are generated to calculate the hydrodynamic forces and ship motions. And the amounts of grids (coarse mesh, medium mesh and fine mesh) are listed in Table 3. In all the cases, the Froude number is 0.2, the Reynolds number is identical. The number of grids, coarse mesh, medium mesh and fine mesh are $35 \times 10 \times 14$, $50 \times 15 \times 20$, $70 \times 21 \times 28$, and the grid refinement ratio is $\sqrt{2}$.

Table 3. The amounts of grids

<table>
<thead>
<tr>
<th></th>
<th>coarse mesh</th>
<th>medium mesh</th>
<th>fine mesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>amounts of grids</td>
<td>0.74M</td>
<td>1.89M</td>
<td>3.51M</td>
</tr>
</tbody>
</table>

The results of total resistance of mesh convergence study are presented in Fig. 4. Therefore the middle level grid is used for the numerical simulation and mesh of wave conditions are modified on the basis of calm water.

![Fig. 4 The total resistance of three meshes](image)

**Time Histories of Forces and Motions**

The total resistance, heave and pitch are non-dimensional by Eq. 11 ~ 13:

\[ C_t = \frac{R_{x,\text{wave}}}{0.5 \rho S U_{\text{ship}}^2} \]  

\[ \text{heave motion} = \frac{z}{A} \]  

\[ \text{pitch motion} = \frac{\theta}{Ak} \]

where, $R_{x,\text{wave}}$ is the time-averaged force along $x$-direction in waves corresponding to the $0^{th}$ harmonic amplitude. $S$ is the wetted surface area and $U_{\text{ship}}$ is the ship speed. $A$ is wave amplitude and $Ak$ is the wave steepness.

Time histories of resistance coefficient, dimensionless heave and dimensionless pitch of case 11 as an example are shown in Fig. 5.

![Fig. 5 Time histories of resistance coefficient, dimensionless heave and dimensionless pitch](image)

**Response Amplitude Operators**

Response amplitude operator (RAO) is actually the transfer function, which describes the relationship between the incident wave parameters and ship motions. The ship is free in heave and pitch. Therefore, there are periodic motion responses (heave and pitch) in the regular head waves. The heave and pitch with the assumption of linear response are analyzed by transfer functions to study the ship motions. The heave transfer function ($TF_3$) and pitch transfer function ($TF_5$) are expressed as:

\[ TF_3 = \frac{X_{3}}{A} \]  

\[ TF_5 = \frac{X_{5}}{Ak} \]

where, $A$ is the wave amplitude and $Ak$ is the wave steepness. $X_{3}$ and $X_{5}$ are respectively the $1^{st}$ harmonic amplitude of heave and pitch, which can be obtained by the Fast Fourier Transform (FFT) algorithm.

Time history $\phi(t)$ of the heave (or pitch) can be expanded into the sum of Fourier series as Eq. 16.

\[ \phi(t) = \phi_0 + \sum_{n=1}^{\infty} \phi_{n} \cos(\omega t + \gamma_{n}), \quad n = 1, 2, 3, \ldots \]

where, $\phi_n$ is the $n^{th}$ harmonic amplitude and $\gamma_n$ is the corresponding phase. $\omega$ is the encounter frequency. Besides, $\phi_0$ denotes average value of
sinkage (or trim) in waves.

The $n^{th}$ ($n = 0, 1, 2 \ldots$) harmonic amplitude and phase can be obtained by following Eq. 17 and Eq. 18.

$$\phi_n = \arctan \left( \frac{b_n}{a_n} \right), \text{ if } b_n \geq 0$$

$$\gamma_n = \begin{cases} \arctan \left( \frac{b_n}{a_n} \right) + \pi, \text{ if } b_n < 0 \\ \end{cases}$$

Based on the above description, the $n^{th}$ harmonic amplitudes of $X_3, X_5$ are expressed as $X_3^{(n)}, X_5^{(n)}$. Additionally, the $0^{th}$ harmonic amplitude is the time-averaged value. The $1^{st}$ harmonic amplitude is the linear component. The $0^{th}$ and $1^{st}$ harmonics are the primary linear components. The $2^{nd}$ and higher order harmonic amplitudes are the non-linear terms of unsteady quantities, which play a crucial role in non-linear problems.

$$\frac{X_3}{X_3^{(0)}} = R_{X_3}, I_{X_3}$$

$$\frac{X_5}{X_5^{(0)}} = R_{X_5}, I_{X_5}$$

The $1^{st}$ harmonic amplitude and corresponding phase for heave and pitch motions are analyzed at different wave conditions ($\lambda/L$) and shown in Fig. 6 and Fig. 7. In the meantime, due to the lack of experimental results corresponding to our simulated conditions, the test results ($H=0.02m$ and $0.05m$, $L=2.5m$) from Kashiwagi (2013) are also presented, which focuses on explaining the trend of motion responses changed with the wave length.

The results show the $1^{st}$ harmonic amplitude of heave motion increases continuously as $\lambda/L$ increases, and reaches to 0.8 at $\lambda/L=2$. However, the $1^{st}$ harmonic amplitude of pitch motion reduces at $\lambda/L > 1.4$ while it increases at $0.3 < \lambda/L < 1.4$, then it reaches to the maximum value at $\lambda/L = 1.4$. It is worth mentioned that the $1^{st}$ harmonic amplitudes for both of heave and pitch motions increase sharply at $0.8 < \lambda/L < 1.2$ but they are small at $0.3 < \lambda/L < 0.8$ and can be near neglected. It is considered that severe motions happen then the effect of radiation is large under the resonance condition, which leads to a large increase of heave and pitch motion responses.

The phases of heave and pitch show the large fluctuations. The phase of heave motion reduces sharply from $0^\circ$ to $-180^\circ$ at $\lambda/L=0.5$, then increases to about $0^\circ$ in resonance regions ($0.9 < \lambda/L < 1.1$). After that, the phase remains a constant value ($0^\circ$). For the heave motion, its phase reduces sharply from $90^\circ$ to $-90^\circ$ at $\lambda/L=0.8$. Then it decreases again to $-180^\circ$ in resonance regions, then increases slowly to $-90^\circ$.

### Added Resistance

The $x$-directional total resistance coefficient of the ship in waves is periodic and expressed as Eq. 11.

$$C_{aw} = \frac{R_{x,\text{wave}} - R_{x,\text{calm}}}{\rho g A B_{pl} / L_{pp}}$$

where, $A$ is the wave amplitude. $B_{pl}$ is the ship breadth and $L_{pp}$ is the length between perpendiculars.

The results of added resistance coefficient is shown in Fig. 8 and Table 4 (“-" denotes the absence of the data), where the experimental results
provided by Kashiwagi (2013) are also presented. In Fig. 8, the horizontal axis stands for $\lambda/L$ that changes from 0.3 to 2.

Table 4 Results of added resistance coefficient

<table>
<thead>
<tr>
<th>$\lambda/L$</th>
<th>EFD H=0.02m, L=2.5m</th>
<th>EFD H=0.06m, L=2.5m</th>
<th>EFD H=0.06m, L=2.5m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>3.094</td>
<td>2.474</td>
<td>1.833</td>
</tr>
<tr>
<td>0.4</td>
<td>2.819</td>
<td>2.474</td>
<td>1.981</td>
</tr>
<tr>
<td>0.5</td>
<td>2.154</td>
<td>1.925</td>
<td>1.829</td>
</tr>
<tr>
<td>0.6</td>
<td>3.016</td>
<td>2.238</td>
<td>2.865</td>
</tr>
<tr>
<td>0.8</td>
<td>2.81984</td>
<td>-</td>
<td>3.124</td>
</tr>
<tr>
<td>1.0</td>
<td>8.616</td>
<td>7.225</td>
<td>5.884</td>
</tr>
<tr>
<td>1.1</td>
<td>-</td>
<td>8.756</td>
<td>8.743</td>
</tr>
<tr>
<td>1.2</td>
<td>10.104</td>
<td>-</td>
<td>9.549</td>
</tr>
<tr>
<td>1.4</td>
<td>5.483</td>
<td>8.757</td>
<td>4.568</td>
</tr>
<tr>
<td>1.6</td>
<td>4.503</td>
<td>2.160</td>
<td>3.127</td>
</tr>
<tr>
<td>2.0</td>
<td>2.624</td>
<td>-</td>
<td>1.572</td>
</tr>
</tbody>
</table>

The trend of added resistance coefficient agrees well with the experimental results. For $0.3 < \lambda/L < 1.2$, the added resistance coefficient increases with the increase of the wave length. Furthermore, the added resistance coefficient on the whole maintains Table growth for $0.3 < \lambda/L < 0.8$. However, for $0.8 < \lambda/L < 1.2$, the most obvious change is the rapid increase of added resistance coefficient and the coefficient quickly reaches peak 9.55. The peak value of added resistance coefficient occurs around $\lambda/L = 1.2$. This behavior is related to the fact that the resonance occurs when the encounter frequency is close to the natural frequency of the ship. For $1.2 < \lambda/L < 2.0$, the coefficient decreases with the increase of the wave length.

The trend of added resistance coefficient may be explained according to heave and pitch RAOs for the corresponding wave length. The added resistance is dependent on the ship motions of heave and pitch. The ship motions under short wave conditions ($0.3 < \lambda/L < 0.8$) is small and can be near negligible. Added resistance in this range is primarily caused by wave reflections. As the increase of wave length, especially for $0.8 < \lambda/L < 1.2$, the ship motions become significant. Simultaneously, added resistances rise to peak. Then, the added resistance of ship under long wave conditions decreases dramatically and is close to that of short waves. However, the motions of ship ($1.2 < \lambda/L < 2.0$) are still larger than that of short waves. It is because the radiation component caused by the wave reflection occupies a small part of added resistance and diffraction component caused by the ship motions is dominant.

Wave Patterns

In order to show the relationship between the ship's motions and the wave patterns as a whole, the relative position snapshots of wave patterns and corresponding motions of heave and pitch for four different moments in an encounter period at $\lambda/L = 1$ are analyzed (shown in Fig. 9). There are two wave systems, including a bow wave system and a stern wave system. The wave pattern generated by the hull includes the incident waves, the steady wave, the diffraction wave, the radiation wave and the interaction of different types of waves.

Wave Patterns

When a crest of the incident wave passes the bow of the ship, $t/T_e$ is set as 0. At $t/T_e = 0$, wave crests are in bow and stern, and a wave trough is in mid-ship. The whole ship sinks and trim by head. At $t/T_e = 0.25$, there are two wave crests behind bow and stern due to the backward movement of relative positions for wave patterns. And the bow is uplifted by the incident wave. At $t/T_e = 0.5$, the positions of wave crests and wave trough, as well as the motions of heave and pitch is opposite to that of $t/T_e = 0$.

Vorticity Fields of the Bow Wave

The vorticity fields near the bow in one encounter period are shown in Fig. 10. The generated vortices cause a pressure reduction and vibration of the ship. It is significant to study the vorticity fields. Overall, the figure clearly indicates many pairs of counter-rotating vortices induced by the bow wave. The counter-rotating vortices are formed from the hull surface and evolve downstream. At $t/T_e = 0$ or $t/T_e = 0.75$, there are distinct scars along the ship waves.
The axial vortices is related to the crest position. For instance, at \( t/T_e = 0 \) or \( t/T_e = 0.75 \), there are crests at the shoulder of ship, and the axial vortices in the crest area are larger than that of other time. The positive and negative vortices appears alternately near the free surface.

**Velocity Fields of the Bow Wave**

The velocity field is significant for resistance and hydrodynamic performance of a ship. The velocity fields near the bow in one encounter period are shown in Fig. 11. The velocity consists of three components: axial velocity \( (u) \), horizontal velocity \( (v) \) and vertical velocity \( (w) \). It is obvious that the \( u \) and \( w \) are more significant and \( v \) is small. That is to say, the velocity field is mostly directed outward and upward. The magnitudes of \( v \) and \( w \) decrease along the downstream while the magnitude of \( u \) increases along the downstream. The velocity fields at \( t/T_e = 0 \) and \( t/T_e = 0.75 \) show more unsteady than that of \( t/T_e = 0.25 \) and \( t/T_e = 0.5 \). This phenomenon is mainly caused by the different relative position between the hull and the wave.

Overall, the solver naoe-FOAM-SJTU is able to solve the added resistance of a ship at a constant speed and capture the vorticity and velocity of the bow wave.

**CONCLUSIONS**

In the present work, ship motions, added resistance and bow waves of the Blunt modified Wigley with eleven wavelengths at a constant wave steepness are predicted by an in-house computational fluid dynamics solver, naoe-FOAM-SJTU. The VOF method is used to capture the free surface and ship motions are considered using the dynamic grid technology.

The heave, pitch and resistance are decomposed by Fast Fourier Transform (FFT) algorithm. The results show that the 1\(^{st}\) harmonic amplitudes for both of heave and pitch motions increase sharply at \( 0.8 < \lambda/L < 1.2 \) but they are small at \( 0.3 < \lambda/L < 0.8 \) and can be near neglected. At \( \lambda/L = 1.4 \), the resonance occurs, which leads to a large increase of heave and pitch motion responses. Then, the phases show the large fluctuations.

The added resistance is dependent on the ship motions of heave and pitch. The ship motions for \( 0.3 < \lambda/L < 0.8 \) is small and can be near negligible. Added resistance in this range is primarily caused by wave reflections. At \( 0.8 < \lambda/L < 1.2 \), the ship motions become significant and added resistances rise to peak. Under long wave conditions the added resistance of ship decreases sharply and is close to that of short waves.

It is observed that pairs of counter-rotating vortices induced by the bow wave and the velocity components of \( v \) and \( w \) is larger than \( u \). The counter-rotating vortices are formed from the hull surface and evolve downstream. And the positive and negative vortices appears alternately near the free surface.

Overall, the solver naoe-FOAM-SJTU is able to solve the added resistance of a ship at a constant speed and capture the vorticity and velocity of the bow wave.

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