

## **Sloshing Flows in an Elastic Tank with High Filling Liquid by MPS-FEM Coupled Method**

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### **ABSTRACT**

As the size of liquid cargo ship enlarges, the hydro-elastic behavior induced by the sloshing flow should be taken into account for the design of a safer tank. In this paper, the response of elastic tank walls induced by the sloshing impact loads is numerically investigated. The in-house solver MlParticle-SJTU, which is developed base on the Moving Particle Semi-Implicit (MPS) method for fluid analysis and expanded with the finite element method (FEM) for structure analysis, is employed for the simulation. The coupling strategy between the fluid and structural analysis modules of the solver, together with the data transformation on the interface, are introduced in detail. Behaviors of the elastic tank walls under the impulsive forces of sloshing flow with high filling ratio of water are simulated and characteristics regarding the roof impact event, including the evolutions of free surface, dynamic responses of the structures in both time and frequency domains, are presented.

**KEY WORDS:** Sloshing; fluid structure interaction (FSI); MlParticle-SJTU solver; moving particle semi-implicit (MPS) method; finite element method (FEM).

### **INTRODUCTION**

In the past decades, the sloshing flow in partially filled tanks of liquid cargo ship has been studied intensively. Most contributions are focused on the extreme impact pressures on the tank walls, the coupling mechanism between ship motions and internal sloshing flows (Zhao et al., 2014; Mitra et al., 2012), techniques to minimize the sloshing (Liu and Lin, 2009) in rigid tanks with model scale, etc. In these works, the assumption that the tank walls are rigid structures is justified while the size of tank is relatively small. Actually, the resonance frequency of fluid motion in cargo tanks would decrease to the same order of the ship motion frequency as the size of ship enlarges, which may result in structural deformation or even serious damage (Lee and Choi, 1999). Hence, the elasticity of tank walls should be taken into account for the design of a larger and safer tank.

Until now, numerical approach is more popular in investigating the fluid structure interaction (FSI) problem of sloshing involving tank's deformation in comparison with the experimental method which is expensive and limited in scope and theoretical analysis method which is difficult to describe the violent free surface evolution. For the FSI problem in an oil/water tank, deformation of the whole tank wall results from the impact loads of sloshing flow is dominant. To investigate the relevance between the structural oscillation and impact pressure patterns, many numerical approaches are proposed. For instance, Lee et al. (1995) utilized a finite element (FE) method coupled with compressible finite volume method (FVM) to analyze the structure and sloshing fluid behaviors in a tank of VLCC. Fossa et al. (2012) investigated the possible effects of a deformable structure on the sloshing phenomenon with the help of ADINA software which is based on the finite element method (FEM) for both fluid and structural analysis. Liao and Hu (2013) developed a coupling finite difference method (FDM) and the finite element method (FEM) for simulating the interaction between liquid sloshing flow in a rolling tank and a thin elastic plate. However, applications of these grid-based methods face some challenges, e.g. inefficient process of grids generation for complex shape of structure, requirement of dynamic mesh technologies for moving boundary or large structural deformation, simulation of free surface with large deformation or breaking, etc. In view of these points, the Lagrangian meshless methods, which are the new generation computational methods, are quite suitable to tackle these challenges. For example, the Moving Particle Semi-Implicit (MPS) method, which is originally proposed by Koshizuka and Oka (1996) for incompressible flow, has been integrated with the FEM and exhibits good performance in FSI problem according to the numerical benchmark tests of dam break flow interacting with flexible structure (Mitsume et al., 2014; Sun et al., 2015). However, the application of the MPS-FEM coupled method for the interaction between violent sloshing flow and elastic tank is rarely reported.

In this paper, a fully Lagrangian FSI solver MlParticle-SJTU is implemented for the numerical investigation of interaction between sloshing flow and elastic tank. The coupling Moving Particle Semi-Implicit (MPS) method and the finite element method (FEM) employed in the proposed FSI solver is introduced firstly. A partitioned coupling strategy used for the data transformation on the interface between the

fluid and structural domain is presented. Then, the FSI solver is employed to investigate the behaviors of elastic tank walls resulting from the impulsive forces of sloshing flow with high filling ratio of water.

## COUPLING STRATEGY FOR FSI PROBLEMS

In the present study, the MPS-FEM coupled method is proposed to address the FSI problems. Details of the MPS method for the fluid domain analysis and FEM for the structural domain analysis have been introduced in the previous published papers (Zhang and Wan, 2012; Zhang et al., 2014; Tang et al., 2015; Tang et al., 2016; Zhang et al., 2016a, 2016b). Herein, only the coupling strategy and data transformation approach on the fluid-structure interface used in the FSI solver MLParticle-SJTU will be presented.

### Coupling strategy

Until now, the coupling strategies for the FSI simulations can be classified into the strong coupling approaches and the weak coupling approaches. In the present study, the weak coupling strategy is employed for the FSI analysis since this approach allows the use of separated fluid and structure codes for each computational domain, be flexible in describing engineering problems with mathematical model and efficient in calculation.

The concept of the coupling system is shown in Fig.1. The coupling process within each cycle of FSI simulation can be summarized as follows:

- (1) For the fluid domain analysis, the fluid module of the FSI solver works at every fluid time step  $\Delta t_f$  to obtain the particle pressure on the interface between fluid and structure boundary.
- (2) For the structure domain analysis, the structure module of the FSI solver only works at the structural time step  $\Delta t_s$  which is usually set to  $k$  multiples of  $\Delta t_f$ , where  $k$  is an integer. Then, the positions of both fluid and structure particles will be updated based on the displacement response  $\mathbf{y}_{n+k}$  calculated by the structure module.
- (3) To obtain more accurate external sloshing loads for the structural response analysis, mean values of particle pressures on the fluid-structure interface are calculated by

$$\bar{p}_{n+k} = \frac{1}{k} \sum_{i=1}^k p_{n+i} \quad (1)$$

where  $p_{n+i}$  is pressure of the fluid particle on wall boundary at the instant  $t_{n+i}$ .

- (4) Though the structural displacement response is calculated only at the structural time step  $\Delta t_s$ , positions of particles should be updated by Eq.(2) at every fluid time step  $\Delta t_f$  to avoid the instability of fluid field produced by the large displacements of structure particles within a larger time interval  $\Delta t_s$ .

$$\dot{\mathbf{y}}_{n+i} = \dot{\mathbf{y}}_{n+i-1} + \ddot{\mathbf{y}}_{n+i-1} \times \Delta t_f, \quad i \in [1, k] \quad (2)$$

where  $\dot{\mathbf{y}}_{n+i-1}$  and  $\ddot{\mathbf{y}}_{n+i-1}$  are the structural nodal velocity and accelerate at the time  $t_{n+i-1}$ , respectively.

### Data transformation on the fluid-structure interface

For a two-way FSI problem, accuracy of the numerical result is tightly associated with the manner of data transformation between the fluid and structure domain.

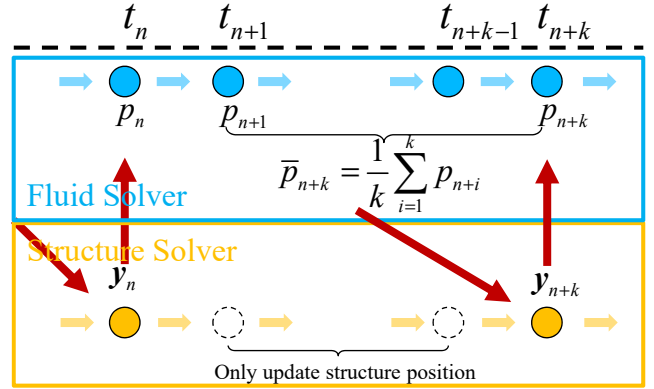


Fig.1 The concepts of the coupling strategy

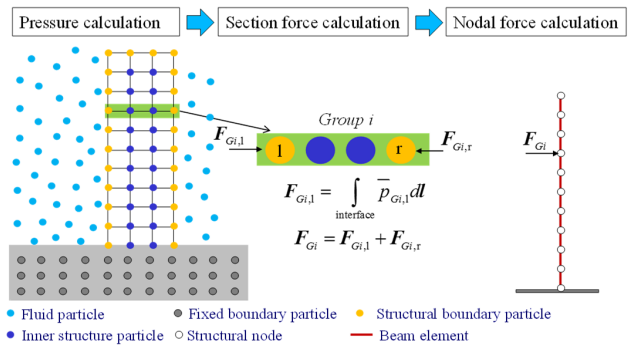


Fig.2 The concepts of the numerical considerations for force

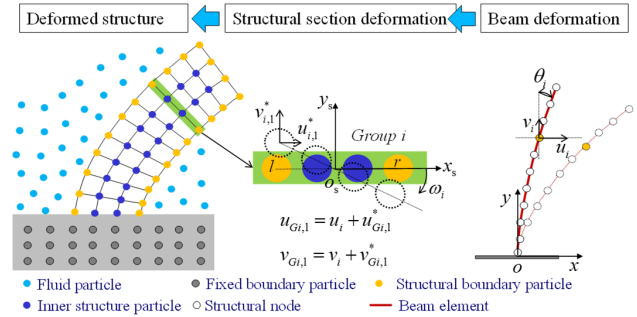


Fig.3 The concepts of the numerical considerations for deformation of structural particle model

In the present paper, two-dimension FSI problems are considered and the tank walls will be dispersed to beam elements for the analysis of structural physics. As a result, special treatments need to be applied for data transformation on the fluid-structure interface, including the application of external force onto the beam nodes and the deformation of structural particle model corresponding to the displacements of the beam elements. Here, a particle group scheme (Hwang et al., 2016) is considered. Structural particles located within the same section are grouped. For the force transformation, the concepts of the numerical considerations are shown in Fig.2. Herein, the vector  $\mathbf{F}_{Gi,l}$  and  $\mathbf{F}_{Gi,r}$  represent the force acting on left and right boundary particle of the

structural group  $i$ , respectively. As mentioned previously, the pressure of boundary particle is calculated by MPS method initially. Then, force acting on the structural boundary particle within the structural group is calculated by the integration of average pressure acting on the interface. After this, the resultant of forces of particles within the same group are applied onto structural FEM node as the external load for the structural physics analysis. For the deformation of structural particle model, particles within a group move as one body based on the nodal linear velocities  $u_i$  and  $v_i$  which represent the velocities of beam nodes. Then, the final position of structure particles can be updated according to the rotation of group around the center of the section based on the angular velocity  $\omega_i$ . The concepts of the numerical considerations for the deformation of structural particle model are shown in Fig.3.

## NUMERICAL CONDITIONS

For the sloshing in a tank with high filling ratio of water, roof impacts events are quite common phenomenon during the operation period of liquid cargo ship. This phenomenon should be investigated since the tank roof, which is often less reinforced than the bottom part, will subject to high risks of structural vibration or even damage.

In the present paper, the behaviors of elastic tank walls under the impulsive forces of sloshing flow with high filling ratio (70%) of water is investigated by the FSI solver MParticle-SJTU. Capability of the solver for simulation of sloshing flow and structural response under impact loads have been validated in previous published papers (Zhang et al., 2016a, 2016c). Here, the numerical application of the solver for the FSI problem is carried out.

Fig.4 shows the schematic sketch of the sloshing tank. Length ( $L$ ) and height ( $H$ ) of the rectangular tank model are 0.9 m, 0.508 m, respectively. The tank is partially filled with water with the depths  $h=0.356$  m. The tank is forced to roll around the center of bottom ( $O$ ) with the resonant frequency ( $\omega=5.38$  rad/s) corresponding to the filling heights of water. For this FSI problem, structural elasticity of both lateral walls and roof of the rectangular tank is taken into consideration in the simulations. The vibrations of tank walls are measured at the points S1 and S2, which are the centers of left lateral wall and roof wall, respectively. The time history of pressure is measured at the point P1. The tank is dispersed by particles with three initial spacing sizes ( $l_0$ ) of 0.003 m, 0.004 m and 0.005 m to check the convergence of the numerical model. Correspondingly, the total number of particles is 38120, 25351 and 14425. For the fluid analysis, time step size is set as 0.0001 s which satisfy the Courant–Friedrichs–Lewy (CFL) condition.

$$\Delta t_f \leq \frac{Cl_0}{V_{\max}} \quad (3)$$

where the upper bound of Courant number  $C$  is considered as 0.2,  $V_{\max}$  is the maximum velocity of particle.

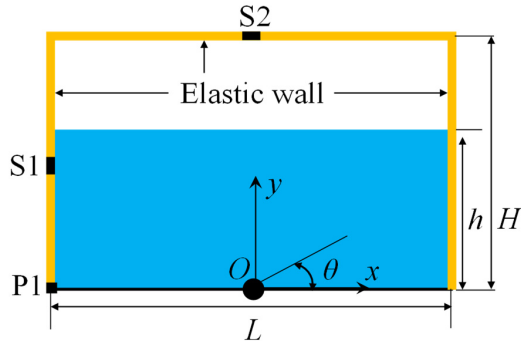


Fig.4 Schematic sketch of the sloshing tank

For the structural analysis, the material properties are set similar to those of magnesium-aluminium alloy. The structural density is 1800 kg/m<sup>3</sup> and the Young's modulus is 40 GPa. Rayleigh's damping of structure has been taken into consider with a factor of  $\alpha_1=0.0128$  for the mass-proportional contribution to the damping and  $\alpha_2=5.01e-7$  for the stiffness-proportional contribution. Detailed parameters for fluid and structural analysis are presented in Table 1.

Table 1. Parameters for fluid and structural analysis

	Parameters	Values
Fluid	Fluid density (kg/m <sup>3</sup> )	998
	Kinematic viscosity (m <sup>2</sup> /s)	$1 \times 10^{-6}$
	Gravitational acceleration (m/s <sup>2</sup> )	9.81
	Particle spacing (m)	0.004
	Total number of particles	25351
	Fluid time step size (s)	$1 \times 10^{-4}$
Structure	Structure density (kg/m <sup>3</sup> )	1800
	Young's modulus (GPa)	40
	Thickness of tank wall (m)	0.004
	Elements per lateral wall	126
	Elements roof wall	229
	Damping coefficients $\alpha_1$	0.0128
	Damping coefficients $\alpha_2$	$5.01 \times 10^{-7}$
	Structural time step size (s)	$1 \times 10^{-4}$

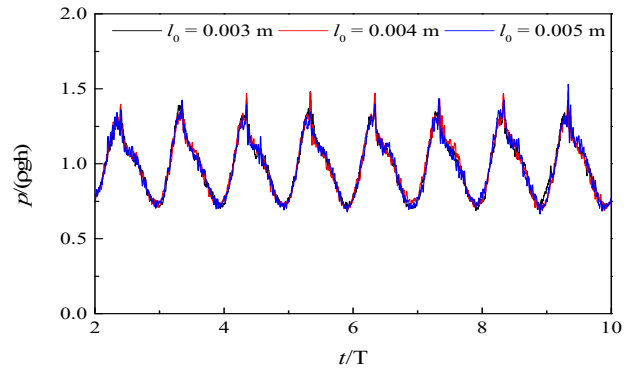


Fig.5 Time history of pressure at the measuring point P1

In the convergence study, sloshing in the rigid tank is simulated and the pressure histories of the three different spatial resolutions corresponding to the point P1 are shown in Fig.5. It can be noticed that both results of the models with initial spacing size of 0.005 m and 0.004 m agree well with that of the fine resolution model ( $l_0=0.003$  m). Considering the computational efficiency, initial distance between fluid particles is set to be 0.004 m for the following FSI simulations in this paper.

## NUMERICAL RESULTS

### Evolution of free surface

Before we investigate the behaviors of sloshing flow in an elastic tank, the simulation regarding to a rigid tank with the same numerical condition is carried out as a comparison study. Fig.6 shows the evolutions of free surface in an elastic tank in comparison with those in rigid tank, including the instant before impact event, impact instant, formation of jet, drop of fluid front and travelling of sloshing wave. It can be noticed that there is no significant difference corresponding to

the shapes of free surface between the two tanks in general, though the deformations of the lateral walls of the elastic tank are observed.

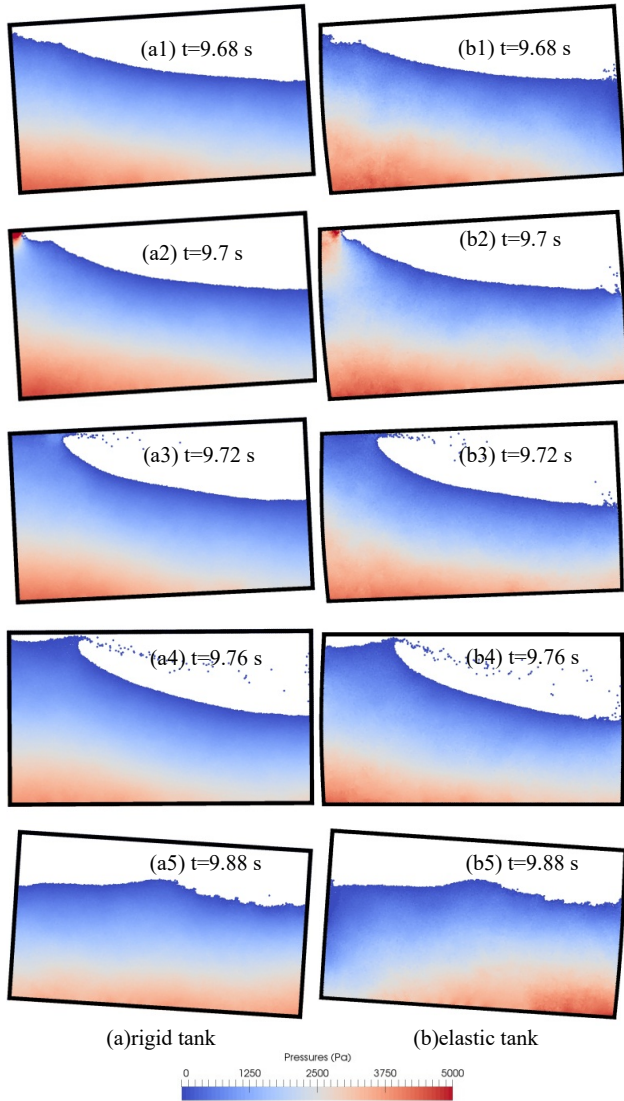


Fig.6 Evolution of free surface

### Response of lateral walls

In one cycle of the tank's motion, lateral walls of the elastic tank will deform due to the periodic variation of pressures which are composed of hydrostatic pressure and hydrodynamic pressure result from the travelling of sloshing wave in the partially filled tank.

The dimensionless displacement time history of the measuring point S1 on the left lateral tank wall is shown as Fig.7. The displacements ( $d$ ) and time ( $t$ ) have been made dimensionless with the height of lateral tank wall ( $H$ ) and excitation period ( $T$ ), respectively. It can be appreciated from Fig.7 (a) that the lateral wall vibrates periodically with the deformation towards outside of the tank. The large oscillation amplitudes will present with a large period approximating to the excitation period of sloshing. Meanwhile, the oscillations with small amplitudes and small period, due to the resultant forces of structural

elastic restoring force and hydrostatic force, are also observed. From the enlarged signal shown as Fig.7 (b), it can be find that the wall's deflection increases due to the increase of water lever near the measured lateral wall during the first half period of tank roll motion, while it will decrease due to the decrease of water lever.

According to the spectrum of displacement response of lateral wall as shown as Fig.8, the structural deformation is mainly associated with the low frequency components. Here, we mark the fundamental response frequency as  $f_1$  (0.863 Hz). It can be noticed that the response frequencies below 8 Hz present integer times of the fundamental frequency. By comparison with the excitation frequency of tank's roll motion  $f_e$  (0.857 Hz), it proved the statements that the period of large oscillation amplitudes approximates to the excitation period of sloshing.

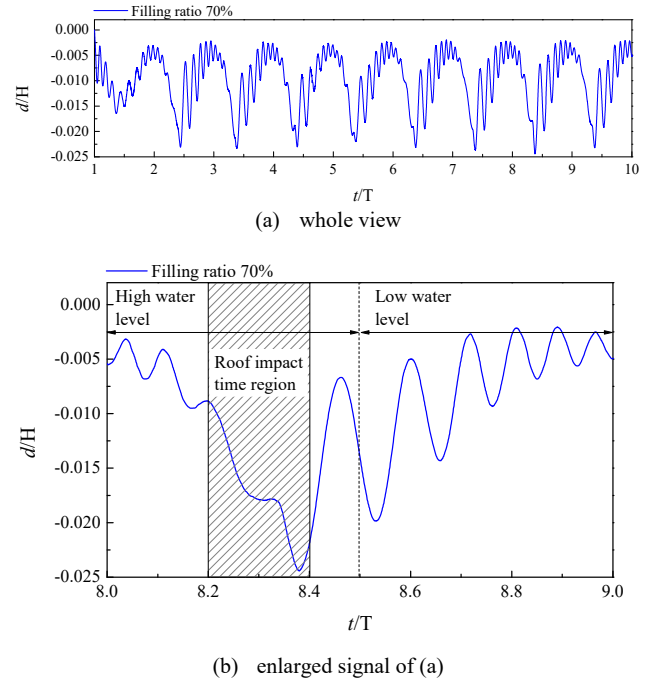


Fig.7 Dimensionless time history of calculated displacement at measuring point S1

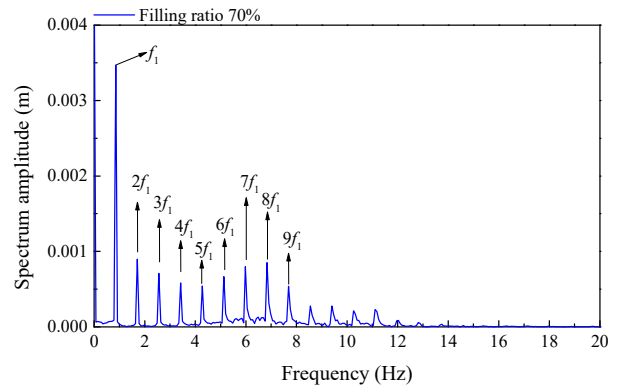


Fig.8 Spectrum of displacement response of lateral wall

## Response of roof wall

For a partially filled tank, the structural response of roof is much different with that of lateral walls. For instance, the roof vibrates symmetrically around the initial position, as shown in Fig.9 (a). The difference is mainly resulting from that the lateral walls will be subjected to both hydrostatic and hydrodynamic pressures while the roof wall is only subjected to the impact loads of sloshing.

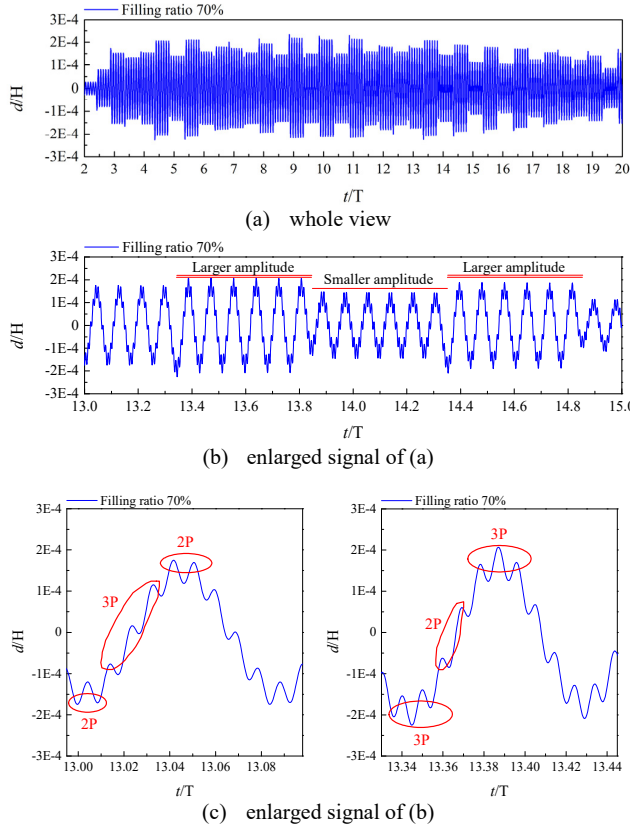
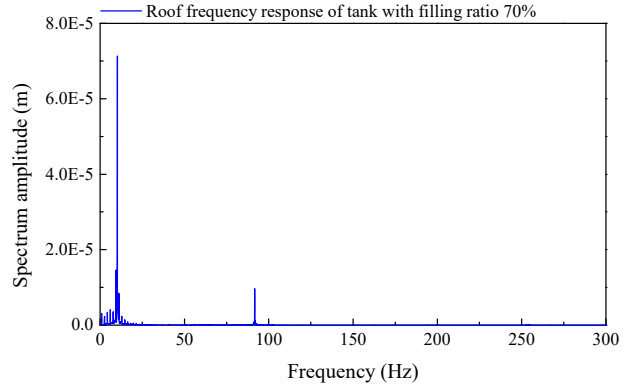
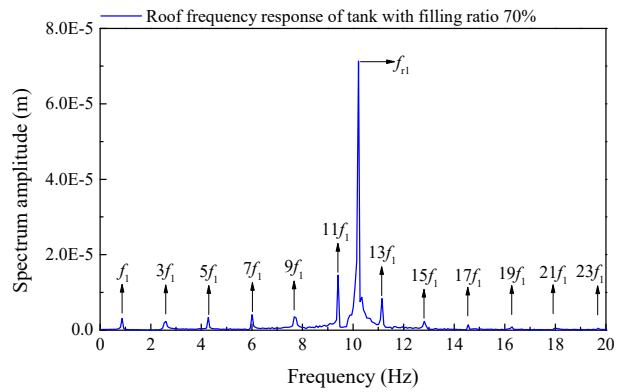


Fig.9 Dimensionless time history of displacement at measuring point S2

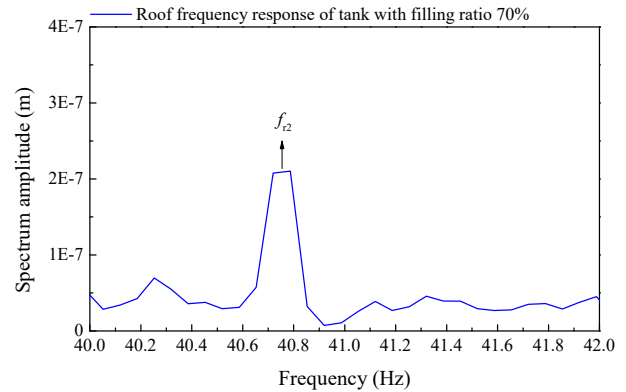
Besides, the response presents the step shape with small amplitudes, as shown in Fig.9 (b). The step shape response is consisted of six cycles of vibration with relatively smaller amplitude and following six cycles of vibration with relatively larger amplitude. Accordingly, the shape of vibration transforms from “2P+3P+2P” pattern to “3P+2P+3P” pattern, as shown in Fig.9 (c). Here, “P” represents the peaks of vibration.



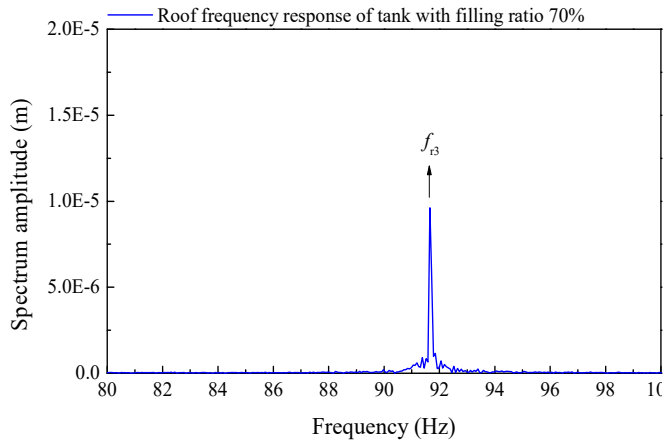
(a) whole view



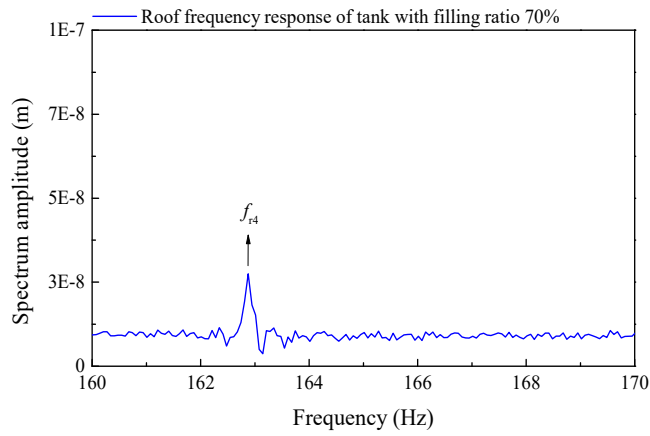
(b) enlarged signal NO.1



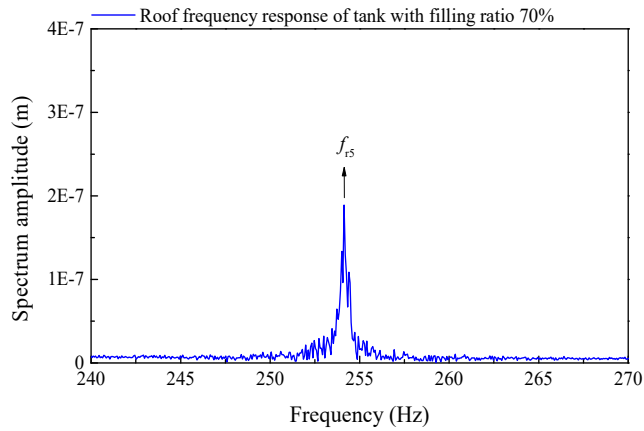
(c) enlarged signal NO.2



(d) enlarged signal NO.3

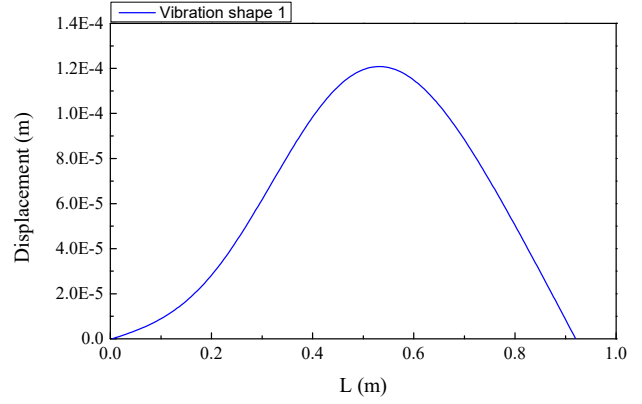


(e) enlarged signal NO.4

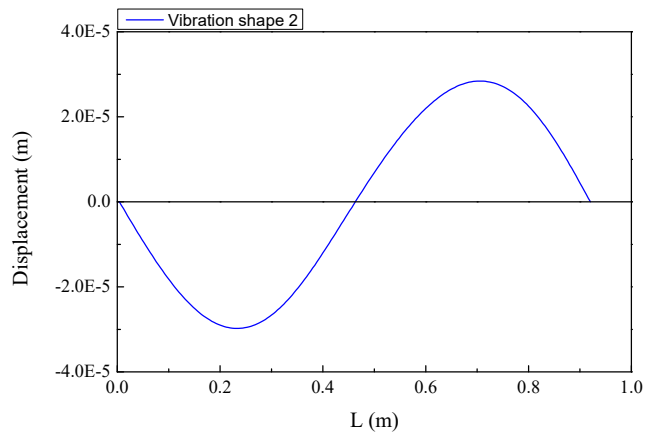


(f) enlarged signal NO.5

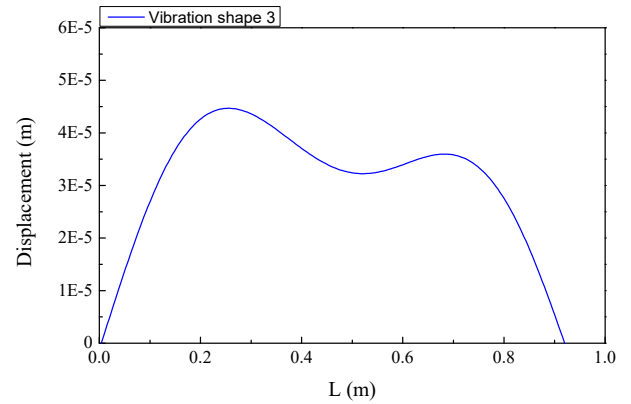
Fig.10 Spectrums of vibration response of roof wall



(a) Shape 1



(c) Shape 2



(c)Shape 3

Fig.11 Vibration shapes of roof wall

Fig.10 shows the spectrums of vibration response of roof wall, including the whole view and the enlarged signals of the spectrums. According to the whole view of the spectrum shown in Fig.10 (a), the structural response covers a wide frequency region. According to the enlarged signal of low frequency region shown as Fig.10 (b), the

predominant component of structural vibration presences at the frequency  $f_{f1}$  (10.22 Hz) which is almost 12 times the fundamental response frequency  $f_1$ , while that presences at the frequency  $f_1$  for the response of lateral wall. The interval between the neighbouring response frequencies is 2 times  $f_1$  since the roof impact event occurs twice in one cycle of tank's roll motion. It should be noticed that the response frequency  $f_{f1}$  closely approximates the first order natural frequency  $f_{N1}$  (10.56 Hz) of roof wall. Besides, the responses with the frequencies approximating higher order natural frequencies are also observed, including the natural frequencies NO.2-5 shown as Fig.10 (c)-(f). Accordingly, the first three order vibration shapes of roof wall are shown as Fig.11, which corresponding to the first three order natural frequencies. Herein, the structural natural frequency is calculated by

$$f_{Ni} = \frac{\pi i^2}{2l^2} \sqrt{\frac{EI}{\rho_l}} \quad (4)$$

where  $f_{Ni}$  denotes the  $i^{\text{th}}$  order natural frequency,  $l$  and  $\rho_l$  denote the length and line density of the structure, respectively.  $E$  and  $I$  denote the Young's modulus and cross-sectional moment of inertia. The structural stiffness is represented by the product of  $E$  and  $I$ .

## CONCLUSIONS

In this paper, the fluid structure interaction problem of sloshing flow in a fully elastic tank with high filling ratio of water is numerically studied using the in-house solver MParticle-SJTU which is developed based on the MPS-FEM coupled method. Characteristics regarding the roof impact event, including the evolutions of free surface, dynamic responses of the structures in both time and frequency domains, are presented. Based on the results of simulations, the following conclusions can be summarized:

- For the sloshing in an elastic tank with small deformation, the evolution of free surface is similar to that in the rigid tank.
- As the results of structural response under the periodic variation of pressures, the lateral wall vibrates with the relatively large amplitude and the period approximates to the excitation period of sloshing. Meanwhile, the vibration with small amplitudes and small period, due to the resultant forces of structural elastic restoring force and hydrostatic force, are also observed.
- The displacement time history of roof wall presents the step shape with small amplitudes, and the shape of vibration transforms between the so called "2P+3P+2P" pattern and "3P+2P+3P" pattern.
- According to the spectrums of vibration response, the roof wall vibrates with the predominant frequency  $f_{f1}$  which closely approximates its first order natural frequency  $f_{N1}$ . Besides, the responses with the frequencies approximating higher order natural frequencies are also observed, including the natural frequencies NO.2-5.

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