Numerical Analysis of Atmospheric Turbulence Effects on Wind Turbine

Dezhi Wei¹, Decheng Wan¹*, Shengxiao Zhao²

¹ Computational Marine Hydrodynamics Lab (CMHL), State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai, China
² Key Laboratory of Far-Shore Wind Power Technology of Zhejiang Province, PowerChina Huadong Engineering Corporation Limited, Hangzhou, China

*Corresponding author

ABSTRACT

Different from the uniform inflow condition, the wind turbine actually operates in the atmospheric boundary layer flow, wind shear and turbulence intensity in the ABL flow are considered to have great impacts on the behavior of wind turbines. In this paper, neutral stability boundary layer with different surface roughness height (0.001 m and 0.2 m) were simulated, large-eddy simulation (LES) coupled with the actuator line model (ALM) was used to study the performance of wind turbines under the neutral conditions. Focus is placed on mean wake characteristics and structural response. The results show that as the surface roughness increase, the mean shear and turbulence intensity of the inflow become larger, which makes the wake recovery and the dissipation of span-wise vorticity faster under the high roughness condition. In addition, maximum turbulence intensity and momentum flux are found to be located around the top-tip level as the stronger mean shear at that place. Long coherent turbulent structures in the high roughness condition also significantly affect the structural response of wind turbines, making an increase in the Root-Mean-Square (RMS) values and the Standard-Deviation (STD) values for the representative aerodynamic loads. What’s more, as the low speed shaft torque is resulted from the tangential force acting on the blade but not the axial force, its quantity is less sensitive to changes in the inflow conditions.

KEY WORDS: actuator line model; large eddy simulation; atmospheric turbulence; wake characteristics; structural response.

INTRODUCTION

For large commercial wind turbine, it actually operates in the lower part of the atmospheric boundary layer, instead of the uniform inflow condition studied in previous reports (Troldborg et al., 2010; Adaramola et al., 2011). What’s more, performance of the wind turbine is supposed to be greatly influenced by the characteristics of the ABL flows, which depend on many factors, such as the atmospheric stability, topography and surface roughness height, which is the focus of the present work. In the past, a lot of experimental and numerical studies have been carried out to explore the effects of atmospheric turbulence on wind turbines. Chamorro et al. (2009) performed a wind-tunnel experiment to explore the wake for a model wind turbine operating in atmospheric boundary layer with different roughness surfaces, as a result of the wind shear in the inflow, the spatial distribution of turbulence statistics is found not axisymmetric, similar conclusions are also drawn by other researchers (Porte-Agel et al., 2011; Maeda et al., 2011; Miguel et al., 2017). Hansen et al. (2012) simulated the flow inside a wind farm, emphasis is placed on the power generation, they found that power deficit is closely related to the ambient turbulence intensity, the higher turbulence intensity in the inflow, the smaller power deficit. Troldborg et al. (2007) used the 3D flow solver EllipSys3D together with actuator line model to simulate the wake of a wind turbine under the atmospheric turbulent flow without mean shear, the result reveals that large coherent structures in atmospheric wind is the main reason for the weak meandering phenomenon. Frandsen et al. (1997) studied loads acting on wind turbines in onshore and offshore conditions, and found that both the fatigue loads and extreme loads are higher in the offshore case. Churchfield et al. (2012) applied LES to generate atmospheric winds and simulate the wind turbine flows, they stressed the important impact of turbulent structures on the loads experienced by wind turbines.

However, among the above mentioned researches, most are limited to the partial behavior of wind turbines under the atmospheric boundary layer, few has studied systematically.

In this article, a more comprehensive study is conducted on the interaction between wind turbines and the atmospheric boundary layer flow. A brief introduction of the numerical methods and setup are firstly described; then, neutral atmospheric boundary layers with different surface roughness lengths are generated, and the detailed information about key features of the incoming flow, spatial distribution of mean wake characteristics and structural responses of wind turbine are displayed and analyzed; at last, conclusions are provided.
NUMERICAL METHODS

Actuator Line Model
Actuator line model is firstly proposed by Sørensen and Shen (2002) which is a simplify method to parameterize the turbine-induced forces. Compared with conventional method for resolving the real geometry of wind turbine, a large amount of computational resources is reduced by applying the ALM. Within such model, the blade is divided into several elements, by introducing the BEM theory with the information about the relative wind velocity and tabulated airfoil data, the lift and drag forces acting on each 2-D airfoil can be given by:

\[ L = \frac{1}{2} C_l \rho V_{\infty}^2 c d r \]
\[ D = \frac{1}{2} C_d \rho V_{\infty}^2 c d r \]

Where, \( C_l \) and \( C_d \) are the lift and drag coefficient, respectively. \( \rho \) is the air density, \( c \) is the chord length, \( d r \) is the blade segment width, \( V_{\infty} \) is the local relative velocity, which is expressed as:

\[ V_{\infty} = \sqrt{V_e^2 + (\Omega r - V_p)^2} \]

Where, \( V_e \), \( V_p \) and \( \Omega \) are the axial velocity, tangential velocity and angular velocity, respectively. \( r \) is the radial distance where the blade element is located.

The schematic of the velocities and forces acting on cross section of a blade element are depicted in Fig.1.

![Fig. 1 A cross-sectional airfoil element](image)

Governing Equation
In the present work, the large eddy simulation (LES) technique is used to solve the continuity equation and the filtered momentum equation, which are written as below:

\[ \frac{\partial \tilde{\rho}}{\partial t} + \nabla \cdot (\tilde{\rho} \tilde{u}) = 0 \]  
(5)

\[ \frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \nabla \tilde{u}_i = -\frac{1}{\rho} \nabla \cdot \tilde{p} + \frac{\partial}{\partial x_j} \left[ \frac{1}{2} \left( \nabla \tilde{u}_j + (\nabla \tilde{u}_j)^T \right) - \frac{\partial}{\partial x_j} \right] - \frac{1}{\rho} \tilde{F}_i \]  
(6)

Where, the overbar denotes spatial filtering, \( \tilde{u}_i \) is the resolved-scale velocity, \( i \) represents three orthogonal directions related to the inflow. What’s more, on the right hand side of the filtered momentum equation, Term I represents the background driving pressure gradient, which adjusted at every time step for generating a desired wind at the specified height; Term II is the modified pressure gradient; \( \tau_{ij} \) in Term III is fluid stress tensor, its divergence part is denoted by \( \tau_{ij}^D \), which consists of two parts: viscous part and SGS part; the Coriolis force is modeled in Term IV, which arising from the Earth rotation; \( F_i \) in Term V is the body force, it reveals the effect of wind turbines in the flow. The divergence part of stress tensor is unknown, and due to Reynolds number for the ABL flow is high, only the place near the solid surfaces need to take the viscous effects into account. That is to say, in the flow field except at the lower boundary surface, \( \tau_{ij}^D \) can be completed modeled by SGS effect, and it can be computed by:

\[ \tau_{ij}^D = -2\nu_{SGS} \hat{S}_{ij} \]

Where, \( \nu_{SGS} \) is the SGS viscosity, which can be calculated based on Smagorinsky model (Smagorinsky, 1963):

\[ \nu_{SGS} = (C_s \Delta)^2 (2 \hat{S}_{ij} \hat{S}_{ij})^{1/2} \]

Where, \( C_s \) is the Smagorinsky coefficient, which is set to 0.14, \( \Delta \) is the filter scalar, it is a deviation standard to distinguish the resolved scale and unresolved small scale; \( \hat{S}_{ij} \) is the resolved strain-rate tensor, \( P_f \) is turbulent Prandtl number, which is taken to be 1/3, a commonly used value for the simulation the neutral atmospheric boundary layer flow. While in the place near the lower boundary surface, it is necessary to consider the viscous effect, but directly resolving the large scales is computationally expensive for the mesh must be fine enough. Hence, in order to avoid such restriction, Moeng’s model (Moeng, 1984) is applied in this study, which has been proven to be efficient in characterize the stress tensor and temperature fluctuates. Surface roughness height \( z_0 \), horizontally averaged temperature flux \( q_t \) and friction velocity \( u_* \) are the three required inputs for Moeng’s model, and the first two parameter can be specified directly, the last one need to be estimated according to Monin–Obukhov similarity theory.

Simulation Setup
In the present work, the entire simulation is divided into two phases, namely the “precursor simulation” and the “wind turbine simulation”, respectively.

(1) Precursor simulation: in this stage, neutral stability boundary layer
with different surface roughness values are simulated, the surface roughness heights are set to 0.001 m and 0.2 m, which are typical of the offshore and onshore conditions, respectively, from here, they are referred to by "NBL-L" case and "NBL-H" case, respectively. The computational domain extends 3000 m in both horizontal directions and 1000 m in the vertical direction, and it is divided uniformly into $300 \times 300 \times 100$ grid points. In addition, the incoming wind is not set parallel to the x or y coordinate direction of the simulation domain, but deflects 30° to the southwest relative to x-axis, as apparent in Fig.2, this is done to avoid “stuck” phenomena caused by the application of periodic boundaries in lateral. What’s more, from 0 m to 700 m in the vertical direction, the potential temperature is taken to be 300 K and it increases linearly to 308 K in the next 100m upwards; the averaged wind speed at hub height level is driven to 8 m/s. All precursor simulations in the present work are run for 18000 s at first, to guarantee the generated flow field attains a quasi-steady state. Then the simulations run for another 1000 s, during that time, instantaneous data about velocity and temperature on the south and west lateral boundaries related to the inflow direction are saved at every time step, which are used as the inflow conditions for the wind turbine simulations.

RESULTS AND DISCUSSIONS

Key features of the incoming ABL flows

After the run of percursor simulations, several results about the incoming wind are obtained. Fig.3 presents the vertical profile of wind speed and turbulence intensity, which are the two important characteristics of the inflow. As evident in Fig.3(a), we can see that wind speed at hub height level is almost the same as the set value, with the ratio around 1.0, but for different cases, there is a large discrepancy in wind shear, for example, vertical change in wind velocity across the rotor plane is about 1.59m/s for the “NBL-L” case but 2.52m/s for the “NBL-H” case. In addition, it is also clear that turbulence intensity is decreased with height, as shown in Fig.3(b), and its value is larger in the high roughness condition. Specifically, the turbulence intensity is about 4.7% and 8.6% at the hub height level for the “NBL-L” case and “NBL-H” case, respectively.

Table 1. Gross properties of NREL-5MW Turbine

<table>
<thead>
<tr>
<th>Rating</th>
<th>5 MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Orientation, Configuration</td>
<td>Upwind, 3 Blades</td>
</tr>
<tr>
<td>Rotor Diameter, Hub Diameter</td>
<td>126 m, 3 m</td>
</tr>
<tr>
<td>Hub Height</td>
<td>90 m</td>
</tr>
<tr>
<td>Cut-in, Rated, Cut-out Wind Speed</td>
<td>3 m/s, 11.4 m/s, 25 m/s</td>
</tr>
<tr>
<td>Cut-in, Rated Rotor Speed</td>
<td>6.9 rpm, 12.1 rpm</td>
</tr>
<tr>
<td>Overhang, Shaft Tilt, Precone Angles</td>
<td>5 m, 5°, 2.5°</td>
</tr>
</tbody>
</table>

(a) Horizontal section at hub height

(b) Vertical section

Fig. 2 Overview of turbine layout and mesh refinement

(2) Wind turbine simulation: In this phase, wind turbines are immersed in the flow field, and corresponding to the first stage, two cases with a stand alone wind turbine under the inflow conditions generated in the precursor simulations are performed. The size of computational domain and background mesh in these wind turbine simulations are the same as that in the “precursor” cases, two refinement processes are carried out, Fig.2 shows the detail of turbine layout and mesh refinement. In particular, within the buffer zone, which extends 19D (where D is the rotor diameter) in the stream-wise direction and 2D outwards based on the center plane of wind turbine in lateral, the mesh resolution is 5 m in all directions. While for the inner box region surrounding the wind turbine and turbine wake, the mesh is refined again, to a uniform 2.5-m resolution. What’s more, for all cases in this paper, the wind turbine used is the NREL 5MW baseline wind turbines (Jonkman, 2009), the gross properties of it are listed in Table 1.
Apart from averaged quantities, we also pay some attentions on the instantaneous turbulent structures in the flow field, they are evolved in time and significantly affect the structural responses and the fatigue performance of wind turbines, which will be illustrated later. Visualizations of turbulence structures are presented in Fig. 4, iso-surface of vertical velocity fluctuations with value of +0.8 m/s are rendered in red and horizontal fluctuations with value of -1 m/s are colored by blue. Obviously, compared with the “NBL-L” case, much more low velocity stream-wise structures are observed in the “NBL-H” case and they are almost aligned with the inflow wind. In addition, a horizontal contour plane about temperature is plotted at the lower height level, with light color representing the warmer air and dark color denoting the cooler air, it can be seen that the temperature change is quite small in both cases, indicating that the inflow wind is less disturbed, which is consistent with the few vertical structures in the flow field.

Mean Wake Characteristics
Firstly, let us take a look at characteristics of mean stream-wise velocity in both the central wake plane and the horizontal hub height plane, which are shown in Fig.5 and Fig.6. Since the turbine extracts wind energy from the inflow, a region with reduced velocity is produced behind the rotor plane, which is the wake region. With increasing downstream distance, the wake expands outwards in radial direction, and different to that in lateral, the wake expansion is blocked by the ground surface in the far wake region in vertical, which will slow the wake recovery process. In addition, it is also clear that the velocity in wake region is lower near the turbine rotor and then it gradually recovers as the wake propagates downstream. What’s more, we can find that the wake region is smaller in the “NBL-H” case, due to the stronger ambient turbulence, which will enhance the mixing effect and make more momentum to be transferred into the wake. Moreover, in order to have a quantitative understanding about the stream-wise velocity in wake region, both lateral and vertical profiles of normalized velocity deficit at different downstream locations are analyzed, which are displayed in Fig 5(c) and Fig 6(c), starting from 1D ahead of the turbine rotor. Where, Δu is the averaged velocity, u₀ is the mean inflow wind speed at hub height. As shown in these figures, it is clear that as the surface roughness increases, the wake recovery process becomes faster. In addition, velocity deficit profiles for both the two cases studied in the present work show self-similarity and axial symmetry with a maximum value near the turbine hub.
In most existing empirical wake models, only wake characteristics on the horizontal hub height plane are considered for simplicity, and wake width is one of critical parameters for building an efficient analytical model, in the past, many possible ways have been proposed to define the wake width, for example:

(I) take the position where the wake velocity equals 95% of the inflow speed (Qian et al., 2018);

(II) take the distance between the central wake plane and the point where the velocity deficit is half of its maximum (Bastankhah et al., 2014; Abkar et al., 2015).

However, for the above two methods, measurement uncertainty is a big matter, which makes it difficult to accurately identify the wake boundary. In order to circumvent such limitation, another way is adopted in the present work, the standard deviation of Gaussian fitting to the velocity-deficit profile is regarded as the characteristics wake width. Before analyzing the width of turbine wake, let us take a look at Fig.7, which displays the velocity profile at different downstream locations in the neutral, low-roughness condition. From Fig.7, it can be seen that the velocity distribution in turbine wake is approximately fitted to a Gaussian curve except at the edge. Therefore, it is reasonable to apply a Gaussian fitting to the velocity profile and take the standard deviation of it as wake width. What’s more, according to such method, wake boundaries are plotted in Figure 6, donated by black dash lines. After determining the wake width, properly estimate the variation of it over downstream distance is also important for predicting the velocity distribution in wake region. And for a better comparison and analysis, standard deviations of turbine wake at different downwind locations in the range of 5 < x/D < 10 are drawn in Fig. 8. From the figure, it can be clearly seen that the wake expansion is around linearly with x/D, and for a high roughness condition, a faster growth rate of wake width is found, which provides a basis for establishing and verifying a new wake model in the future.

In addition to the velocity distribution, difference in aerodynamic surface roughness height also affects the vortex behavior in wake region. Fig.9 shows the contours of the averaged span-wise vortex in difference cases. From the figure, it can be seen that positive and negative vorticities are distributed at the upper and lower part of the rotor plane, respectively, which indicates the alternate shedding of the blade vortex. What’s more, behind the nacelle, an opposite distribution of span-wise vorticity appears, with negative in the upper and positive in the lower, it reveals that the shearing behavior of hub vortex is different from that of the blade vortex. Moreover, one can also observe that blade tip vortex dissipates slower in the top height level, as a result of the lower turbulence intensity in that place. Also because of the lower ambient turbulence, the span-wise vorticity in the “NBL-L” case is found persisting over a longer distance.
increases, the turbulence intensity decreases. What’s more, it is also clear that there is a close relationship between the magnitude of the turbulence intensity and the inflow conditions. Specifically, a higher ambient turbulence in the “NBL-H” case leads to a stronger turbulence level in the turbine wake, then causing a faster wake recovery shown in Fig.5(b) and Fig.6(b). And the location of maximum turbulence intensity in turbine wake is also closer to the rotor plane under high roughness condition, for instance, in the “NBL-H” case, the maximum appears at approximately x/D=2, while in the “NBL-L” case, it occurs at about x/D =3.

What’s more, it is also clear that there is a close relationship between the magnitude of the turbulence intensity and the inflow conditions. Specifically, a higher ambient turbulence in the “NBL-H” case leads to a stronger turbulence level in the turbine wake, then causing a faster wake recovery shown in Fig.5(b) and Fig.6(b). And the location of maximum turbulence intensity in turbine wake is also closer to the rotor plane under high roughness condition, for instance, in the “NBL-H” case, the maximum appears at approximately x/D=2, while in the “NBL-L” case, it occurs at about x/D =3.

(A) The neutral, low-roughness case

(B) The neutral, high-roughness case

Fig. 10 Contours of the time-averaged turbulence intensity in the central wake plane (y=0)

Fig. 11 Contours of the time-averaged turbulence intensity in the horizontal plane at hub height

In order to further explore the effect of atmospheric turbulence on the wake filed of wind turbines, momentum flux is also investigated. Fig. 12 and 13 display contours of the vertical momentum flux in the central wake plane and the lateral momentum flux in the horizontal hub height plane, respectively. As evident in these figures, the vertical momentum flux is negative in the upper part of the wake and positive in the lower part, the span-wise momentum flux is also negative at one side and positive at the other side, which reveals that the movement of momentum flux is from the ambient wind to the wake center. In addition, similar as the distribution of turbulence intensity, the absolute values of momentum flux are also higher at the top-tip level, where the mean shear is stronger, and the location of maximum in “NBL-H” case is also closer to the turbine rotor compared with that in the “NBL-L” case. What’s more, under the high roughness condition, the magnitude of the momentum flux is also larger, indicating that the momentum exchange is stronger. As a result, more kinetic energy is entrained into the turbine wake, which accelerates the recovery process of velocity deficit. It is consistent with the shorter wake region and wider wake expansion as presented in the “NBL-H” case.

(A) The neutral, low-roughness case

(B) The neutral, high-roughness case

Fig. 12 Contours of the time-averaged vertical momentum flux in the central wake plane (y=0).

Fig. 13 Contours of the time-averaged span-wise momentum flux in the horizontal plane at hub height

Structural response

As described above, the turbulent structures presented in “NBL-L” and “NBL-H” conditions are quite distinct, which is expected to have great impacts on wind turbines, especially the structural response. In this part, three representative aerodynamic loads of wind turbines are examined, which are defined mathematically as:

\[
M_{\text{op}} = \sum_{i} (\frac{1}{r_i} \times \bar{F}_{i,j}) \cdot \bar{e}_{\theta,i} \\
M_{\text{yaw}} = \sum_{i} (\bar{b}_{i,j} \times \bar{F}_{i,j}) \cdot \bar{e}_{a} \\
M_{\text{hub}} = \sum_{i} (\bar{f}_{i,j} \times \bar{F}_{i,j}) \cdot \bar{e}_{\text{hub}}
\]

where, \(i\) is the index of radial blade sections, \(j\) is the index of blades,
$N_b$ is the total number of blades, $N_r$ is the total number of radial blade sections, $\vec{F}_{i,j}$ is the force vector located at the $i$ th radial blade section on the $j$ th blade, $\vec{r}_{i,j}$ is the distance vector between the rotor hub and the $i$ th radial blade section on the $j$ th blade, $\vec{b}_{i,j}$ is the distance vector between the intersection of tower axis and low-speed shaft axis and the $i$ th radial blade section on the $j$ th blade, $\vec{e}_{d,j}$ is a unit vector tangent to the rotation direction of the $j$ th blade, $\vec{e}_t$ is a unit vector aligned with the tower axis, $\vec{e}_{ls}$ is a unit vector aligned with the low-speed shaft.

$M_{oop}$: a bending moment acting at the root of each blade, which affects the fatigue characteristics of blade and its attachment to the turbine hub.

$M_{ls}$: a torque generated by the low-speed shaft, which is proportional to the power generation.

$M_{yaw}$: a side-side bending moment experienced by the nacelle yaw bearing, which has a great impact on the yaw driving system.

In order to better quantify the structural response of wind turbines, the Root-Mean-Square (RMS) values and Standard-Deviation (STD) values of the time history of the above three loadings are calculated, and the results are presented in Fig 14 and Fig 15, respectively. In general, it can be seen that the RMS values and the STD values for all loads examined here are higher in “NBL-H” case than that in “NBL-L” case, which is attributed to the stronger ambient turbulence intensity and long coherent structures in the high roughness condition. Specifically, due to the existence of long lines of low speed stream-wise structures in the “NBL-H” case, the turbine rotor is often partially engulfed, which makes the blades enter and leave turbulent structures periodically, thus, causing an increase in the RMS and STD values of $M_{oop}$. What’s more, with the exception of $M_{yaw}$, its statistic values are also increased as a result of the occurrence of lateral asymmetric flow. In addition, one can also observe that the STD of $M_{oop}$ is relatively higher, for only the aerodynamic force acting on a single blade is considered in its calculation, instead of the entire rotor, which makes $M_{oop}$ being more sensitive to changes in the inflow. Conversely, through comparison, the statistical values of $M_{ls}$ are relatively smaller, as such moment is resulted from the tangential force but not the axial force, with the former being much smaller than the last. Hence, both the RMS and STD quantities of $M_{ls}$ are lower than that of the other two loads examined in this paper, which also reveals that $M_{ls}$ is less sensitive to aerodynamic turbulence.

CONCLUSIONS

In this paper, LES coupled with the actuator line model was used to investigate the effect of aerodynamic turbulence on the performance of a stand alone wind turbine. Neutral stability boundary layer with different surface roughness height (0.001 m and 0.2 m) were simulated, which are typical of offshore conditions and onshore conditions, respectively.

At first, we analyzed the mean wake characteristics, the results reveal that inflow conditions have significant impacts on the distribution of key turbulence statistics of turbine wake. In the “NBL-H” case, as a result of high level of ambient turbulence, both the wake recovery and dissipation of the span-wise vorticity are faster. In addition, due to the stronger mean shear at the top tip level, maximum turbulence intensity and momentum flux are located around that place, and the location is closer to the rotor plane as the surface roughness increase. What’s more, wake width on the horizontal hub height plane was also analyzed by adopting the Gaussian fitting method, standard deviation of the fitting curve to the velocity deficit profile was regarded as the characteristics wake width. The turbine wake is found to expand linearly with the distance to the rotor plane, and under the high roughness condition, a faster growth rate is visible, which provides a basis for establishing and verifying new wake models in the future.

Next, a closer look was taken at the structural response of wind turbines, three representative aerodynamic loads are examined, including the blade-root bending moment ($M_{oop}$), yaw bearing moment ($M_{yaw}$) and low speed shaft torque ($M_{ls}$), they are also greatly affected by the inflow condition, especially the turbulent structures in the flow field. Both the RMS values and the STD values for all three moments are increased as the surface roughness is increased. In addition, since $M_{ls}$ is resulted from the tangential force acting on the blade but not the axial force, with the former being much smaller than the later, hence, the low speed shaft torque ($M_{ls}$) is less sensitive to changes in the inflow.

Apart from aerodynamic roughness height, there are many other factors affect the performance of wind turbines, for example, atmospheric stability and topography. Hence, in the further, more numerical and experimental studies should be performed to investigate the wake characteristics and structural response of wind turbines under different conditions.
ACKNOWLEDGEMENTS

This work is supported by the National Natural Science Foundation of China (51879159), The National Key Research and Development Program of China (2019YFB1704200, 2019YFC0312400), Chang Jiang Scholars Program (T2014099), Shanghai Excellent Academic Leaders Program (17XD1402300), and Innovative Special Project of Numerical Tank of Ministry of Industry and Information Technology of China (2016-23/09), to which the authors are most grateful.

REFERENCES


