Introduction to Marine Hydrodynamics (NA235)

(2014-2015, 2nd Semester)

Assignment No.10

(5 problems, given on June 18th, need not submit)

Problem 1: A test for drag force on a submarine is performed in a wind tunnel. The length ratio of the model to prototype is $1/10$. Following parameters of the test have been measured. Pressure in the wind tunnel is 20 atmospheric pressures. Wind speed in the tunnel equals $12\,m/s$, and the drag force on the model is $120N$. If flow in the tunnel is laminar, please ① estimate the corresponding speed of the prototype submarine, and ② estimate the corresponding power needed to drive the prototype submarine at that speed.

Solution: ① Under 1 atmospheric pressure at 15 degrees Celsius, kinematic viscosity of water and air are equal to

$$\nu_w = 1.14\times10^{-6}\left(m^2/s\right) \qquad \text{and} \qquad \nu_a = 1.46\times10^{-5}\left(m^2/s\right)$$

respectively. Assuming air in the wind tunnel is isothermal, under 20 atmospheric pressure air density increases to 20 times of the one under 1 atmospheric pressure, accordingly its kinematic viscosity decreases to one 20th of the one under 1 atmospheric pressure. That is,

$$\nu_{a(20atm)} = \frac{\nu_{a(1atm)}}{20} = \frac{1.46\times10^{-5}}{20} = 7.30\times10^{-7}\left(m^2/s\right).$$

According to Reynolds similarity,

$$\frac{u_a \cdot l_a}{\nu_a} = \frac{u_w \cdot L_w}{\nu_w}$$

corresponding to the speed $12\,m/s$ in wind tunnel speed of the prototype submarine is immediately evaluated

$$u_w = u_a \cdot \frac{\nu_w}{\nu_a} \cdot \frac{l_a}{L_w} = 12 \times \frac{1.14\times10^{-7}}{7.30\times10^{-6}} \times \frac{1}{10} = 1.874\left(m/s\right).$$

② Since the flow state is laminar, drag coefficient is determined by Reynolds number. Accordingly drag coefficient of the prototype submarine is identical to the one of the model, that is,

$$C_{Da} = \frac{R_a}{\frac{1}{2}\rho_a u_a^2 A_a} = \frac{R_w}{\frac{1}{2}\rho_w u_w^2 A_w} = C_{Dw}$$

Drag force of the prototype submarine is thus obtained

$$R_w = R_a \frac{\rho_w}{\rho_a}\frac{u_w^2}{u_a^2}\frac{A_w}{A_a} = 120 \times \frac{999.1}{1.226\times 20} \times \frac{1.875^2}{12^2} \times (\frac{10}{1})^2 = 11924\left(N\right) = 11.924\left(kN\right),$$

and the corresponding power is

$$P = R_w u_w = 11.925 \times 1.874 = 22.345\left(kW\right).$$

Problem 2: Given ratio of a ship model to its prototype, $1:50$. In a model test, when the model is towed at a speed of $u_m = 1.33 m/s$, measured the drag force, $r_m = 9.81N$. Please estimate the speed of and the drag on the prototype ship for the following two cases. ① If the dominant drag is due to wave generation. ② If the dominant drag is due to viscous friction.

Solution: ① In case of wave generation is dominated, Froude similitude is satisfied, that is,

$$\frac{u_m}{\sqrt{gl_m}} = \frac{u_s}{\sqrt{gL_s}}$$

from which speed of the prototype ship is evaluated

$$u_s = u_m\sqrt{\frac{L_s}{l_m}} = 1.33\times\sqrt{\frac{50}{1}} = 9.40\left(m/s\right)$$

and the corresponding drag force is estimated in terms of identical drag coefficient, that is,

$$R_s = \frac{\frac{1}{2}\rho_s u_s^2 L_s^2}{\frac{1}{2}\rho_m u_m^2 l_m^2}\cdot r_m = \frac{L_s^3}{l_m^3}\cdot r_m = (50)^3 \times 9.81 = 1226250\left(N\right) = 1226.25\left(kN\right).$$

② In case of viscosity is dominated, Reynolds similitude is satisfied, that is,

$$\frac{u_m \cdot l_m}{\nu_m} = \frac{u_s \cdot L_s}{\nu_s}.$$

Since fluid used in model test is water, the same as the one in prototype ship voyage, speed of prototype ship is immediately estimated from above Reynolds similarity

$$u_s = u_m \cdot \frac{l_m}{L_s} = 1.33 \times \frac{1}{50} = 0.0266\left(m/s\right)$$

similar to ①, the corresponding drag force is

$$R_s = \frac{\frac{1}{2}\rho_s u_s^2 L_s^2}{\frac{1}{2}\rho_m u_m^2 l_m^2} \cdot r_m = r_m = 9.81\left(N\right).$$

Problem 3: A resistance test on a ship model, scaled from the prototype at ratio $1:40$, in towing tank was performed. Total resistance of the ship model measured is $3.2kgf$. It is known that frictional resistances of the ship model and the prototype can be estimated from expressions $0.37u^{1.95}(kgf/m^2)$ and $0.29u^{1.8}(kgf/m^2)$ respectively, where u is the ship speed in m/s. Now the prototype ship is traveling at sea (salt water) at speed $12\,m/s$, please estimate its total resistance. Area of the wetted hull surface of the prototype ship is equal to $2500m^2$. The towing tank is filled with fresh water.

Solution: In ship resistance test, Froude similitude is satisfied. From it speed of the ship model is evaluated as follows

$$u_m = u_s \lambda^{-1/2} = 12 \times \frac{1}{\sqrt{40}} = 1.897\left(m/s\right)$$

According to the expressions given in the problem, frictional resistances of the ship model and the prototype ship are estimated

$$r_f = 0.37u_m^{1.95}\Omega_m = 0.37 \times 1.897^{1.95} \times \frac{2500}{40^2} = 2.01\left(kgf\right)$$

and

$$R_f = 0.29 u_s^{1.8} \cdot \Omega_s = 0.29 \times 12^{1.8} \times 2500 = 63513.2(kgf) = 63.513(tonf).$$

Residual resistance of the ship model is the difference of total resistance to the frictional resistance, that is,

$$r_w = r_t - r_f = 3.2 - 2.01 = 1.19(kgf).$$

It is assumed that residual resistance is mainly due to wave generation, and Froude similitude govers, that is,

$$\frac{u_s}{u_m} = \sqrt{\frac{L_s}{l_m}}$$

then

$$R_w = r_w \cdot \lambda^3 \cdot \frac{\rho_s}{\rho_m} = 1.19 \times 64000 \times 1.025 = 78064(kgf) = 78.064(tonf).$$

Total resistance of the prototype ship equals the sum of frictional one and the residual one, that is,

$$R_t = R_f + R_w = 63.513 + 78.064 = 141.577(tonf).$$

Problem 4: Resistance of a sphere in an unbounded fluid domain at very low speed is investigated by Stokes. It is found that the resistance varies with the speed V, dynamic viscosity μ and diameter D of the sphere, but independent on the fluid density. Please find the dependence of resistance F on those parameters by means of dimensional analysis.

Solution: Assume dependence of resistance, F, on those parameters as follows

$$F = kV^a \mu^b D^c$$

where k is a dimensionless constant, a, b and c are constants in the powers to be determined. According to basic dimensions of mass $[M]$, length $[L]$ and time $[T]$, from above equation a dimension equation is derived as follows

$$[M][L][T]^{-2} = ([M][L][T])^0 \left([L][T]^{-1}\right)^a \left([M][L]^{-1}[T]^{-1}\right)^b ([L])^c.$$

Due to dimension homogeneity, 3 algebraic equations are derived from the above dimension equation, that is,

$$[M]:\quad 1=b$$
$$[L]:\quad 1=a-b+c$$
$$[T]:\quad -2=-a-b$$

Solution of above linear equations is $a=b=c=1$. Therefore,

$$F=kV^a\mu^bD^c=k\mu VD.$$

Problem 5: An ocean wave of height H is generated by a wind blowing at speed U. Denote densities of air and water as ρ_a and ρ respectively. Water depth is d, and the wave is away from the shore with distance L. Gravitational acceleration is denoted by symbol g. Generally wave height H depends on all those parameters. Please find the general dependence based on Π theorem.

Solution: The dependence is generally written as

$$H=f\left(U,\rho_a,\rho,d,L,g\right).$$

Since this problem is a dynamic problem, the mass $[M]$, length $[L]$ and time $[T]$ are conventionally used as basic dimensions. Among all the 7 parameters, 3 of them are chosen as fundamental parameters, other 4 can be expressed as a dimensionless number, i.e., Π number derived from one of the other parameter and the selected 3 fundamental numbers. Choice of fundamental parameters is relatively arbitrary. Here, ρ, U and d are chosen as the fundamental parameters. Then, 4 Π numbers can be easily derived. As a result, they are

$$\Pi_1=\frac{H}{d},\quad \Pi_2=\frac{\rho_a}{\rho},\quad \Pi_3=\frac{L}{d},\quad \Pi_4=\frac{gd}{U^2}.$$

According to Π theorem, the general dependence is written as

$$\Pi_1=\phi\left(\Pi_2,\Pi_3,\Pi_4\right)$$

or more clearly

$$\frac{H}{d}=\phi\left(\frac{\rho_a}{\rho},\frac{L}{d},\frac{gd}{U^2}\right)$$

where function ϕ can not be explicitly determined from dimension analysis, instead can be determined by experiments in common.