Introduction to Marine Hydrodynamics (NA235)

(2014-2015, 2nd Semester)

Assignment No.7

(8 problems, given on May 18th, submitted on May 28th, 2015)

Problem 1: Given a deep water wave with period, $\tau = 5s$, wave height, $H = 1.2m$. Calculate wavelength, phase velocity (celerity), group velocity and energy transmission rate of the wave.

Problem 2: Given a deep water wave of wavelength $6.28m$. How deep from the free surface, wave height will be reduced to half of the one at the free surface?

Problem 3: In a water field of depth $H = 10m$, there is a surface wave of amplitude, $a = 1m$, and wave number, $k = 0.2m^{-1}$. ① Calculate wavelength, phase velocity and period of the wave. ② Give out the equation of the wave elevation. ③ Write down equation of the path of a water particle at $x_0 = 0$ and $z_0 = -5m$.

Problem 4: Given two deep water waves with wavelength $15m$ and $150m$ respectively. ①Evaluate their wave velocities (*i.e.* phase velocities) and periods. ②Discuss variations when they propagates from deep water into a shore of $10m$ deep.

Problem 5: Set a buoy on a sea of depth $H = 6.2m$. Under the excitation of a water wave, the buoy is moving up and down (*i.e.* heaving) periodically at a rate of 12 times a minute. The wave height is measured of $h = 1.2m$. Calculate the wave length and amplitudes of the velocity and dynamic pressure of a particle at the seabed.

Problem 6: Two kinds of fluid are separated by a horizontal plane. Their thicknesses are assumed great enough. The upper layer fluid is known of density ρ', and the lower layer of density ρ. ①Show that a surface wave of wave length λ on the separation plane will propagate at the velocity

$$c = \sqrt{\frac{g\lambda}{2\pi}\frac{\rho - \rho'}{\rho + \rho'}}$$

②Show that for any group waves, group velocity is just equal to half of the wave velocity.

Problem 7: Show that for deep water waves, hydrodynamic pressure of any fluid particle just equals the hydrostatic pressure of the particle at the equilibrium position in calm water, that is,

$$\frac{P}{\rho} + gz_0 = const\,.$$

Problem 8: A standing wave is formed near a vertical breakwater. Show that a water particle with equilibrium position (x_0, z_0) will trace along the flowing straight line

$$\frac{z - z_0}{x - x_0} = -\tanh k(H + z_0)\cot kx_0$$

where H is the water depth. *Oxz* is a Cartesian coordinate system with *Ox* horizontal and perpendicular to the breakwater, and *Oz* vertical, upward positive.