# Introduction to Marine Hydrodynamics (NA235)

(2014-2015, 2nd Semester)

## Assignment No.6

(9 problems, given on May 4, submitted on May 14th, 2015)

**Problem 1:** Inside a large sphere of radius $R$ fills with incompressible perfect fluid. A small ball of radius $a$ is moving in it at speed $V(t)$. At initial instant $t_0$, the small ball is concentric with the large sphere. Please write down governing equations and boundary conditions that velocity potential of the flow between them obeys.

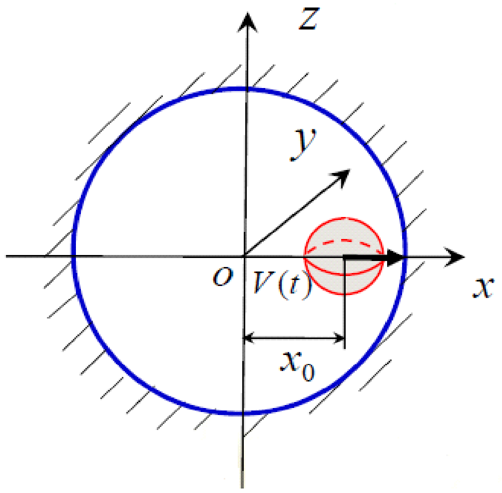


**Figure 6-1**

**Problem 2:** Given a planar potential flow, in which there are a point source of intensity $Q_1$=20$m^3/s$ at point (-1, 0), and a point sink of intensity $Q_2$=40$m^3/s$ at point (2, 0). Density of the fluid is $\rho = 1.8\, kg/m^3$. If pressure at origin is 0, please calculate velocities and pressures at point (0, 1) and at point (1, 1) respectively.

**Problem 3:** Velocity field of a flow is given as follows,

$$u = y + 2z, \quad v = z + 2x, \quad w = x + 2y$$

① Determine the vorticity field and write down the equation of vortex lines; ② Calculate the flux of vorticity in a cross section of area $dS = 0.0001m^2$ on the plane $x + y + z = 1$.

**Problem 4:** A flow is a superposition of a uniform flow of speed $u_0$= 10$m/s$ along positive $x$-axis with a point vortex at origin. If a stagnation point is at (0, -5), please ① determine the intensity of the vortex; ② calculate the velocity at (0, 5); ③ write down the equation of the streamline passing through the stagnation point.

**Problem 5:** A three dimensional axle symmetric flow with velocity potential

$$\varphi = U_0 r \left(1 + \frac{a^3}{2r^3}\right)\cos\theta$$

where $U_0$ and $a$ are constants, $\theta$ is the polar angle from the symmetric axle, and $r$ is the radial distance from origin. ① Prove that for $r \geq a$ it is equivalent to the flow of a uniform flow past a fixed sphere of radius $a$.

② Determine positions on the sphere at which the velocities take maximum value and the value of $U_0$ respectively.

**Problem 6:** An axle symmetric flow is generated by a point source of intensity $m_1 = 60\ m^3/s$ at the origin and another point source of intensity $m_2 = 30\ m^3/s$ at (0, 0, 2). Calculate velocities at (-1, -2, 0) and (1, 1, 1).

**Problem 7:** A sphere is fixed in a uniform flow field. If $a$ is radius of the sphere, $U_0$ and $P_0$ are the speed and pressure of the uniform flow, find the maximum and minimum pressures on the sphere and their positions.

**Problem 8:** Given $\varphi = V_0(r + \frac{a^2}{r})\cos\theta$ the velocity potential of a circular flow. Please calculate the resultant hydrodynamic force on the semi-circle in Figure 6-2.

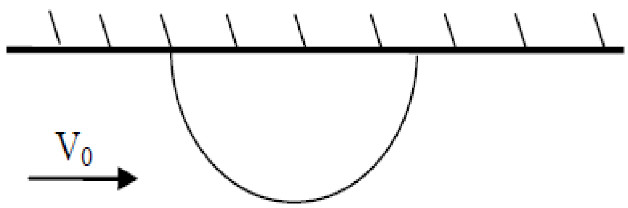


**Figure 6-2**

**Problem 9:** A circular cylinder of radius $R$ and length $L$ is suspended at a fixed point $O$ with thin light ropes *OA* and *OB*. Denote $A$ and $B$ the two end points of the axle of the circular cylinder. Points $O$, $A$ and $B$ forms a isosceles triangle, i.e., $\overline{OA} = \overline{OB}$. Denote $o$ the midpoint of the axle of the circular cylinder, it is given that $\overline{oO} = l$. Now axle *AB* of the circular cylinder is globally rotating around the fixed point $O$ at an angular velocity $\Omega$, and concurrently the circular cylinder is locally rotating around its axle *AB* at an angular velocity $\omega$. Given weight of the cylinder is *G*, fluid density is $\rho$, and $l \gg R$. Calculate tensions applied to the ropes *OA* and *OB* respectively.