Introduction to Marine Hydrodynamics (NA235)

(2014-2015, 2nd Semester)

Assignment No.5

(Seven problems, given on Apr 16, submitted on Apr 27, 2015)

Problem 1: Given velocity field of a flow:

$$u = y + 2z, \quad v = z + 2x, \quad w = x + 2y$$

Determine: (1) Vorticity field of the flow and the equation of vortex lines; (2) Vortex strength passing a cross section with area $dS = 0.0001m^2$ on the plane $x + y + z = 1$.

Problem 2: A planar fluid flow is given in a polar coordinate system:

$$v_r = U_0(1 - \frac{a^2}{r^2})\cos\theta, \quad v_\theta = -U_0(1 + \frac{a^2}{r^2})\sin\theta + \frac{k}{r}$$

where a, k, U_0 are constants. Determine the velocity circulation around an arbitrary closed curve, which encloses the circle centered at the origin of radius $r = a$.

Problem 3: Given velocity distribution of a flow: $u = -\omega y, \quad v = \omega x$.

Determine (1) Velocity circulation around the circle with a radius R and the vortex flux passing through the area surrounded by that circle; (2) Velocity circulation around closed curve *abcd* (see Figure 5-3) and the vortex flux passing through the area bounded by that curve.

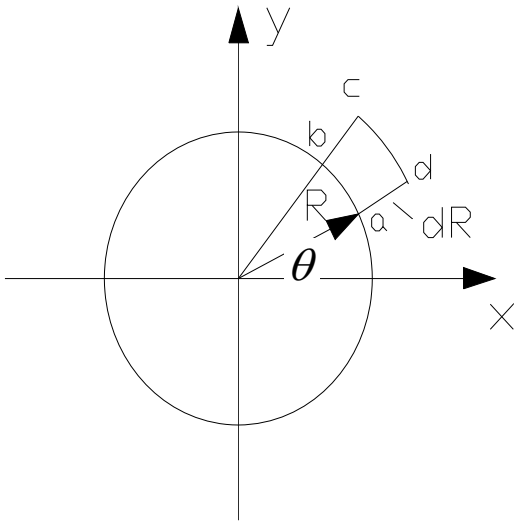

Figure 5-3

Problem 4: Suppose an ideal fluid is barotropic and under the action of body forces with potential Θ. Now if at an instant velocity field $\vec{V}$ of such a flow is irrotational, then verify that the corresponding local acceleration field $\frac{\partial \vec{V}}{\partial t}$ will be irrotational as well at any instant. Furthermore, derive the theorem that in that case vortex can be neither created nor destroyed.

Problem 5: Four vortices with an equal strength Γ initially located at (1, 0), (0, 1), (-1, 0), (0, -1) respectively. Determine the path for each of them.

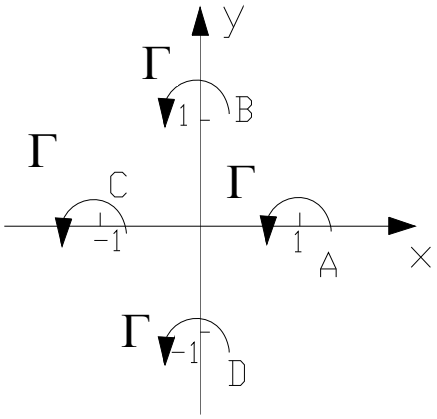

Figure 5-5

Problem 6: Suppose a circular vortex line, whose radius is *a*, and strength is Γ. Determine the induced velocity on the symmetry axis.

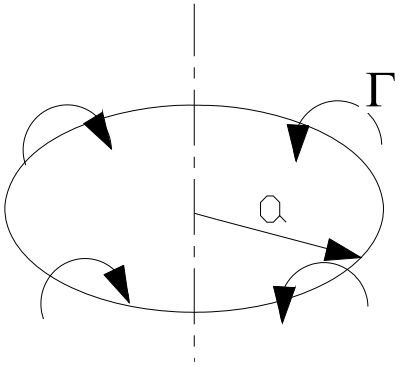

Figure 5-6

Problem 7: Two vortices at a distance *r* with strengths Γ_1 and Γ_2

respectively, of same magnitude $|\Gamma_1| \neq |\Gamma_2|$. Determine motions of these vortices for Γ_1 and Γ_2 with same or opposite signs.

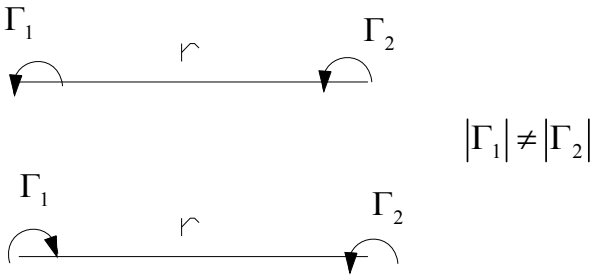

Figure 5-7