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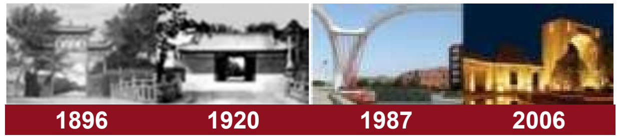


Introduction to Marine Hydrodynamics

(NA235)

Department of Naval Architecture and Ocean Engineering

School of Naval Architecture, Ocean & Civil Engineering

Shanghai Jiao Tong University

Ninth Assignment

- **The ninth assignment can be downloaded from following website:**

Website：ftp://public.sjtu.edu.cn

Username：dcwan

Password：2015mhydro

Directory：IntroMHydro2015-Assignments

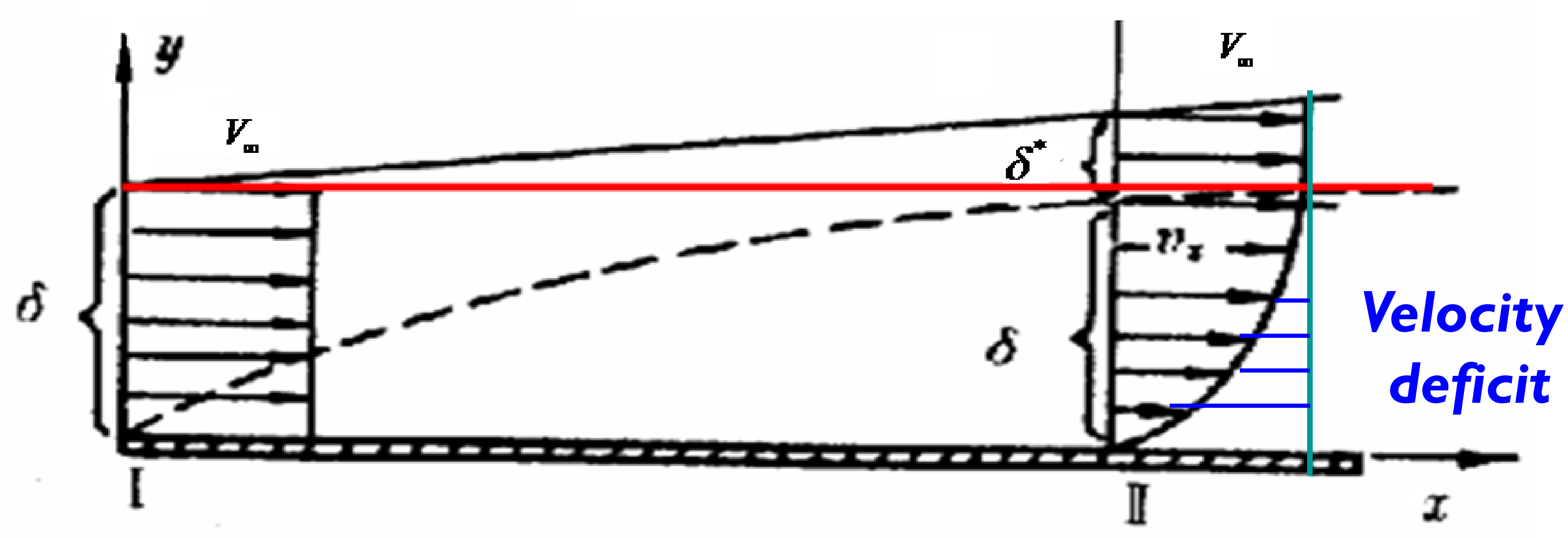
- **Seven problems**

- **Submit the assignment on June 18th (in English, written on paper)**

Chapter 9

Boundary Layer Theory

Boundary Layer Displacement Thickness δ^*



In boundary layer, there is a ***velocity deficit***, accordingly it causes a ***volume deficit***

$$\int_0^{\delta}(V_\infty - v_x)\mathrm{d}y$$

9.5 Momentum Integral Boundary Layer Equation

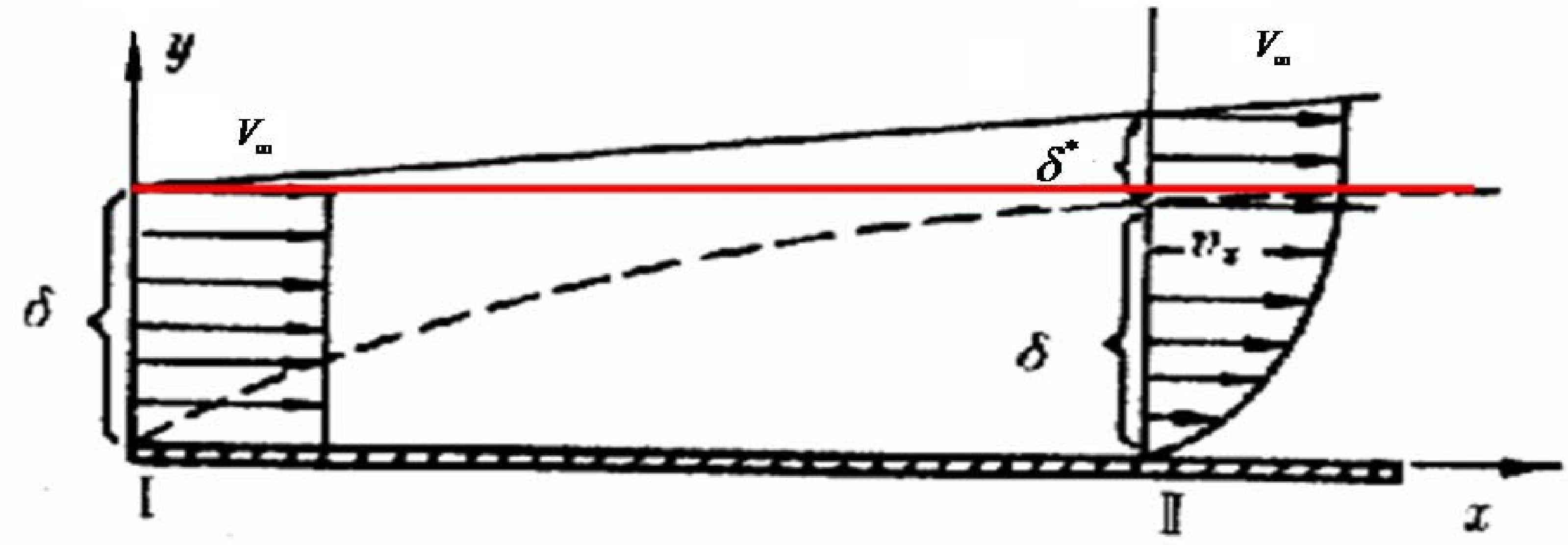
The volume deficit makes the streamline go out a distance, δ^* , namely ***boundary layer displacement thickness***. It is estimated from

$$V_\infty \delta^* = \int_0^\delta (V_\infty - v_x)\mathrm{d}y$$

$$\Rightarrow \quad \delta^* = \int_0^\delta \left(1 - \frac{v_x}{V_\infty}\right)\mathrm{d}y$$

Boundary Layer Momentum Thickness θ

Ⅰ and Ⅱ have the same flow rate, but different momentum. There exists a ***momentum deficit***.



Momentum Deficit

$$K_{\mathrm{I}} - K_{\mathrm{II}} = \int_0^\delta \rho V_\infty^2 \mathrm{d}y - \left(\rho V_\infty^2 \delta^* + \int_0^\delta \rho v_x^2 \mathrm{d}y \right)$$

$$= \int_0^\delta \rho V_\infty^2 \mathrm{d}y - \left(\rho V_\infty^2 \int_0^\delta (1 - \frac{v_x}{V_\infty}) \mathrm{d}y + \int_0^\delta \rho v_x^2 \mathrm{d}y \right)$$

$$= \int_0^\delta \rho v_x (V_\infty - v_x) \mathrm{d}y$$

Let a layer of uniform speed V_∞ and thickness θ have equivalent momentum to the momentum deficit.

$$\rho \theta V_\infty^2 = \int_0^\delta \rho v_x (V_\infty - v_x) \mathrm{d}y$$

$$\theta = \int_0^\delta \frac{v_x}{V_\infty} \left(1 - \frac{v_x}{V_\infty} \right) \mathrm{d}y$$

Boundary Layer Momentum Thickness

9.5 Momentum Integral Boundary Layer Equation

The upper limit can be replaced by ∞ without essential difference.

$$\delta^* = \int_0^{\delta} (1 - \frac{v_x}{V_\infty}) \mathrm{d}y = \int_0^{\infty} (1 - \frac{v_x}{V_\infty}) \mathrm{d}y$$

$$\theta = \int_0^{\delta} \frac{v_x}{V_\infty} \left(1 - \frac{v_x}{V_\infty}\right) \mathrm{d}y = \int_0^{\infty} \frac{v_x}{V_\infty} \left(1 - \frac{v_x}{V_\infty}\right) \mathrm{d}y$$

Three Thicknesses

Boundary Layer Thickness δ

Boundary Layer Displacement Thickness $\delta^* \approx \delta/3$

Boundary Layer Momentum Thickness $\theta \approx \delta/8$

9.5 Momentum Integral Boundary Layer Equation

Integrating boundary layer equation with respect to *y* from 0 to δ, it gives

$$\int_0^\delta \left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} \right) dy = \int_0^\delta \left(V_\infty \frac{dV_\infty}{dx} + \nu \frac{\partial^2 v_x}{\partial y^2} \right) dy$$

$$\int_0^\delta \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) dy = 0 \quad \Rightarrow \quad v_y = -\int_0^\delta \frac{\partial v_x}{\partial x} dy$$

Using previous expression of v_y , we have equality

$$\int_0^\delta v_y \frac{\partial v_x}{\partial y} dy = -\int_0^\delta \left(\frac{\partial v_x}{\partial y} \int_0^\delta \frac{\partial v_x}{\partial x} dy \right) dy$$

$$= -V_\infty \int_0^\delta \frac{\partial v_x}{\partial x} dy + \int_0^\delta v_x \frac{\partial v_x}{\partial x} dy$$

And the next definite integral

$$\int_0^\delta \nu \frac{\partial^2 v_x}{\partial y^2} dy = \int_0^\delta \frac{1}{\rho} \frac{\partial}{\partial y} \left(\mu \frac{\partial v_x}{\partial y} \right) dy = \int_0^\delta \frac{1}{\rho} \frac{\partial(\tau)}{\partial y} dy = \left[\frac{\tau}{\rho} \right]_0^\delta = -\frac{\tau_w}{\rho}$$

9.5 Momentum Integral Boundary Layer Equation

Substitute previous integrals in integral boundary layer equation, it gives

$$2\int_0^\delta v_x \frac{\partial v_x}{\partial x} dy - V_\infty \int_0^\delta \frac{\partial v_x}{\partial x} dy = \int_0^\delta V_\infty \frac{dV_\infty}{dx} dy - \frac{\tau_w}{\rho}$$

$$\frac{\tau_w}{\rho} = -\int_0^\delta \frac{\partial v_x^2}{\partial x} dy + \int_0^\delta \frac{\partial V_\infty v_x}{\partial x} dy - \int_0^\delta v_x \frac{\partial V_\infty}{\partial x} dy + \int_0^\delta V_\infty \frac{dV_\infty}{dx} dy$$

$$\frac{\tau_w}{\rho} = \int_0^\delta \frac{\partial}{\partial x} \left\{ V_\infty^2 \left[\frac{v_x}{V_\infty} \left(1 - \frac{v_x}{V_\infty} \right) \right] \right\} dy + \frac{\partial V_\infty}{\partial x} V_\infty \int_0^\delta \left(1 - \frac{v_x}{V_\infty} \right) dy$$

9.5 Momentum Integral Boundary Layer Equation

According to the definitions of displacement thickness and momentum thickness, ***Karman momentum integral equation*** as below is obtained.

$$\frac{\tau_w}{\rho} = \frac{\partial\left(V_\infty^2\theta\right)}{\partial x} + \frac{\partial V_\infty}{\partial x} V_\infty \delta^*$$

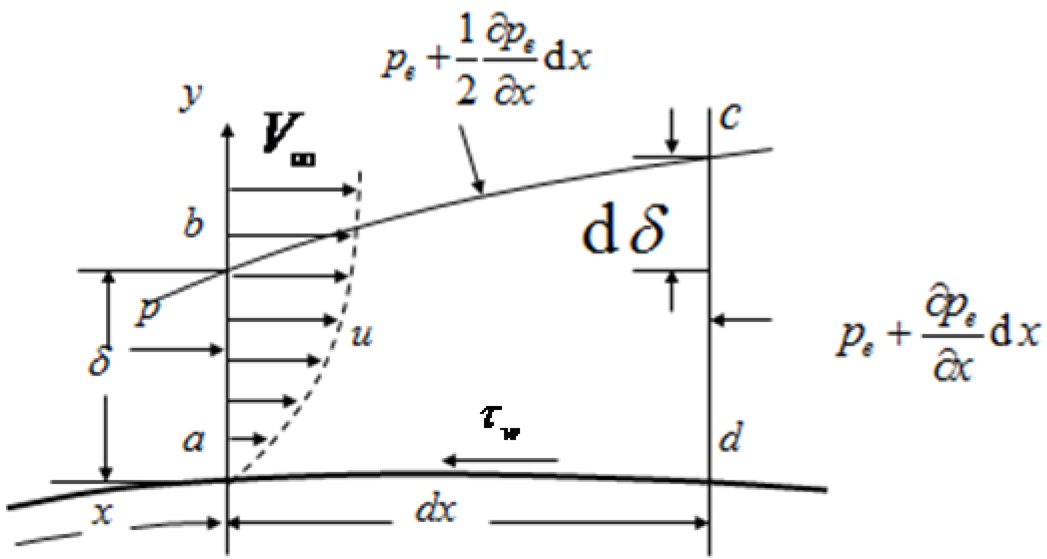
For a uniform flow under favorable pressure gradient, Karman momentum integral equation is further simplified.

$$\frac{\tau_w}{\rho V_\infty^2} = \frac{\partial \theta}{\partial x}$$

9.5 Momentum Integral Boundary Layer Equation

Karman Momentum Integral Equation — A relation between ***wall shear stress*** τ_w, ***boundary layer displacement thickness*** δ^* and ***boundary layer momentum thickness*** θ.

Momentum Conservation Method (another derivation)



At an arbitrary location, x, take a short piece volume of length *dx* with unit depth as a control volume. Surface S_{abcda} is a control surface.

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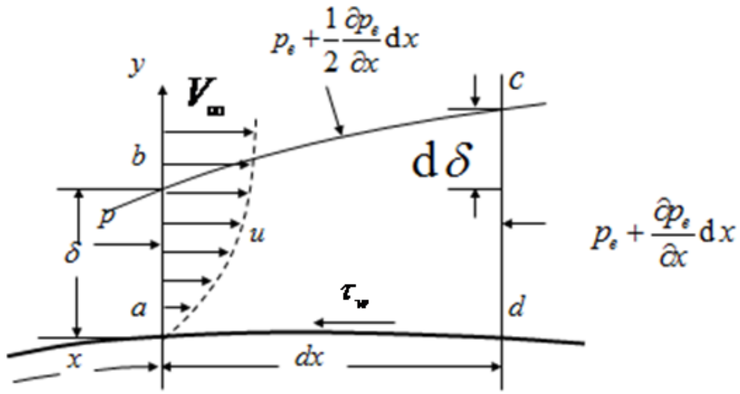
9.5 Momentum Integral Boundary Layer Equation

Momentum Conservation Law（*x*-component）

Momentum into control volume － momentum out of control volume ＝ Resultant force on the surface of control volume

$$K_{cd} - K_{ab} - K_{bc} = \sum F_x$$



Momentum flux into the volume

$$m_{ab} = \int_0^{\delta(x)} \rho v_x dy$$

$$K_{ab} = \int_0^{\delta(x)} \rho v_x^2 dy$$

$$K_{bc} = V_\infty \frac{\partial}{\partial x}\left(\int_0^{\delta(x)} \rho v_x dy\right) dx$$

Momentum flux out of the volume

$$m_{cd} = \int_0^{\delta(x)} \rho v_x dy + \frac{\partial}{\partial x}\left(\int_0^{\delta(x)} \rho v_x dy\right) dx$$

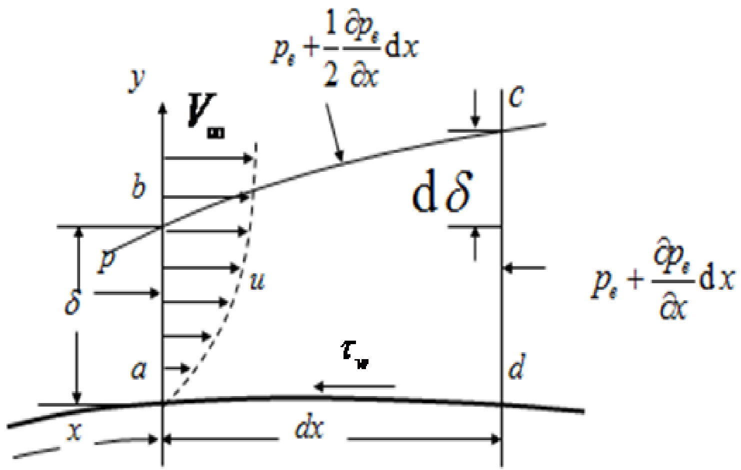
$$K_{cd} = \int_0^{\delta(x)} \rho v_x^2 dy + \frac{\partial}{\partial x}\left(\int_0^{\delta(x)} \rho v_x^2 dy\right) dx$$

Forces:

$$F_{ab} = p_e \cdot \delta \qquad F_{ad} = -\tau_w dx$$

$$F_{bc} = \left(p_e + \frac{dp_e}{dx}\frac{dx}{2} \right) d\delta$$

$$F_{cd} = -\left(p_e + \frac{dp_e}{dx} dx \right) (\delta + d\delta)$$



$$K_{cd} - K_{ab} - K_{bc} = \sum F_x \Rightarrow V_\infty \frac{\partial}{\partial x}\int_0^\delta v_x dy - \frac{\partial}{\partial x}\int_0^\delta v_x^2 dy - \frac{1}{\rho}\frac{dp_e}{dx}\delta = \frac{\tau_w}{\rho}$$

$$\Rightarrow \frac{\partial (V_\infty^2 \theta)}{\partial x} + V_\infty \frac{\partial V_\infty}{\partial x}\delta^* = \frac{\tau_w}{\rho}$$

9.5 Momentum Integral Boundary Layer Equation

Shape factor of velocity profile is defined as the ratio of displacement thickness to momentum thickness. It is closely related to the shape of velocity profile.

$$H = \frac{\delta^*}{\theta}$$

Using shape factor ***Karman momentum integral equation*** can be rewritten as

$$\frac{d\theta}{dx} + (2 + H)\frac{\theta}{V_\infty}\frac{dV_\infty}{dx} = \frac{\tau_w}{\rho V_\infty^2}$$

Discussion on Karman momentum integral equation

1. **Valid** both to laminar and to turbulent boundary layer

2. **Directly unsolvable** due to 3 unknowns(θ, δ^* and τ_w or θ, H and τ_w) in 1 equation. But 3 unknowns are all determined from velocity profile

$$\tau_w = \mu\left(\frac{dv_x}{dy}\right)_{y=0} \qquad \delta^* = \int_0^\infty (1 - \frac{v_x}{V_\infty})\mathrm{d}y \qquad \theta = \int_0^\infty \frac{v_x}{V_\infty}\left(1 - \frac{v_x}{V_\infty}\right)\mathrm{d}y$$

3. **Solution Procedure**

Step 1. Give a velocity profile with parameters tentatively

$$\frac{v_x}{V_\infty} = f(\eta) = a_0 + a_1\eta + a_2\eta^2 + \cdots + a_n\eta^n, \quad (\eta = \frac{y}{\delta})$$

Step 2. Determine the parameters from boundary conditions

Step 3. Solve boundary layer parameters

9.5 Momentum Integral Boundary Layer Equation

Example: Assume a laminar boundary layer of velocity profile

$$\frac{u}{U_\infty} = C_0 + C_1\frac{y}{\delta} + C_2\frac{y^2}{\delta^2}$$

where U_∞ is the outer velocity of boundary layer, δis boundary thickness. Constants, C_0, C_1, C_2, is determine from boundary condition. Then calculate displacement thickness δ^*, momentum thickness θ and wall shear stress τ_w .

Solution:

$$u\big|_{y=0} = 0 \quad \Rightarrow \quad C_0 = 0$$

$$u\big|_{y=\delta} = U_\infty \quad \Rightarrow \quad C_1 + C_2 = 1$$

9.5 Momentum Integral Boundary Layer Equation

Since $$\tau = \mu \frac{\mathrm{d}u}{\mathrm{d}y} = \mu U_\infty \left(\frac{C_1}{\delta} + \frac{2C_2}{\delta^2} y \right)$$

$$\tau \big|_{y=\delta} = 0 \quad \Rightarrow \quad C_1 + 2C_2 = 0$$

therefore $$C_0 = 0, \quad C_1 = 2, \quad C_2 = -1$$

so the velocity profile satisfying boundary conditions is obtained.

$$\frac{u}{U_\infty} = \frac{2y}{\delta} - \frac{y^2}{\delta^2}$$

From this velocity profile, we can calculate displacement thickness

$$\delta^*=\int_0^\delta\left(1-\frac{u}{U_\infty}\right)\mathrm{d}y=\int_0^\delta\left(1-\frac{2y}{\delta}+\frac{y^2}{\delta^2}\right)\mathrm{d}y=\delta-\delta+\frac{1}{3}\delta=\frac{1}{3}\delta$$

momentum thickness

$$\theta=\int_0^\delta\frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)\mathrm{d}y=\int_0^\delta\left(\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right)\left(1-\frac{2y}{\delta}+\frac{y^2}{\delta^2}\right)\mathrm{d}y$$

$$=\delta\text{-}\frac{5}{3}\delta+\delta\text{-}\frac{1}{5}\delta=\frac{2}{15}\delta$$

and wall shear stress

$$\tau_w=\mu\frac{\mathrm{d}u}{\mathrm{d}y}\bigg|_{y=0}=\mu U_\infty\left(\frac{2}{\delta}-\frac{2}{\delta^2}y\right)_{y=0}=\frac{2\mu U_\infty}{\delta}$$

9.5 Momentum Integral Boundary Layer Equation

Substituting them in ***Karman momentum integral equation***

$$\frac{\partial\left(U_\infty^2\theta\right)}{\partial x}+U_\infty\frac{\partial U_\infty}{\partial x}\delta^*=\frac{\tau_w}{\rho}$$

$$\Rightarrow \quad \frac{3}{5}U_\infty\delta\frac{\partial U_\infty}{\partial x}+\frac{2}{15}U_\infty^2\frac{\partial\delta}{\partial x}=\frac{2\mu U_\infty}{\rho\delta}$$

For uniform flow past an infinitely long flat plate, we have

$$\xrightarrow{\partial U_\infty/\partial x=0} \quad \frac{1}{15}\frac{d\delta}{dx}=\frac{\nu}{\delta U_\infty} \quad \Rightarrow \quad \delta=5.49\sqrt{\frac{\nu x}{U_\infty}}$$

9.6 Boundary Layer Separation

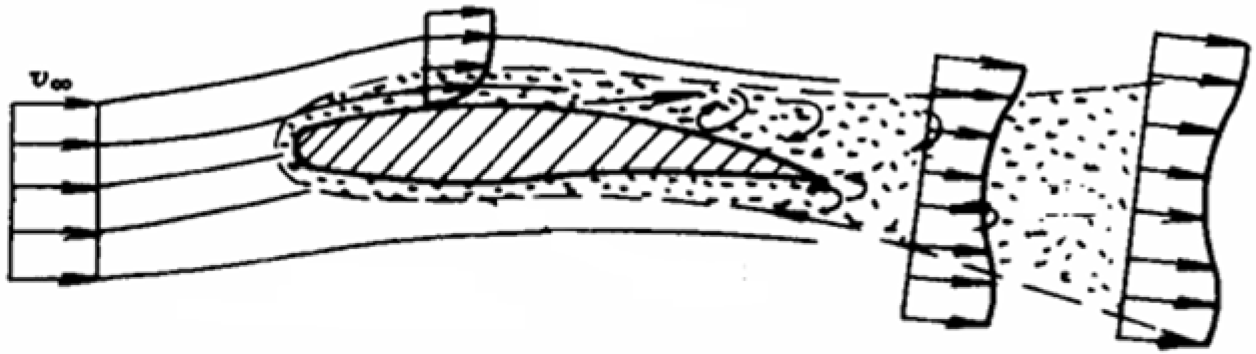
For uniform flow past flat plate, velocity at the outer boundary is of constant value and parallel to the plate, pressure in boundary layer will be a constant.

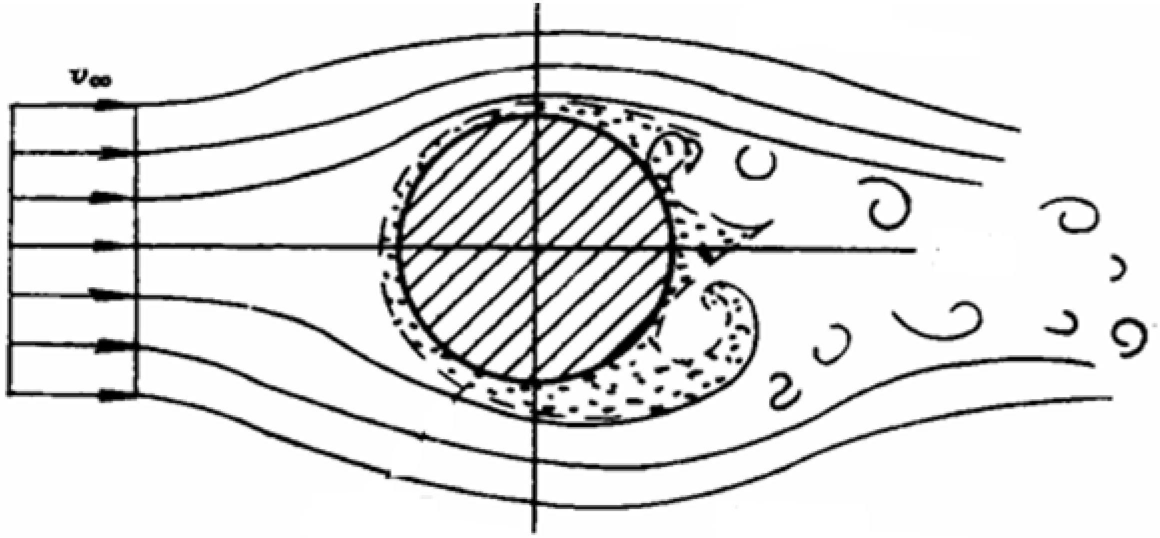
But for uniform flow past a curved body, both direction and magnitude of the outer boundary velocity vary with the body curve. Pressure there also varies. It will change the boundary layer flow field. In the worst case flow will separate from the body. This phenomenon is known as ***boundary layer separation***.

9.6 Boundary Layer Separation

In practice, body surface is often curved. Curved body is usually classified into two classes, ***streamlined body*** and ***non-streamlined body***. Flow past a non-streamlined body, usually it will separate from the body. At downstream, a back-flow may occur, where fluid moves at a direction reversely *against the main flow*. If flow past a streamlined body at a large ***attack angle***, flow will separate from the body surface, and back-flow phenomenon may occur.

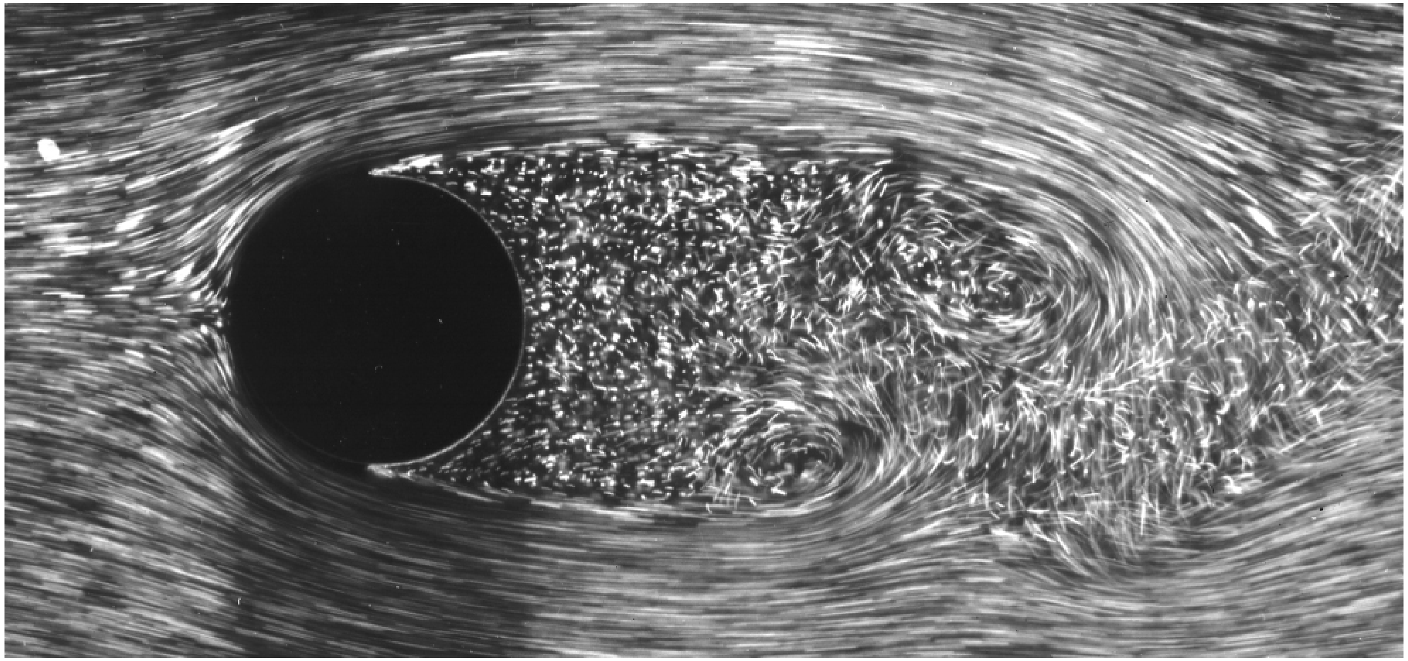
9.6 Boundary Layer Separation





9.6 Boundary Layer Separation

1. The cause ?
2. The criterion ?
3. Characteristics ?

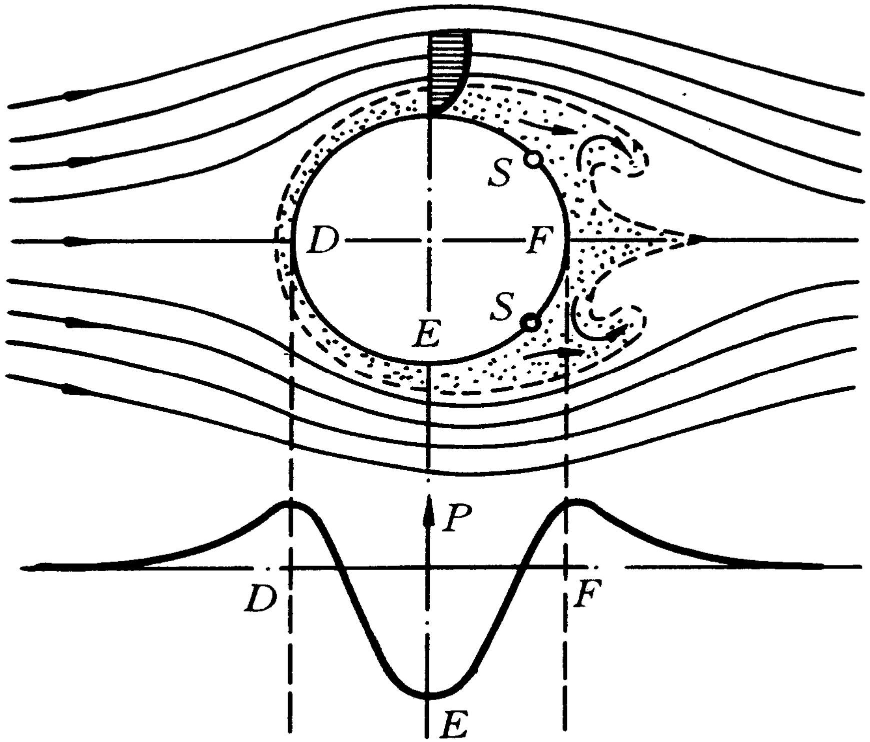


9.6 Boundary Layer Separation

Consider an incompressible uniform flow past a circular cylinder

At front stagnation point, D, there is no boundary layer, where boundary layer thickness is zero.

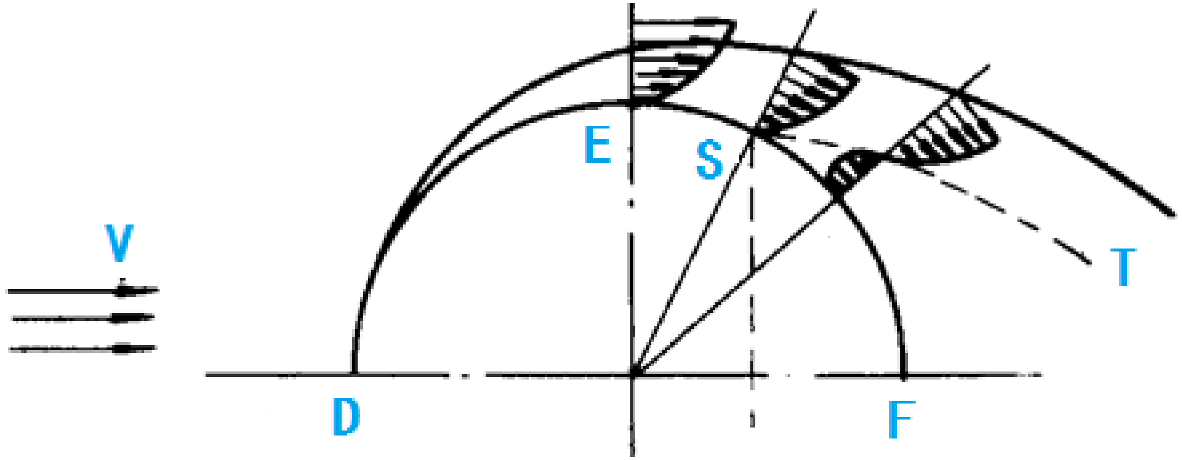
In DE, flow speeds up and pressure reduces. Accordingly a *favorable pressure gradient* forms. At E velocity reaches maximum value. Over E, it begins to speed down and pressure up. An *adverse pressure gradient* forms.



$$\frac{dp}{dx} < 0$$

$$\frac{dp}{dx} > 0$$

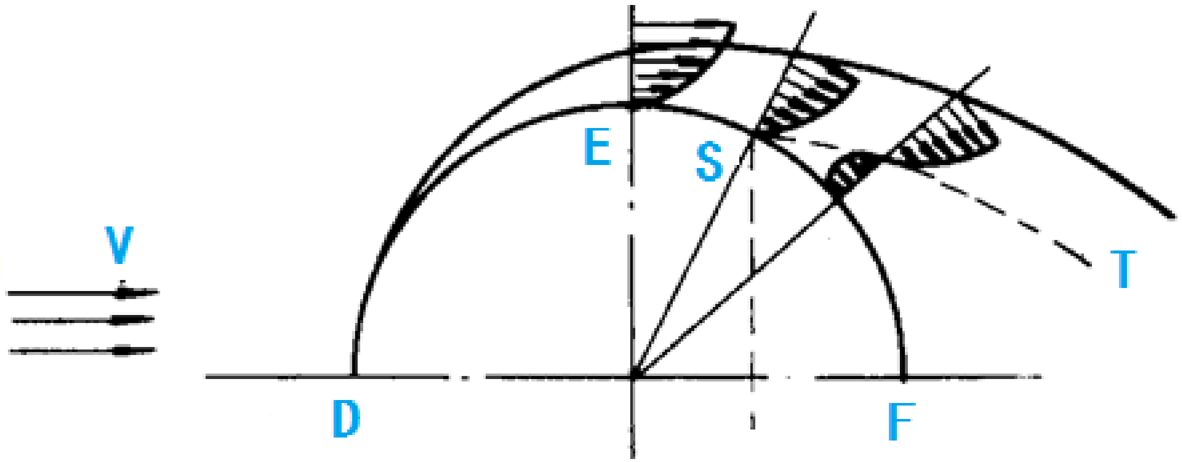
9.6 Boundary Layer Separation



When flow past E, the highest pressure point, pressure gradually increases and velocity decreases, which cause the flow in boundary layer gradually slow down, and the kinetic energy decreases. When flow arrives S, kinetic energy near the wall reduces to zero and the flow is no more able to move forward and it stops at S.

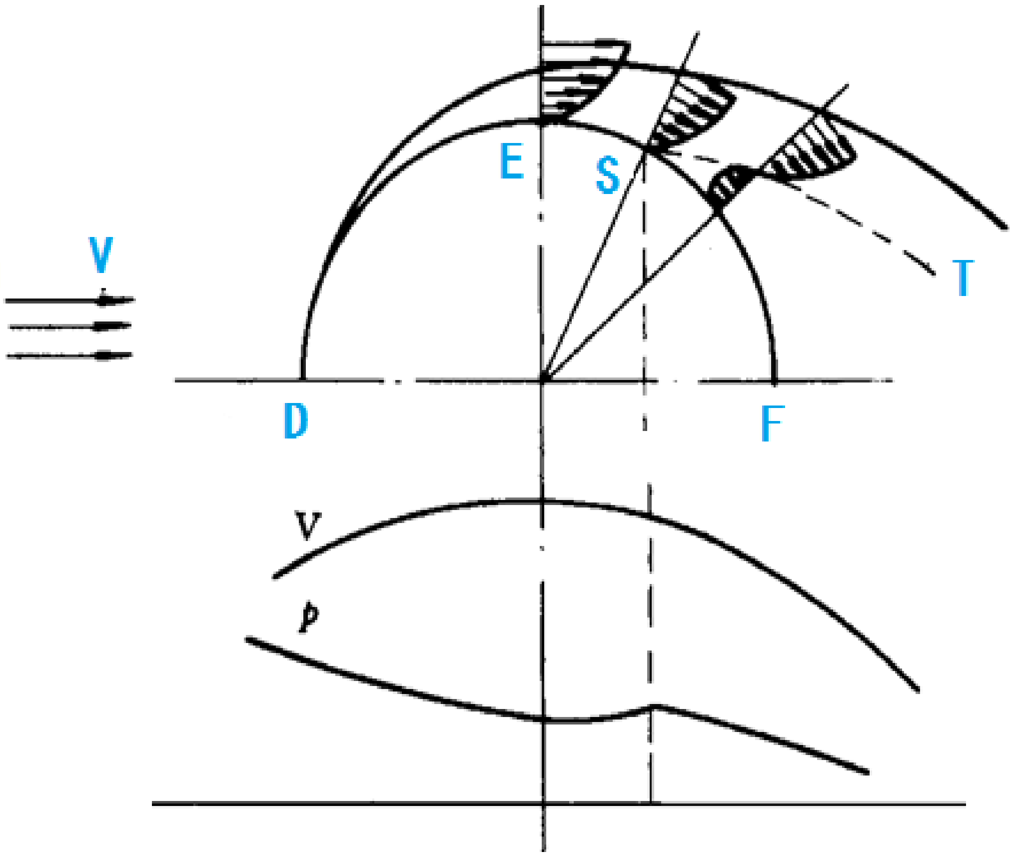
After S, pressure further increases, under action of the *adverse pressure gradient*, flow near the wall will reverse its direction, and generate a backward flow.

9.6 Boundary Layer Separation



In this way, more and more fluid stops, reverses direction, and is accumulated between the wall and the outer main stream. It will dramatically thicken boundary layer and form a remarkable backward flow layer, bounded by ST, out of which is the main forward stream, and inner of which is the backward flow layer. Vortices will generate in this region.

9.6 Boundary Layer Separation



Form drag **is closely related to the body shape.**

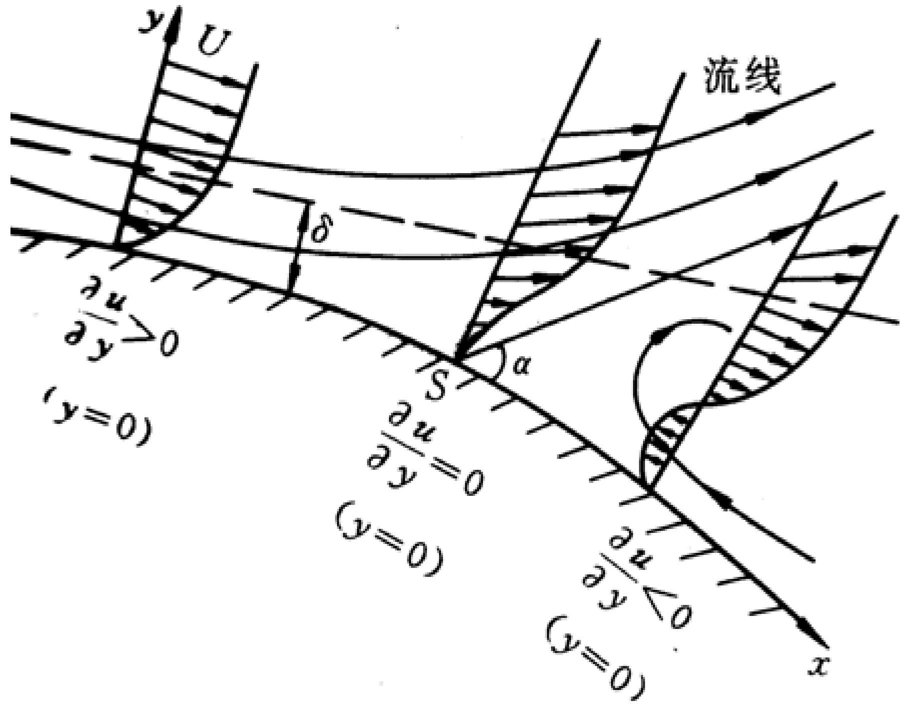
At S, there are 2 branches, SF along body and ST away from it. S is the ***separation location***.

The vortices continuously generate and are brought into the downstream with the main stream, and form a wake area. Due to fluid viscosity, in the wake mechanical energy continuously dissipates. Pressure there further reduces, less than the one in the front surface of the cylinder. Thus, a ***form drag*** results.

9.6 Boundary Layer Separation

Conclusions:

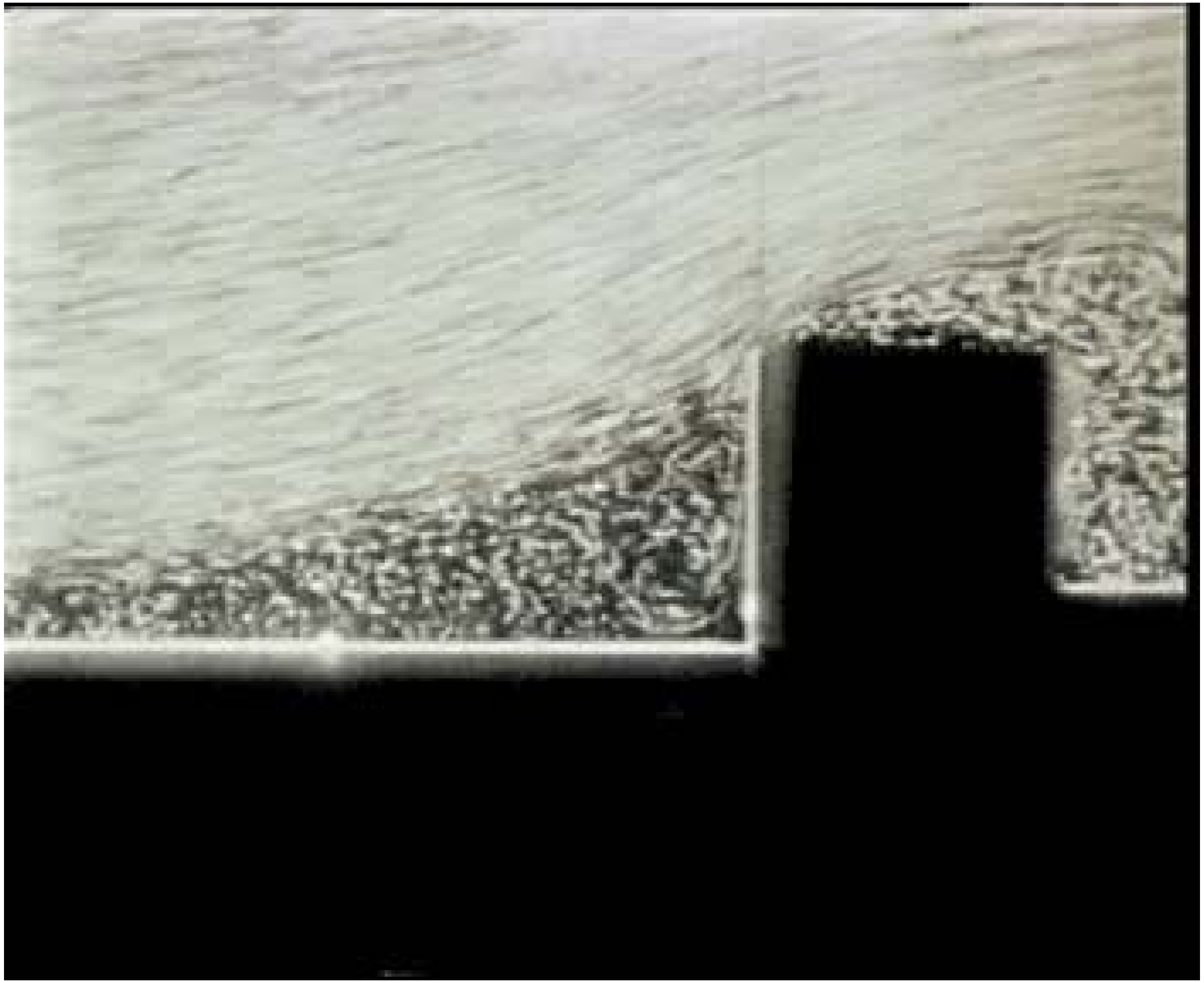
1) **In** ***favorable pressure gradient*** **region, boundary layer would not separate from the wall.**



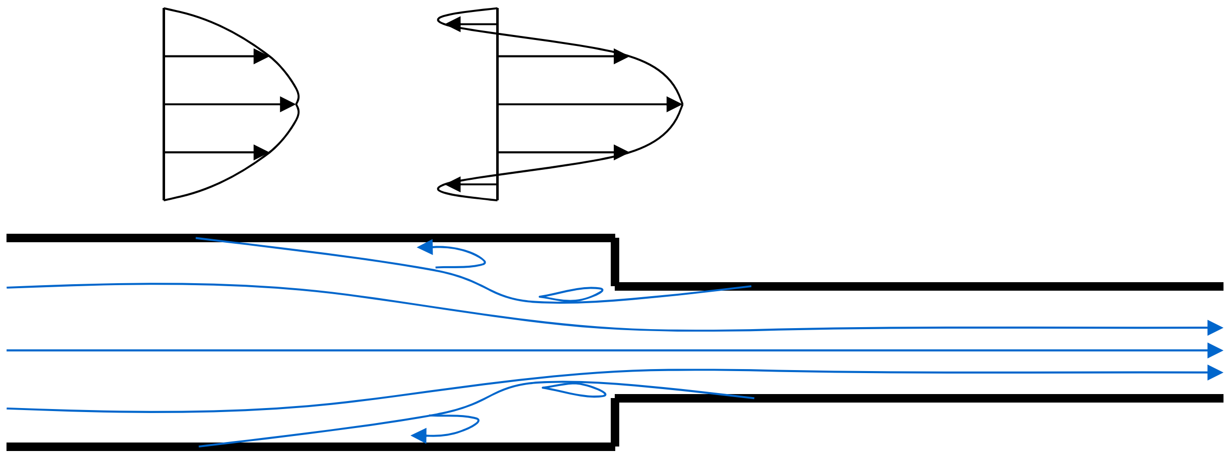
2) **Only in** ***adverse pressure gradient*** **region, boundary layer is possibly separated from the wall and form a wake with vortices.**

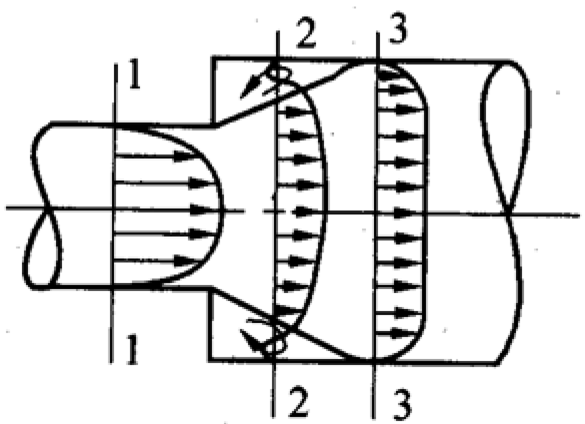
3) **Especially, if in the main stream speed reduction is extremely heavy, separation will definitely occur.**

9.6 Boundary Layer Separation

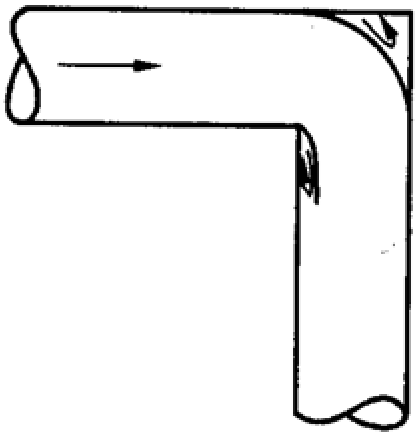


9.6 Boundary Layer Separation





(a)



(b)

(c)

9.6 Boundary Layer Separation

1. Flow Separation and Its Causes

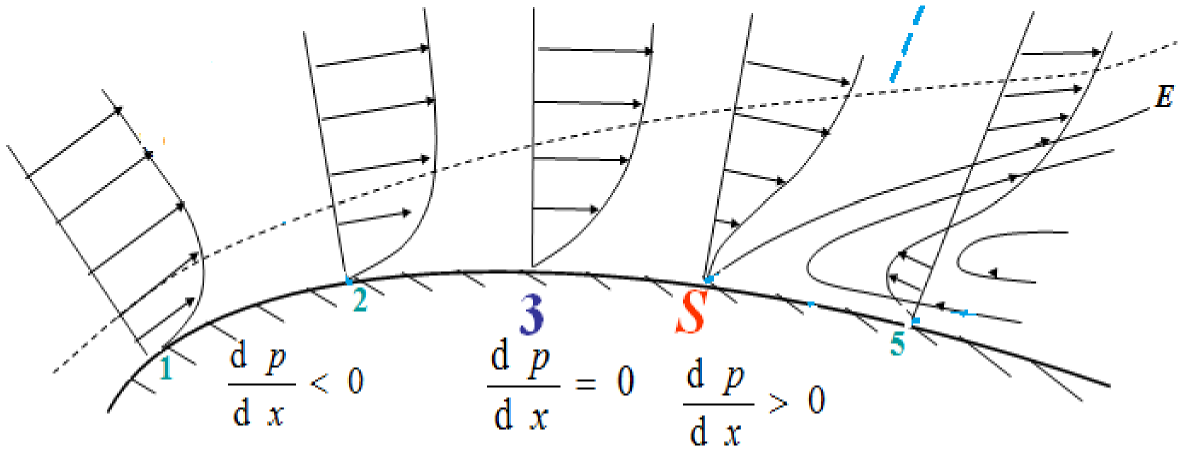
Dynamics of boundary layer: balance of inertial force (kinetic energy), pressure gradient (outer main stream), and viscous force (resistance).

1—3: favorable pressure gradient region

3—5: adverse pressure gradient region

Outer boundary

S: separation location



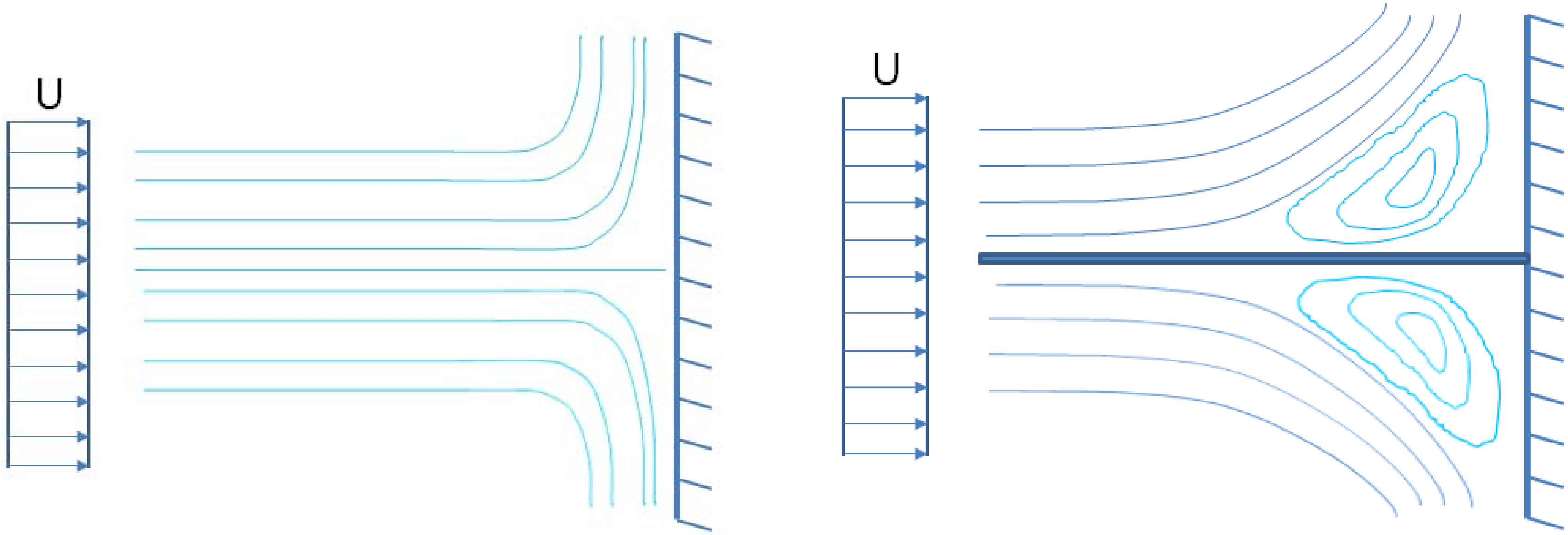
After S: separated regio

9.6 Boundary Layer Separation

Separation Conditions:

① adverse pressure gradient region

② viscous wall resistance



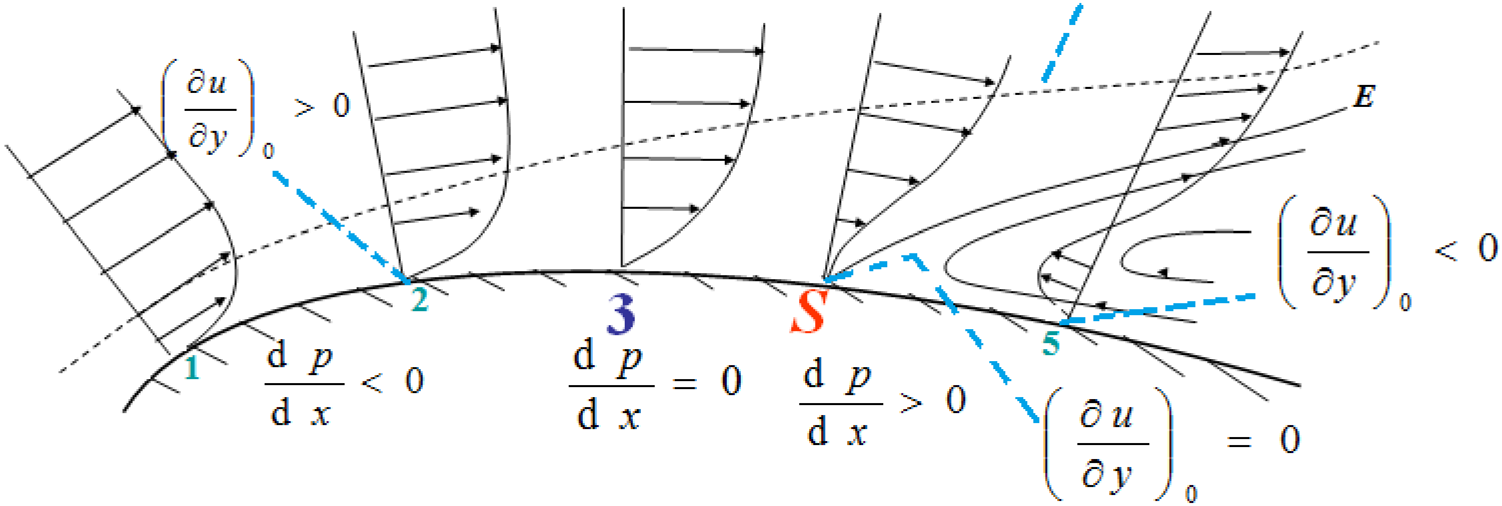
Both conditions, *adverse pressure gradient* and *viscous wall resistance*, are necessary, but not sufficient conditions of boundary layer separation.

2. Criterion of Boundary Layer Separation

—— Plandtl's criterion (2d steady boundary layer)

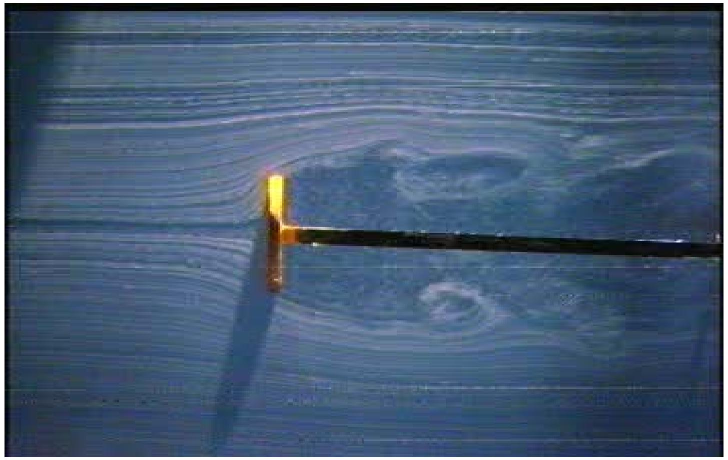
At separation location $\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0$ ⇨ **Location** X_s **of** S.

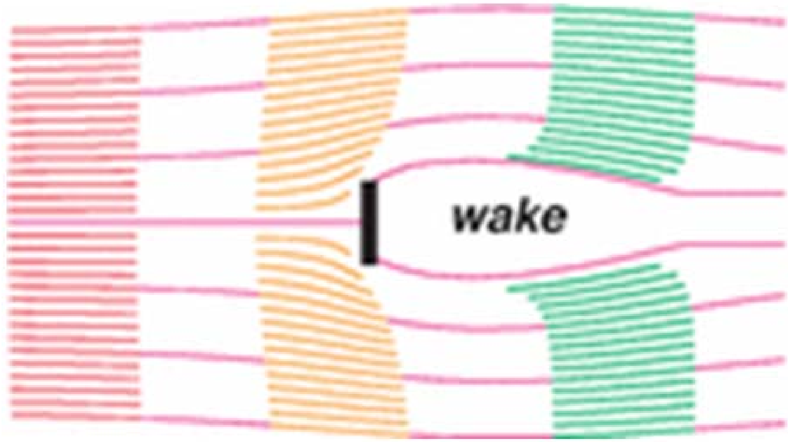
Outer boundary

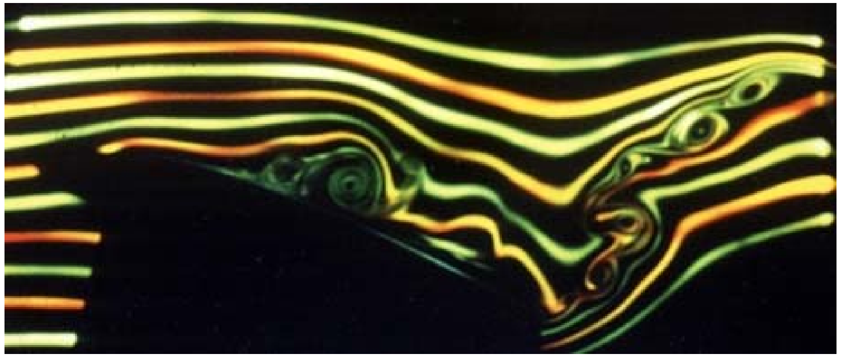


3. Characteristics of Boundary Layer Separation

Boundary layer apart from wall and generate a wake.







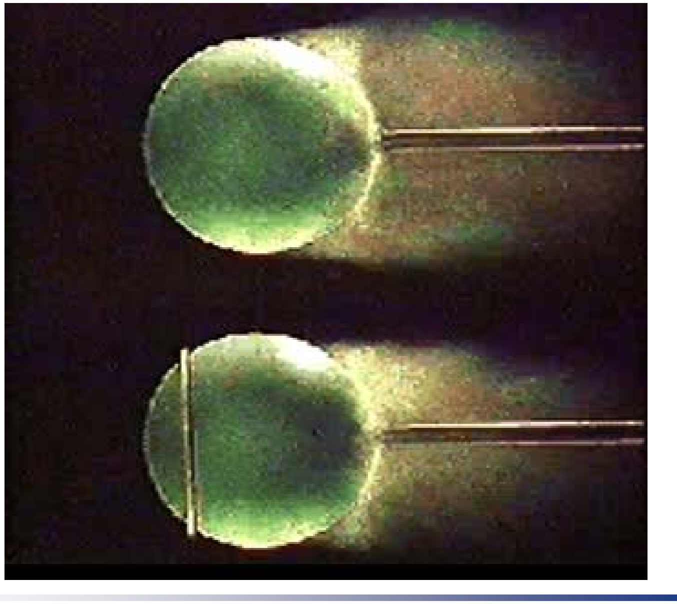
9.6 Boundary Layer Separation

Location x_s of separating point S is closely related to the body shape and state of the boundary layer.

- Laminar boundary layer separates much easier.
- Turbulent boundary layer separates more difficult, and separation location is getting farther. Wake is narrower.

Results of Flow Separation

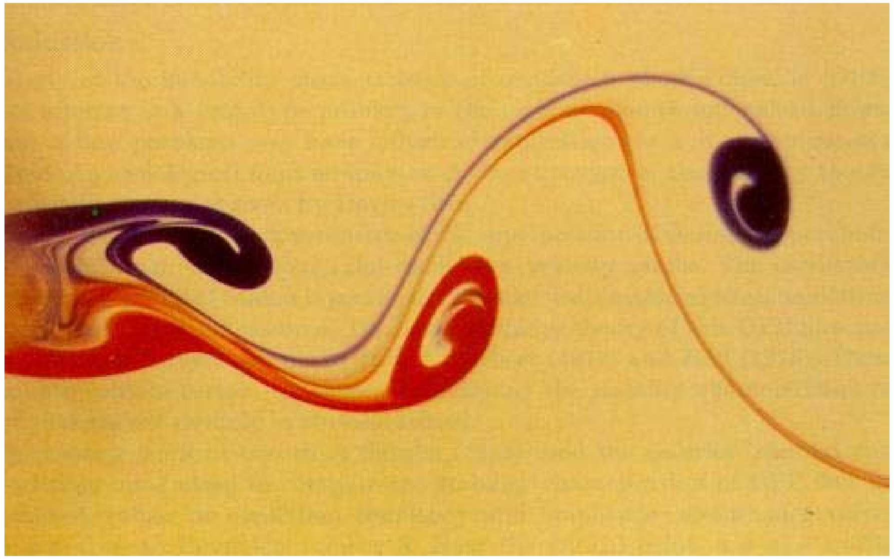
- *Form drag*
- Lift down and drag up
- Noise up
- VIV (vortex induced vibration), longitudinally and transversely

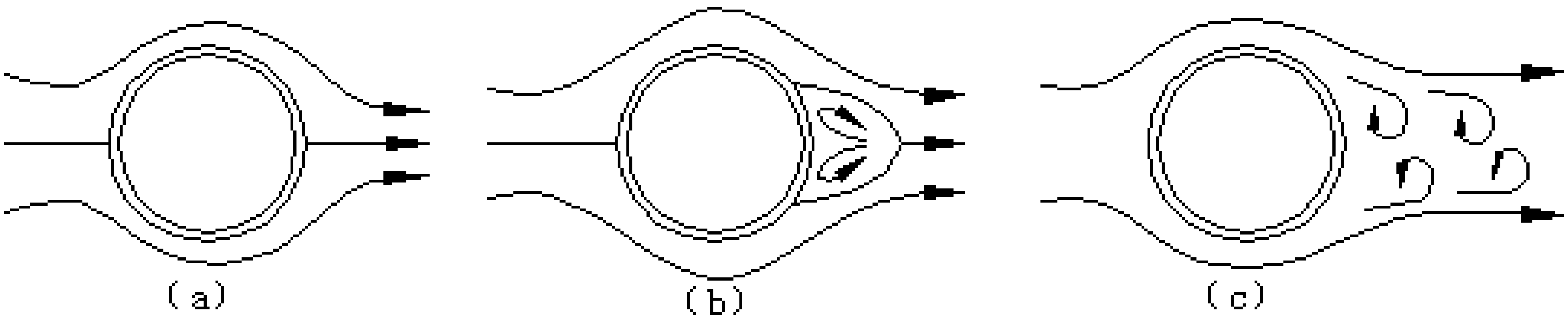


9.6 Boundary Layer Separation

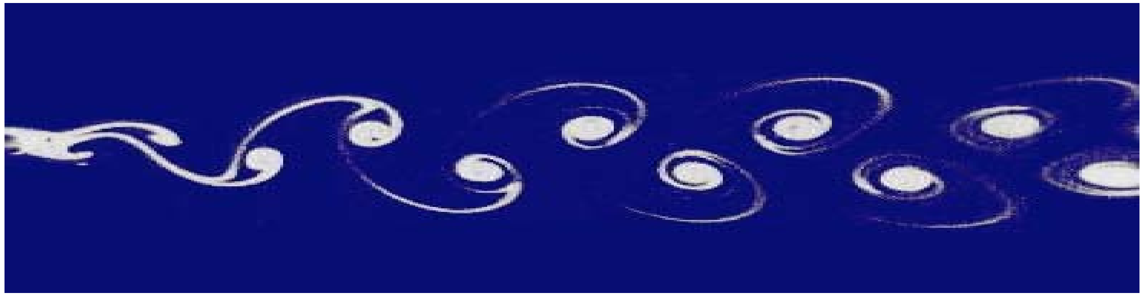
von Karman vortex street

With increase of *Re*, boundary layer is getting to separate from the wall and separation location goes downwards. When *Re* gets large enough, vortices shed from the upper and lower surfaces alternatively and rotate reversely. This regularly shed vortices are called *Karman vortex street*.





9.6 Boundary Layer Separation



For *Re* <40， upper and lower separations are symmetric, and symmetric vortices form.

For *Re*=40～70， wake oscillates periodically.

For *Re*>90， vortices alternatively shed out, and a Karman vortex street forms.

For *Re*>150, vortex street disappears， but vortices still shed out periodically.

With increase of *Re*, periodicity disappears, instead oscillates irregularly with high frequency.

Usage and harm of Karman vortex street

Usage： Measurement of fluid velocity and flow rate.
Vortex street flowmeter.

Harm： Generating vibration and noise.
Possibly cause resonance and acoustic vibration.

The failure of Tacoma bridge, happened in 1940, is an example of VIV due to Karman vortex street.

9.6 Boundary Layer Separation

Flow separation usually annoys engineers!

Examples:

Separation on a wing surface may cause stall.

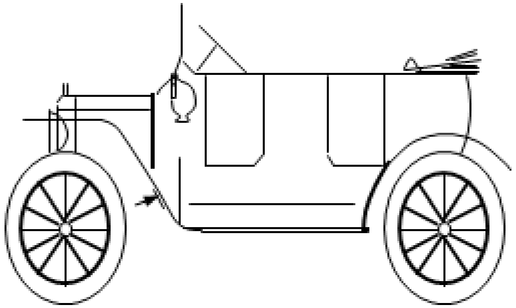
Propeller blade singing may reduce propulsion efficiency, cause cavitation and vibration.

Turbo machine mechanical energy loss, vibration, speed reduction, and even structure failure.

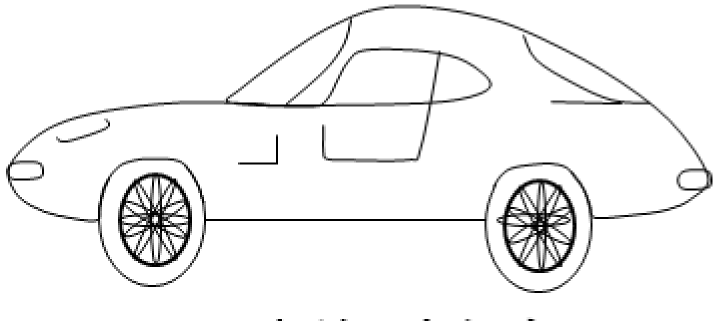
Therefore, control of boundary layer separation is useful in order to increase lift, reduce drag and weaken vibration.

9.6 Boundary Layer Separation

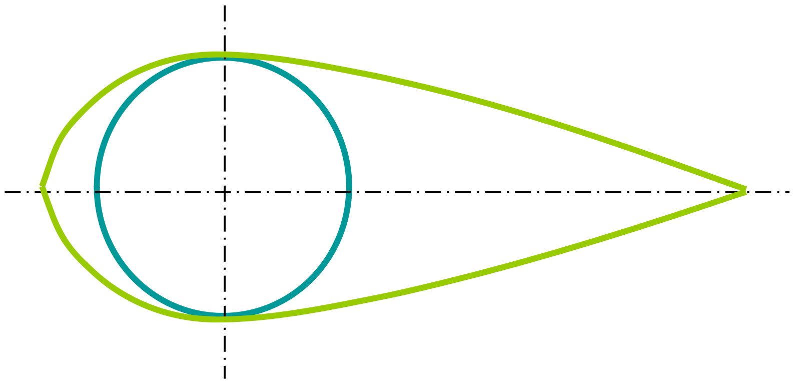
- **Body Shape Modification — Enlarge laminar flow length**



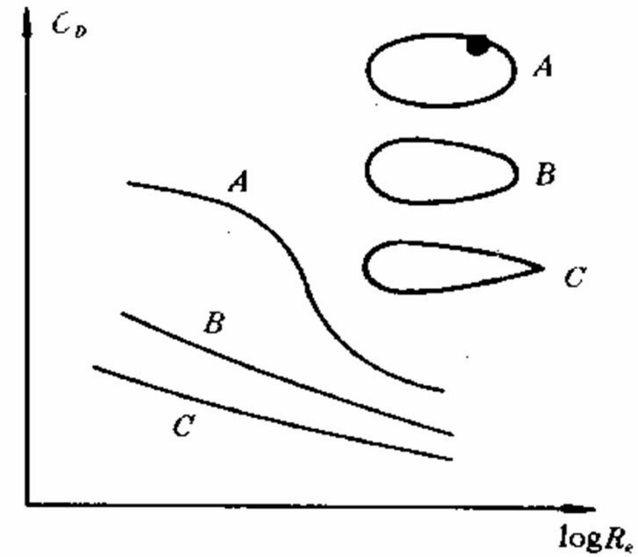
Car in 1920s



Streamlined car



Circular and Streamlined Cylinder



9.6 Boundary Layer Separation

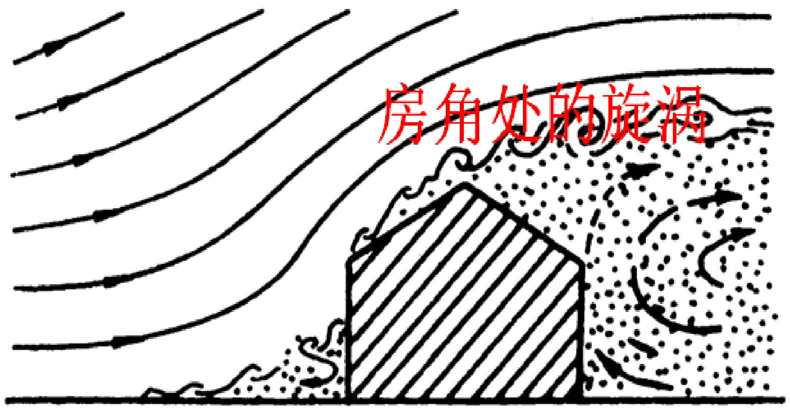
Design of Streamlined Body Shape

Make rear body slender to reduce adverse pressure gradient, and delay separation. Thus, drag due to vortex shedding reduces.

Fin on submarine, rudder and fuselage are streamlined bodies.

Avoid Cusp and Reduce Separation

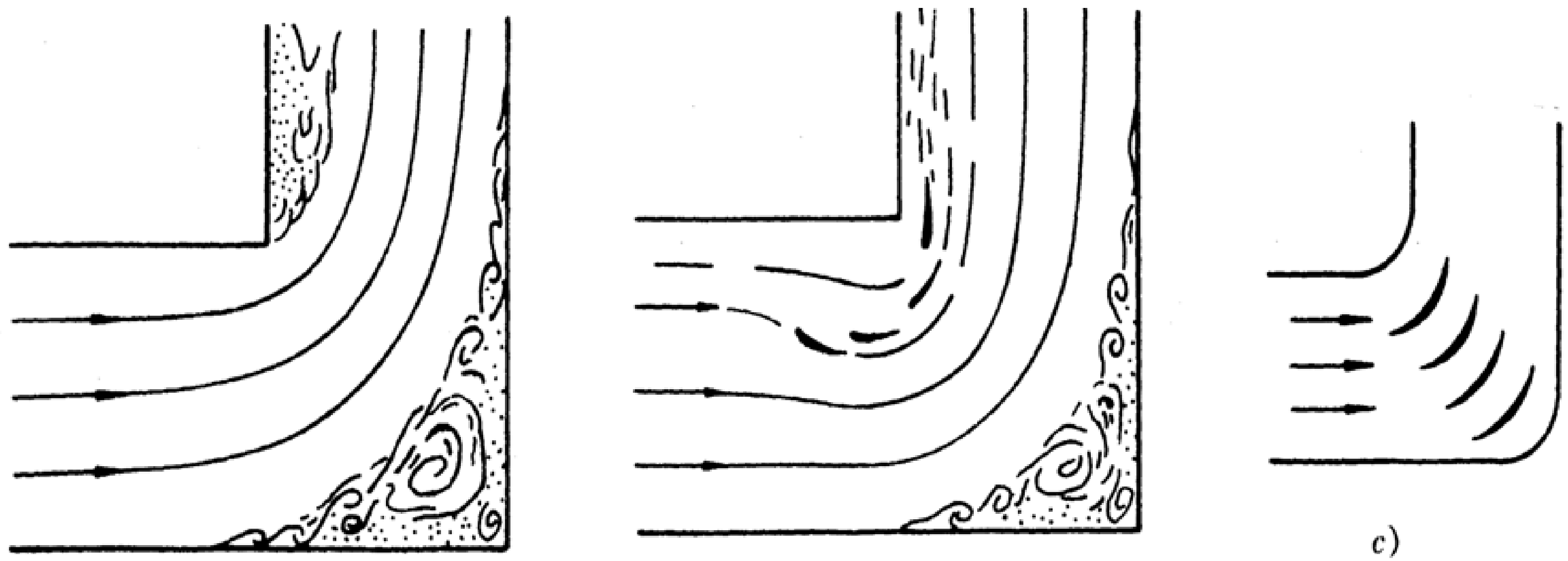
Due to strong adverse pressure gradient, boundary layer may immediately separate after the cusp. Trying to avoid cusp is effective to reduce flow separation.



Deflector

Placing deflector around corners can effectively reduce vortex region and reduce drag due to vortices.

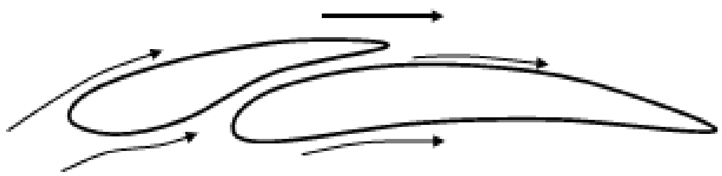
It is often applied to large cross-sectional turn, such as those encountered in wind tunnel and circulating water channel.



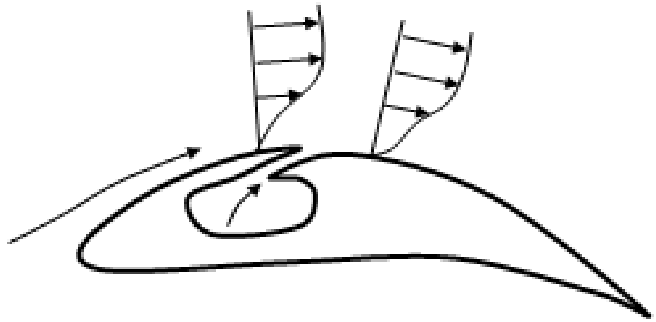
a) 绕尖角的流动　b) 用导流片导流　c)

9.6 Boundary Layer Separation

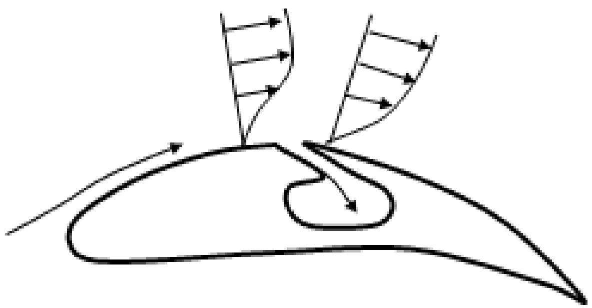
- **Increase Momentum in Boundary Layer — Delay Separation**



Front sub-wing



Blowing out

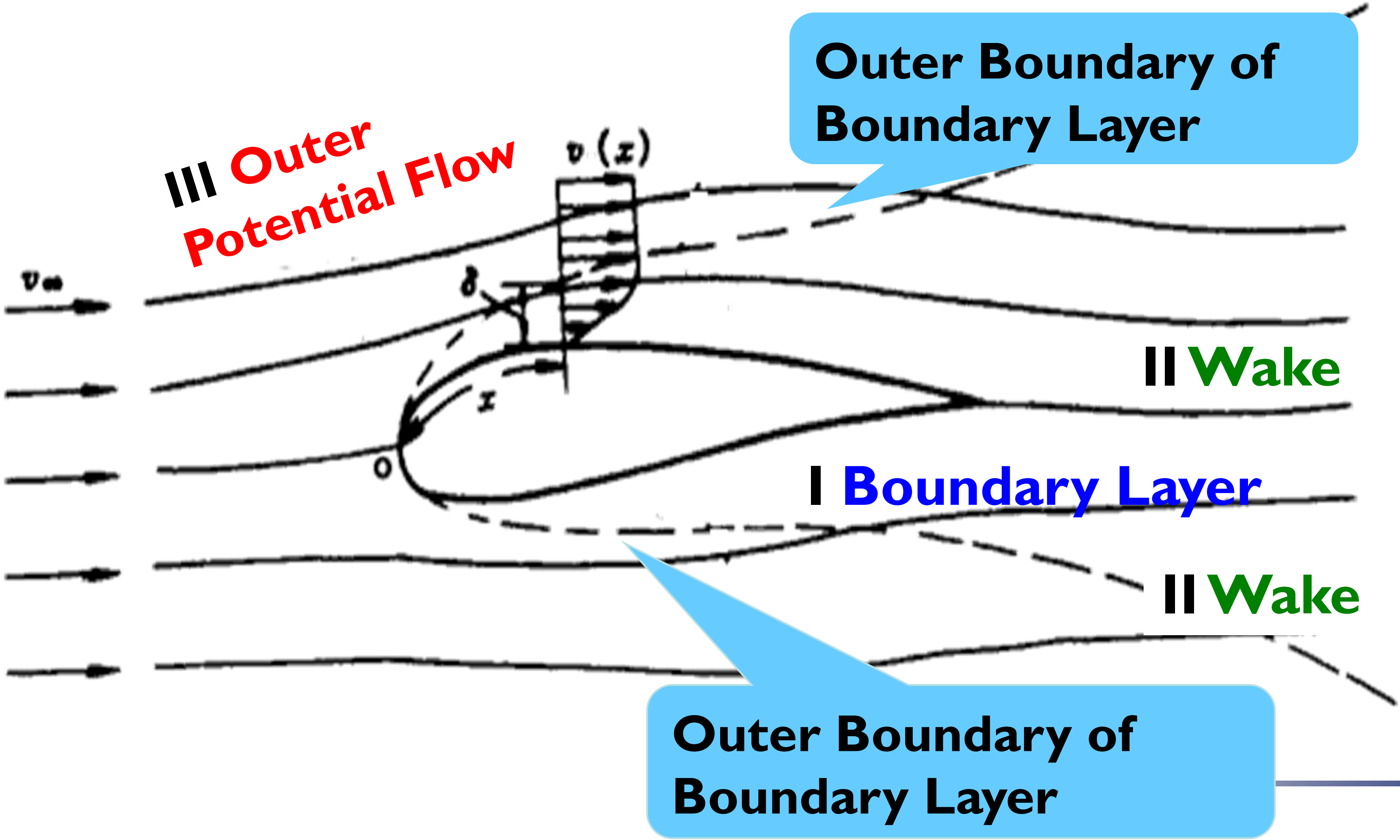


Suction

Review of Chapter 9

Boundary Layer

For large *Re*, when a uniform flow travels past a body, flow field can be divided into 3 different regions, *i.e.* the outer potential flow, near wall boundary layer, and viscous wake.



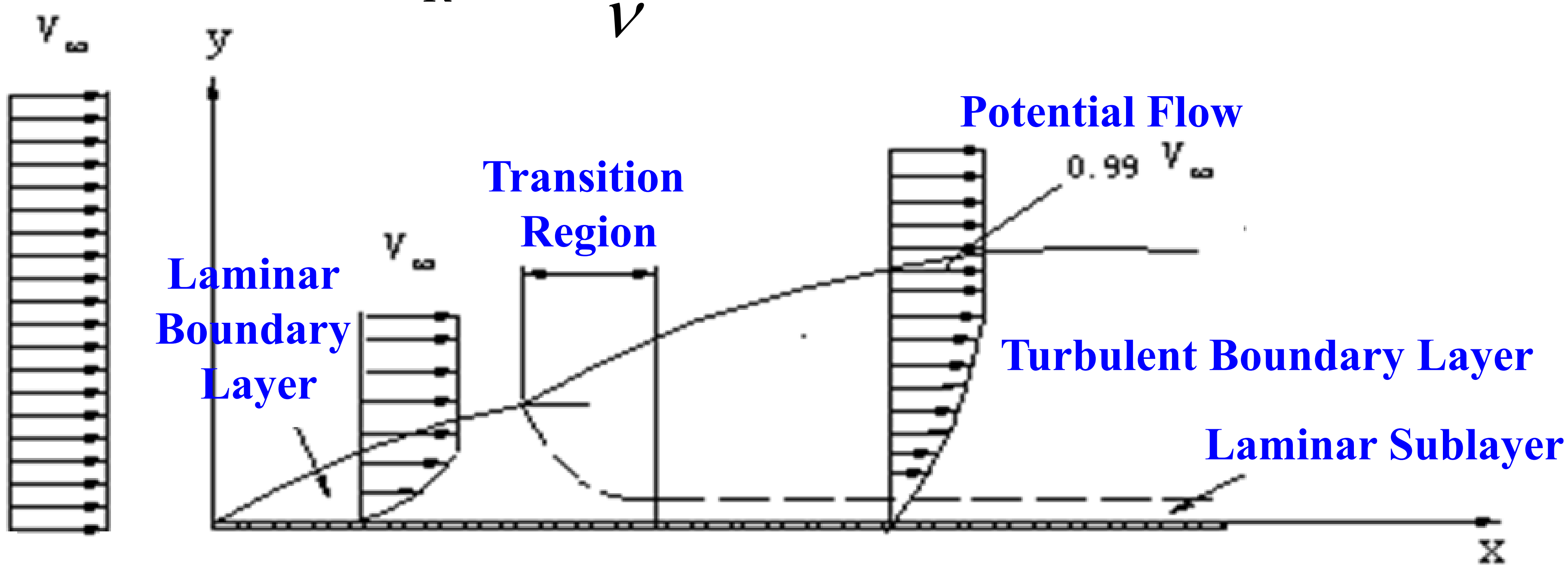
Review of Chapter 9

Flow in Boundary Layer

Laminar, transition, turbulent, and laminar sublayer.

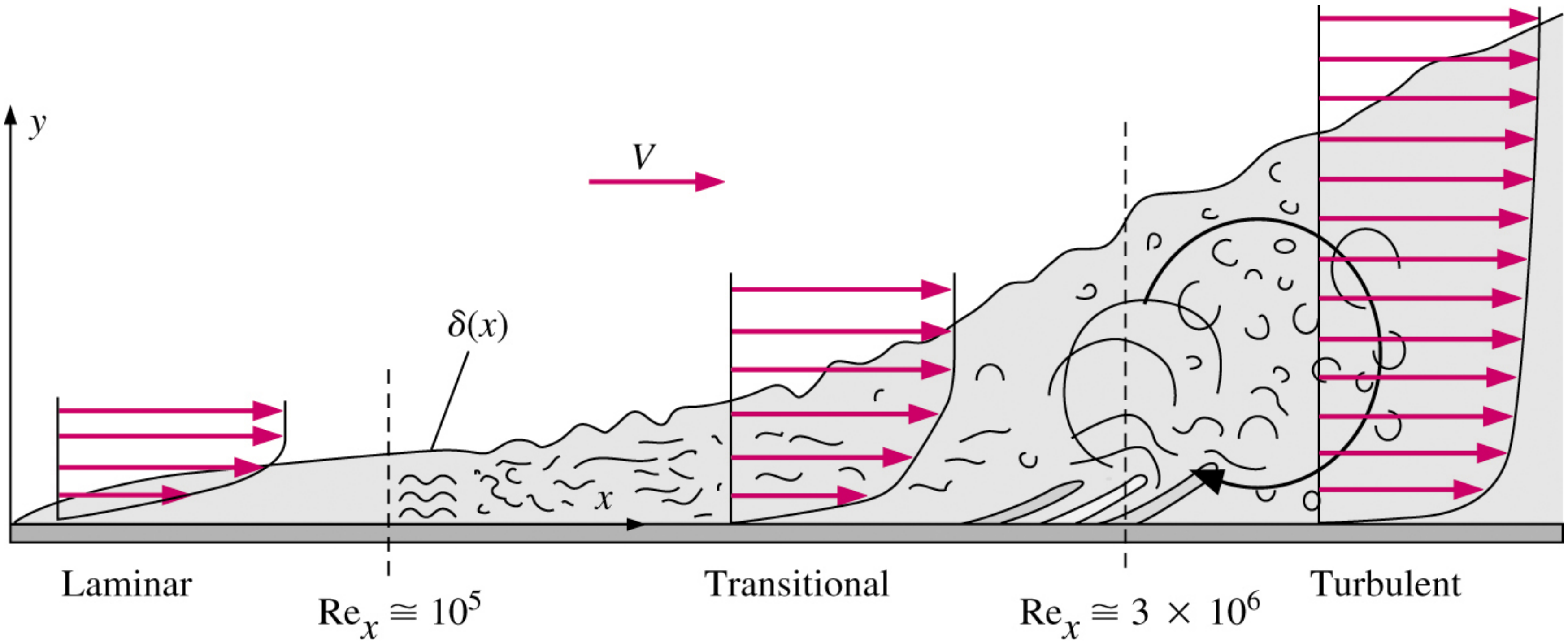
Criterion of flow state is Reynolds number.

$$Re_{xK} = \frac{V_\infty x_K}{\nu} = (3.5 \sim 5.0)\times 10^5$$



Boundary Layer Thickness

Boundary layer thickness, δ, is defined as the distance from the wall (zero velocity) to the 99% potential velocity place. δ increases downstream.



Review of Chapter 9

Displacement Thickness

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

Momentum Thickness

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Review of Chapter 9

Characteristics of Boundary Layer

- Very thin comparing with the length of body, *i.e.*, $\delta \ll L$.
- Large velocity gradient along boundary thickness
- Boundary thickness increases downstream.
- Pressure penetrates boundary layer due to small thickness.
- Viscous force is comparable with inertial force.
- In boundary layer, both laminar flow and turbulent flow exist.

Review of Chapter 9

Boundary Layer Equations of Laminar Boundary Layer

$$\left.\begin{aligned} & v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 v_x}{\partial y^2} \\ & \frac{\partial p}{\partial y} = 0 \\ & \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \end{aligned}\right\}$$

Another Expression of Boundary Layer Equations of Laminar Boundary Layer

$$\left.\begin{aligned} v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} &= V_\infty \frac{dV_\infty}{dx} + \nu \frac{\partial^2 v_x}{\partial y^2} \\ \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} &= 0 \end{aligned}\right\}$$

Review of Chapter 9

Governing Equations of Laminar Boundary Layer near Half Flat Plate under Favorable Pressure Gradient.

$$\left.\begin{aligned} v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} &= \nu \frac{\partial^2 v_x}{\partial y^2} \\ \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} &= 0 \end{aligned}\right\}$$

Review of Chapter 9

Karman Momentum Integral Equation **of Boundary Layer**

$$\frac{\tau_w}{\rho} = \frac{\partial\left(V_\infty^2\theta\right)}{\partial x} + \frac{\partial V_\infty}{\partial x} V_\infty \delta^*$$

Particularly for half flat plate under favorable pressure gradient, Karman momentum integral equation becomes

$$\frac{\tau_w}{\rho V_\infty^2} = \frac{\partial \theta}{\partial x}$$

Review of Chapter 9

Boundary Layer Separation

Viscous flow under the action of *favorable pressure gradient* (speed up) will not separate from wall. Only under the action of *adverse pressure gradient* (slow down) separation may occur and vortices generate. Especially if the **main stream slows down heavily**, flow separation will definitely occur.