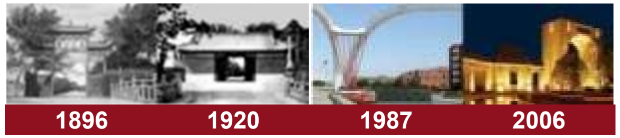



# Introduction to Marine Hydrodynamics

(NA235)

Department of Naval Architecture and Ocean Engineering

School of Naval Architecture, Ocean & Civil Engineering

Shanghai Jiao Tong University

# Eighth Assignment

- **The eighth assignment can be downloaded from following website:**

**Website： ftp://public.sjtu.edu.cn**

**Username： dcwan**

**Password： 2015mhydro**

**Directory： IntroMHydro2015-Assignments**

- **Seven problems**

- **Submit the assignment on June 11th (in English, written on paper)**

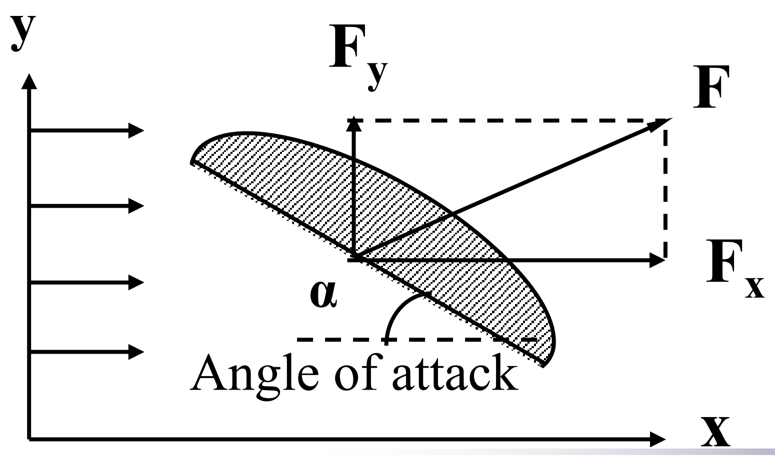
# Chapter 8

## Fundamental Theory of Viscous Incompressible Fluid Flow

# 8.8 Hydrodynamic Force on a Body

- Generally, hydrodynamic force on a body is determined by ***velocity of the flow***, ***shape of the body***, and ***position and attitude of the body relative to the flow***. The resultant force is resolved into two components, parallel and perpendicular to the flow.

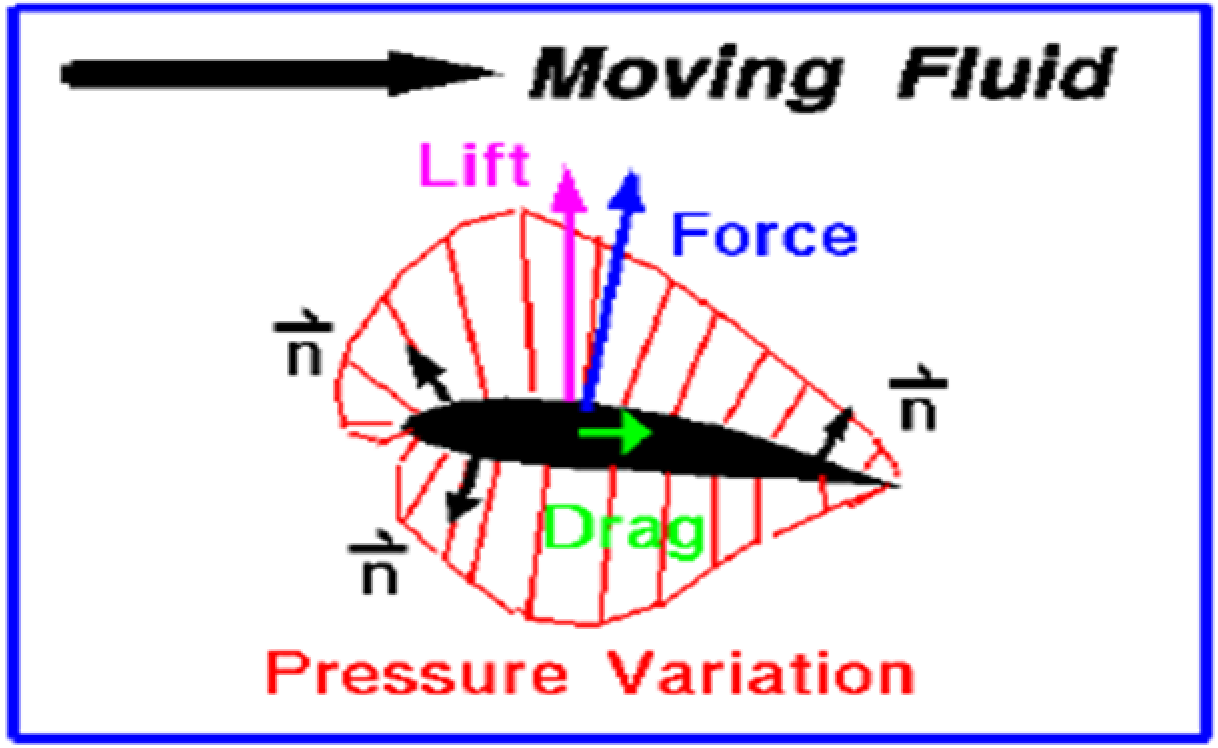
- The component parallel to the flow is called resistance or ***drag***, $\mathbf{F_x}$, while the one perpendicular to the flow is called ***lift***, $\mathbf{F_y}$. If shape of a body is symmetric with respect to the flow, then $\mathbf{F_y = 0}$.



# 8.8 Hydrodynamic Force on a Body

***Drag*** consists of ***frictional drag*** (***viscous drag***) and ***form drag***.

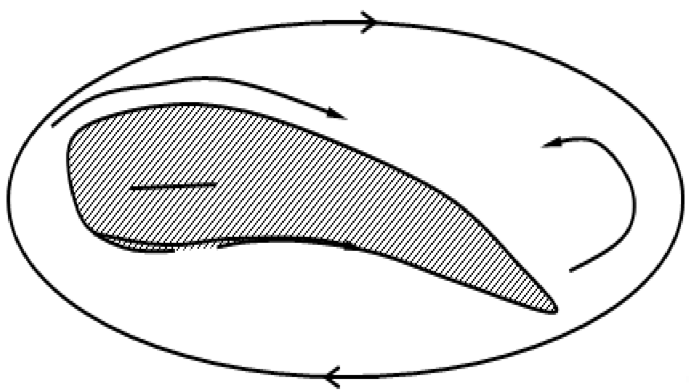
Frictional drag relates closely to the area of the body surface, while form drag relates to the shape of the body. Usually, drag has to be measured.

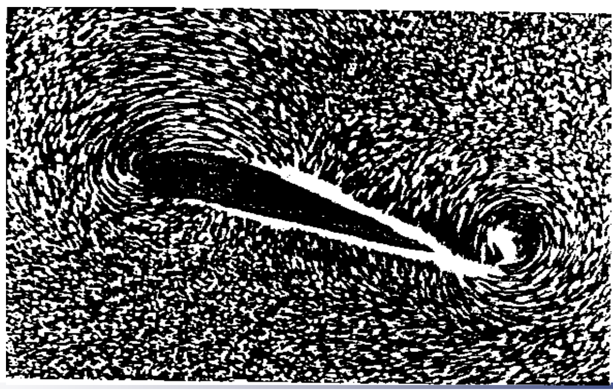


# 8.8 Hydrodynamic Force on a Body

- While a flow travels past an asymmetric object or past a symmetric object with an attack angle, a ***lift*** force, perpendicular to the flow, will result.
- **Qualitative explanation for the mechanism of *lift* generation**

Since a particle along the upper surface travels longer journey than the one along the lower surface to arrive a common end point, it has to run faster and thus pressure will become lower. When the upper and lower streams meet at the end point, deference of speeds between them will generate a vortex. Due to the law of vorticity conservation, it will result a circulation around the body. Thus, according to Joukowski Law, a ***lift*** will result.





# 8.8 Hydrodynamic Force on a Body

- For small ***Re***, **frictional drag** is dominant.

- **Stokes formula**

Frictional drag on a sphere is expressed as

$$\boxed{f = 6\pi\mu r_0 V_\infty}$$

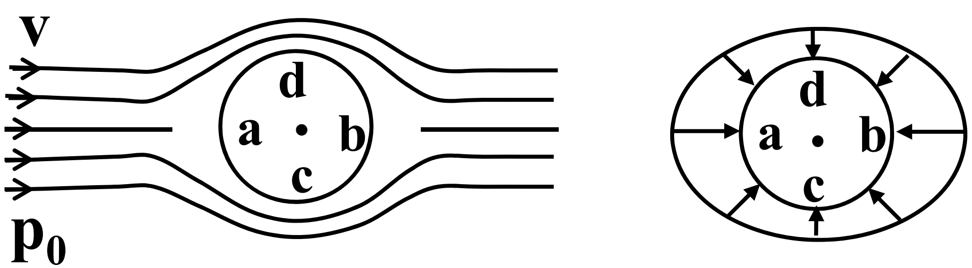
where $r_0$ is the radius of the sphere, $V_\infty$ velocity of flow relative to the sphere and $\mu$ is dynamic viscosity of the fluid.

- For large ***Re***, due to pressure distribution on the surface of the object, **form drag** is dominant.

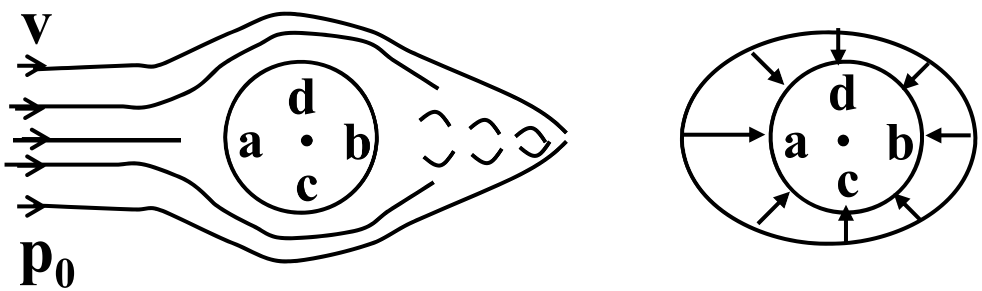
# 8.8 Hydrodynamic Force on a Body

## An explanation on *form drag*

Consider flow past circular cylinder.




**Inviscid fluid, without form drag**




**Viscous flow, with form drag**

## Reduction of *drag force*

### Reduction of *frictional drag*

To make laminar boundary layer longer, and force the transition point downwards as possible, may reduce frictional drag. If wing surface is smooth enough, the transition could be greatly delayed towards downstream. A ***laminar wing***, adopted in aeronautics, is just based on this principle.

### Reduction of *form drag*

To reduce *wake* behind body, that is, delay flow separation, may reduce form drag. A ***streamlined body*** is based on this principle.

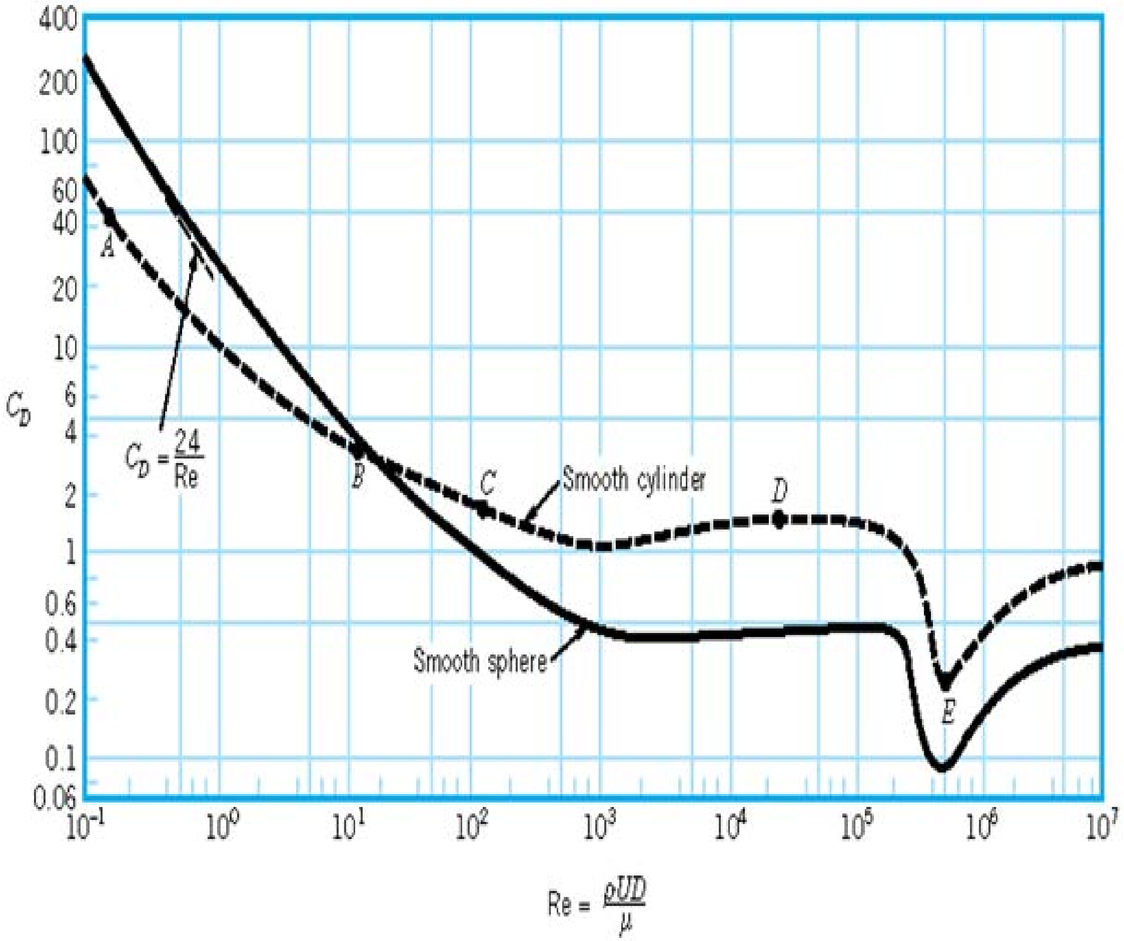
In engineering, following ***drag coefficient*** is extensively used.

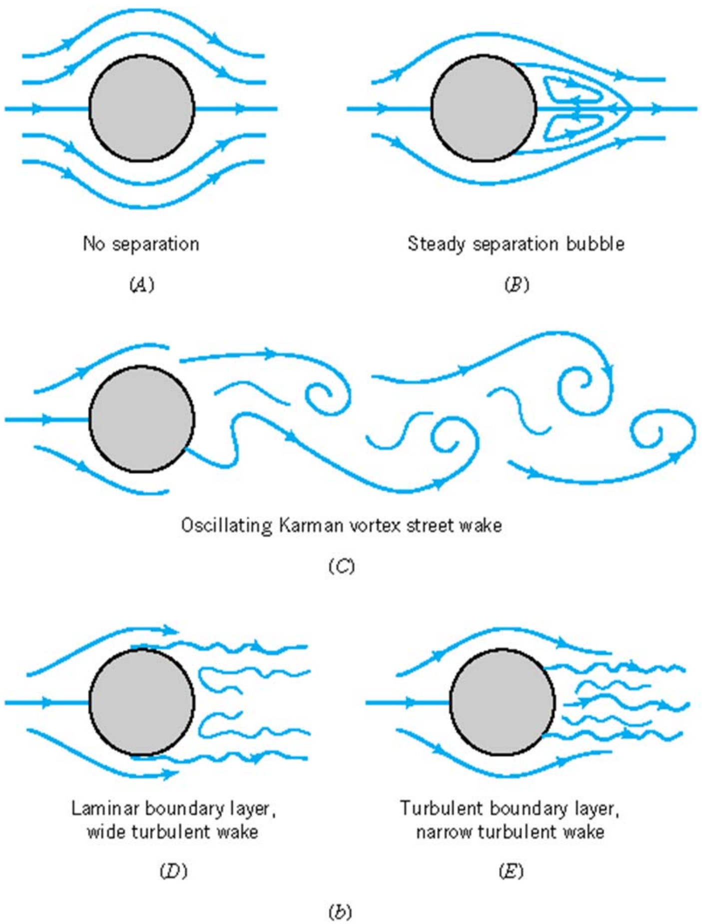
$$\boxed{C_D = \frac{F_D}{\frac{1}{2}\rho V_\infty^2 A}}$$

# 8.8 Hydrodynamic Force on a Body

## *Drag coefficient* of circular and sphere flows

$$C_D = f(R_e)$$



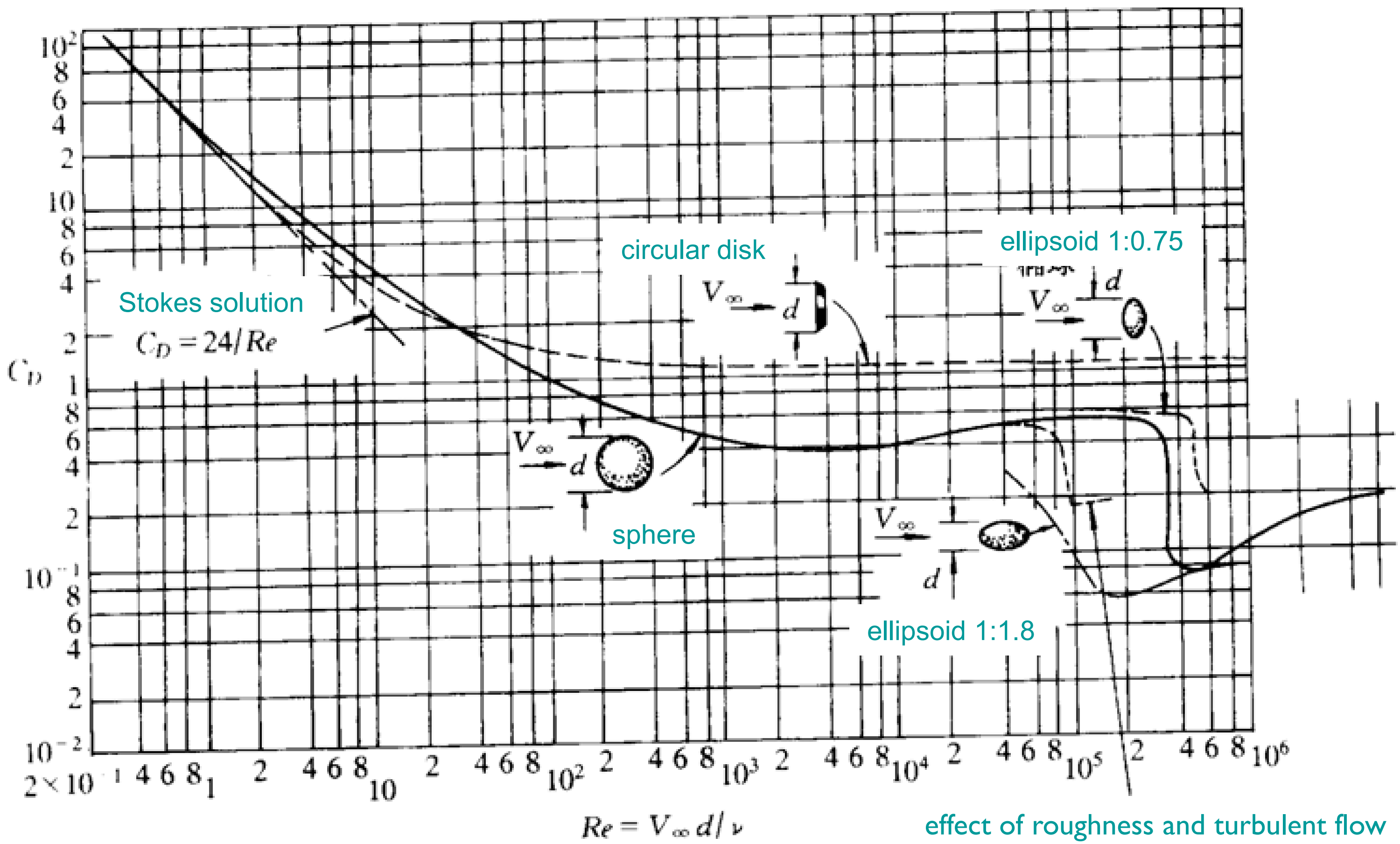



# 8.8 Hydrodynamic Force on a Body

## Drag coefficient of some simple bodies

| 2D Body | | $Re = V_\infty d/\nu$ | $C_D = D/\left(\frac{1}{2}\rho V_\infty^2 A\right)$ |
|---|---|---|---|
| Circular cylinder | | $10^4 \sim 10^5$ | 1.2 |
| Left half tube | | $4\times10^4$ | 1.2 |
| Right half tube | | $4\times10^4$ | 2.3 |
| Square cylinder | | $3.5\times10^4$ | 2.0 |
| Vertical plate | | $10^4\times10^6$ | 1.98 |
| Elliptic cylinder | 2:1 | $1\times10^5$ | 0.46 |
| Elliptic cylinder | 8:1 | $2\times10^5$ | 0.20 |
| 3D Body | | $Re = V_\infty d/\nu$ | $C_D = D/\left(\frac{1}{2}\rho V_\infty^2 A\right)$ |
| Sphere | | $10^4 \sim 10^5$ | 0.47 |
| Left half sphere | | $10^4 \sim 10^5$ | 0.42 |
| Right half sphere | | $10^4 \sim 10^5$ | 1.17 |
| Cube | | $10^4 \sim 10^5$ | 1.05 |
| Cube | | $10^4 \sim 10^5$ | 0.80 |
| Plate (L/h=5) | h | $10^3 \sim 10^5$ | 1.20 |

# 8.8 Hydrodynamic Force on a Body

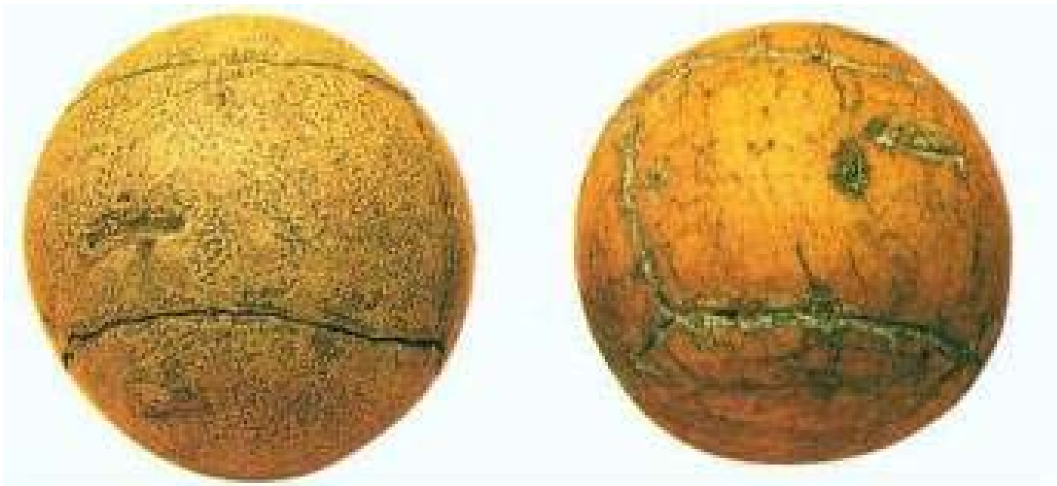



**Drag coefficient of smooth axisymmetric body**

# 8.8 Forces on A Body in Fluid

Patterns
on golf ball surface.
Why?

The earliest golf ball
was a wooden ball.



# 8.8 Forces on A Body in Fluid

- Later leather ball was introduced, but it did not travel as far as the precious wooden ball.

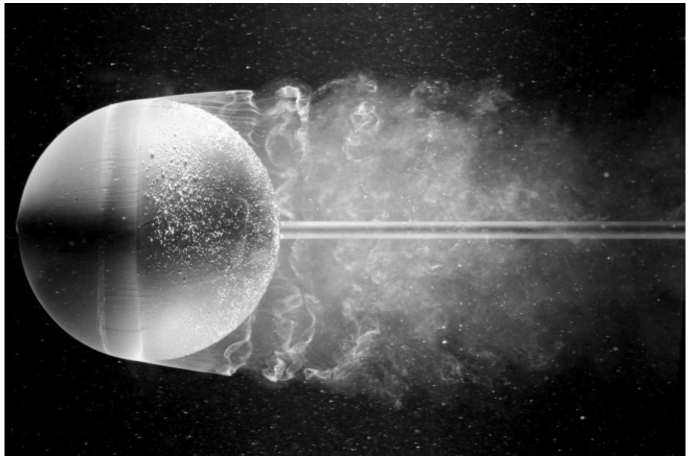
- After that, patterns began to be produced on the ball surface. It is reported that a rough ball, hit by a professional player, travels twice as far as a smooth ball.

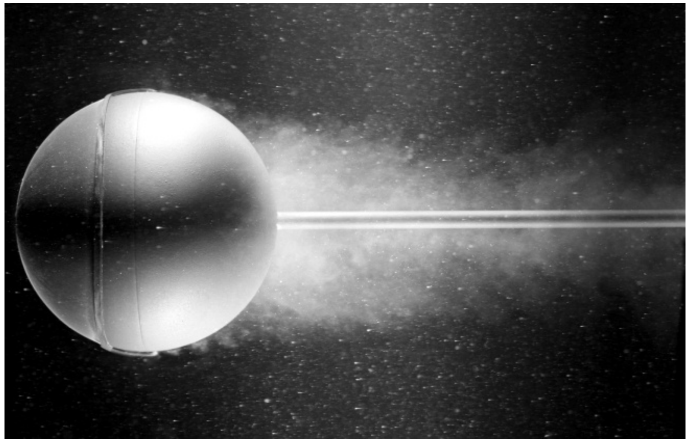
# 8.8 Forces on A Body in Fluid

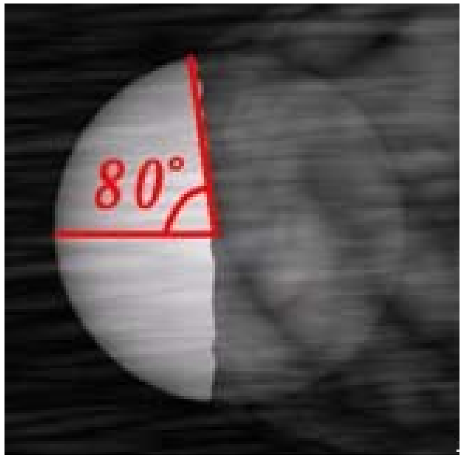
- The surrounding air applies a resultant force to a body travelling in it. Golf ball is just an example. It consists of drag and lift. ***Drag*** acts against its motion, and the ***lift*** is applied perpendicular to the direction of motion, usually upwards. ***Patterns fabricated on golf ball could reduce drag and increase lift.***

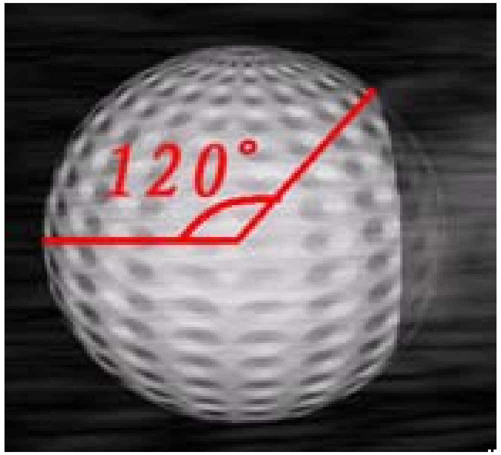
- Patterns fabricated on golf ball will increase lift. For a rotating ball, if bottom backward, it will cause the pressure on the lower part higher than on the upper part, and generate a lift. ***For golf ball, it is found that half of the lift is due to self rotation, another half is due to surface patterns, which can effectively improve the efficiency of rotation.***

# 8.8 Forces on A Body in Fluid







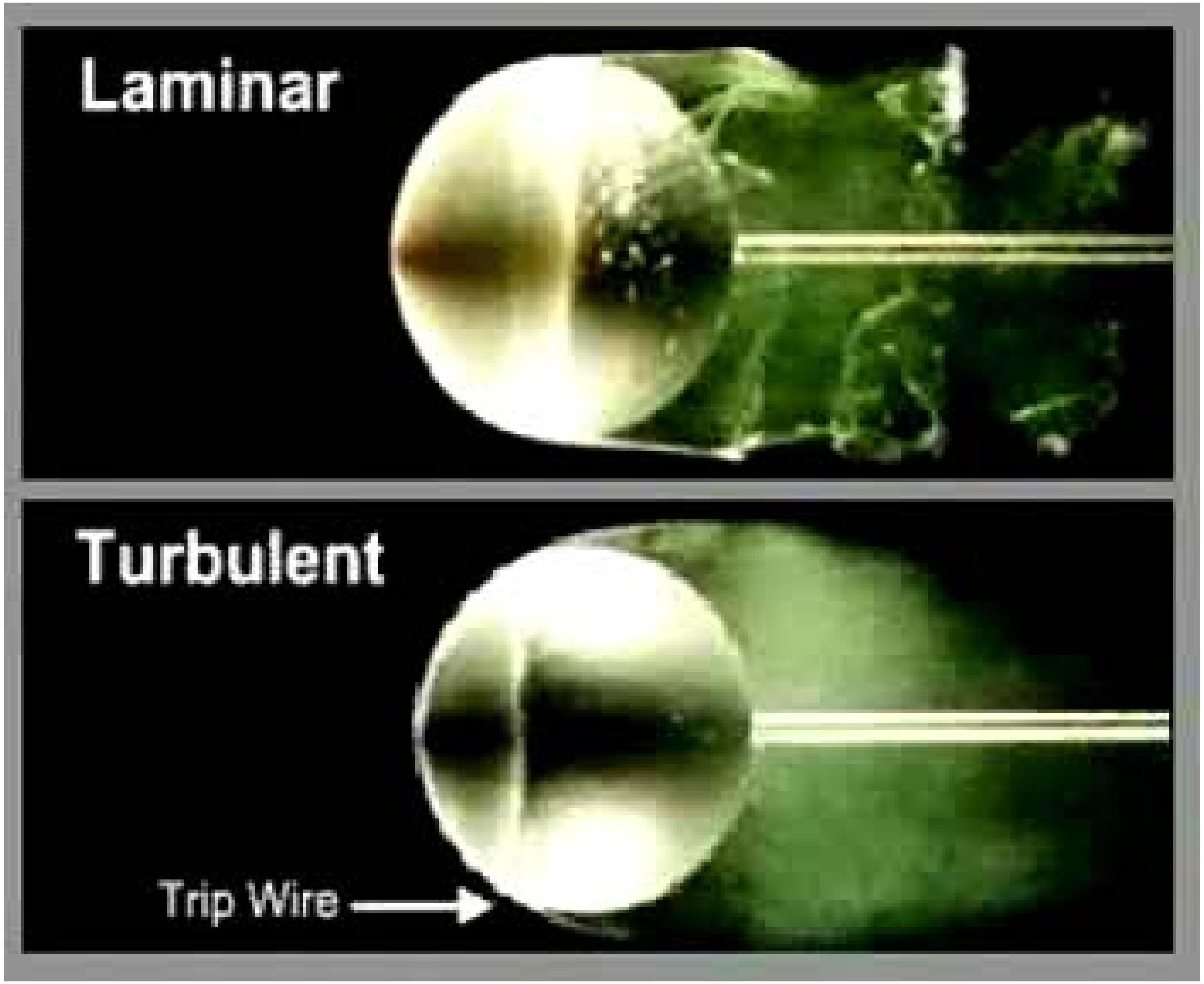

**Smooth ball**

**Rough ball**

# 8.8 Forces on A Body in Fluid

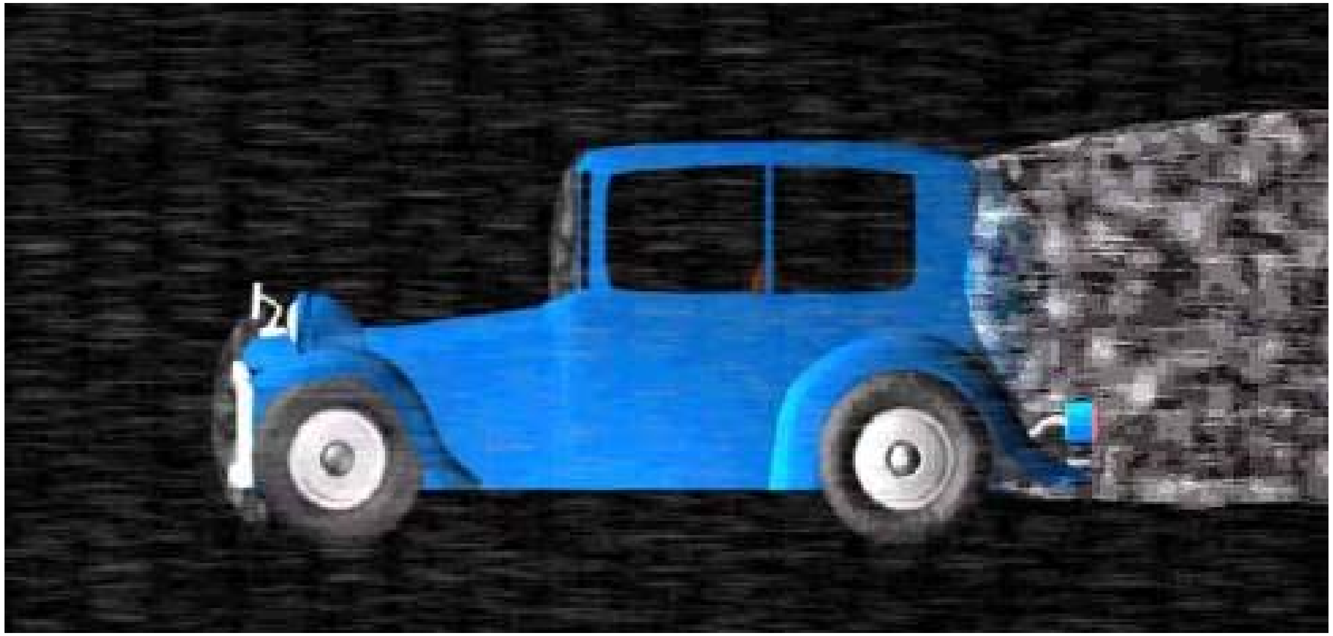
- When a golf ball travels, a high pressure region appears in the forward area. During the course of air flowing past the ball surface towards to the backward, it will separated from the ball surface at some intermediate place. After the separation place, a turbulent wake forms, where pressure is relatively lower. Drag is greatly affected by the size of the wake. *Generally, the smaller the wake area is, the larger the pressure on the rear surface is, and the smaller the drag force is.* As patterns fabricated on the ball surface could help the flow to generate a thin boundary layer and the boundary layer goes farther towards downstream before a flow separation happens. As a result it makes wake narrower, and for a well designed patterned golf ball, drag of it can reduced to be only half of the value for a smooth ball.

# 8.8 Forces on A Body in Fluid



# Car drag – Due to front part or the rear part?

In the end of 19th century, motor car was invented. At that time, car drag was considered to be mainly caused by the action of air on the front part. No attention was paid to the rear part of car. In the earlier car, as its rear part was very steep, it looks like a box, and is called **box-shaped** car. Drag coefficient, $C_D$, for that sort of car was very large, approximately up to **0.8**.



In fact, in car drag, ***form drag*** is dominant, rather than frictional drag, while the wake flow behind the car contributes to the form drag significantly.

# 8.8 Forces on A Body in Fluid

During 1930s, engineers noticed the contribution of wake flow and started to apply aerodynamics to try to improve rear shape of car. As an achievement, **beatlescar** was developed. Drag coefficient of the improved car was greatly reduced, to be only **0.6**.

# 8.8 Forces on A Body in Fluid

Entering into 1950s and 1960s, further improvement on the shape of car body was carried out, and it gradually changed towards **ship-hull-like form**. Drag coefficient of the new shaped car was further dramatically reduced, to be only **0.45**.

# 8.8 Forces on A Body in Fluid

Entering into 1980s, with the help of high performance wind tunnel, pressure distribution on car surface could be accurately measured with great detail, and optimization was able to improve local shape of car body. A **fish-like** car body was developed. Drag coefficient of such a car was much reduced, reached to **0.3**.

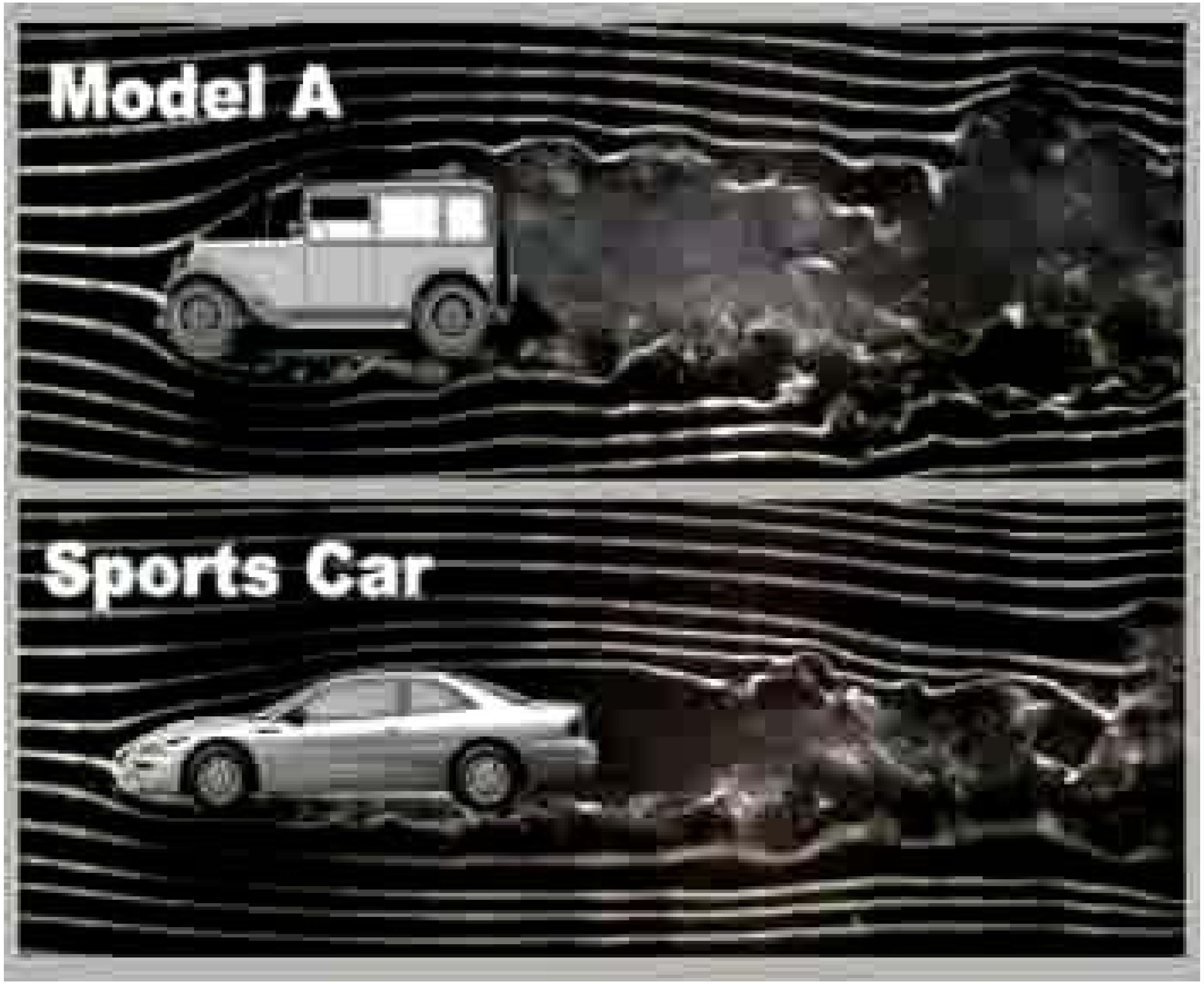
# 8.8 Forces on A Body in Fluid

Then a wedge body car appears. $C_D$ of that modern car is only **0.2**.

# 8.8 Forces on A Body in Fluid

In 1990s, a newest model was developed. It set a $C_D$ record, **0.137**.

# 8.8 Forces on A Body in Fluid



## Summary

During the past 80 years, drag coefficient has been decreased from the initial 0.8 to the recent 0.137, and accordingly drag force has reduced to be 1/6 ~ 1/5 of the original one. Car drag has been dramatically improved. This improvement is greatly related to the knowledge of viscous fluid flow field.

Nowadays, aerodynamic analysis become a necessity and plays an essential roll in car design to optimize the car contour for small drag force to reduce fuel consumption and improve the dynamic performance as well.

# Characters of viscous flows :

1) **Rotational:** vorticity may be non-zero
2) **Dissipation:** mechanical energy may be changed to other energy
3) **Diffusion:** physical quantity may diffuse due to its gradient
4) **Unsteady:** physical quantity may vary with time
5) **Unstable:** physical quantity may change essentially due to some small disturbance
6) **Random:** physical quantity may change in a random fashion and become indeterminable

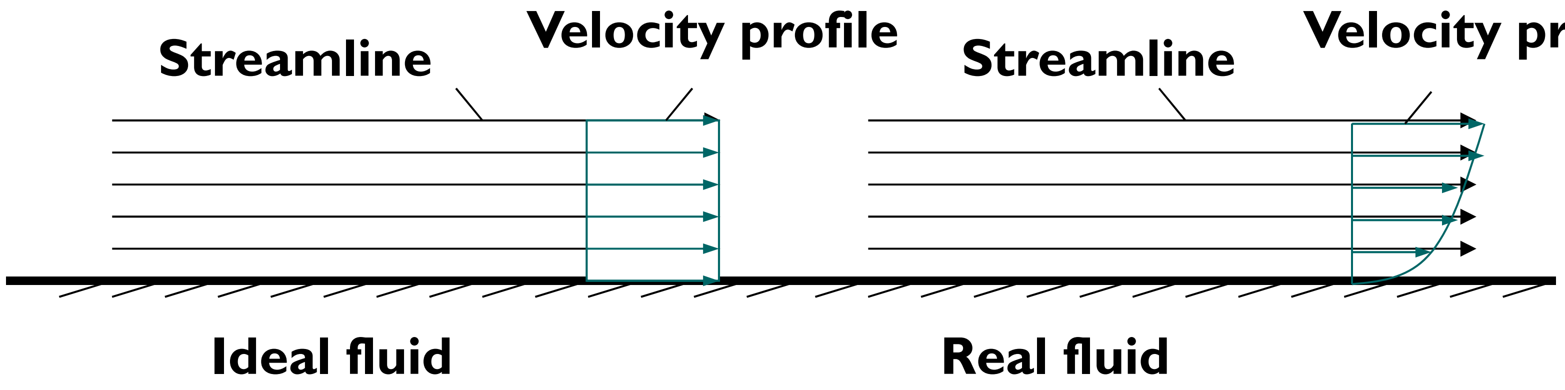
1) **Fluid Viscosity:**

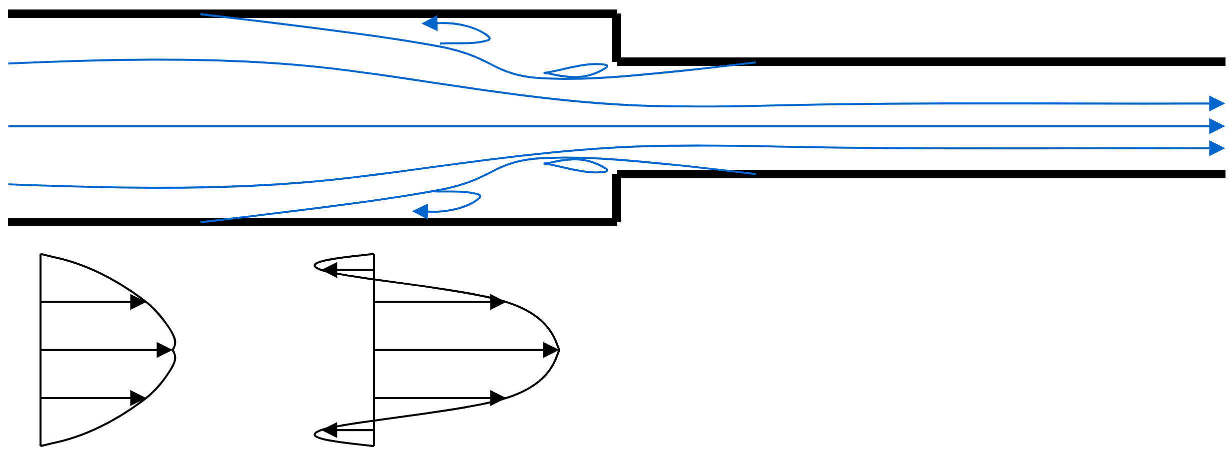
*Viscous flow* – with viscosity

*Ideal flow* (*perfect flow*) – without viscosity

During the course of fluid motion, fluid particles will accompany with deformation, and cause viscous forces, i.e., inner frictions, between adjacent fluid layers, which transfer mechanical energy to irreversible energies, such as thermal energy, and as a result *(mechanical) energy dissipated*.

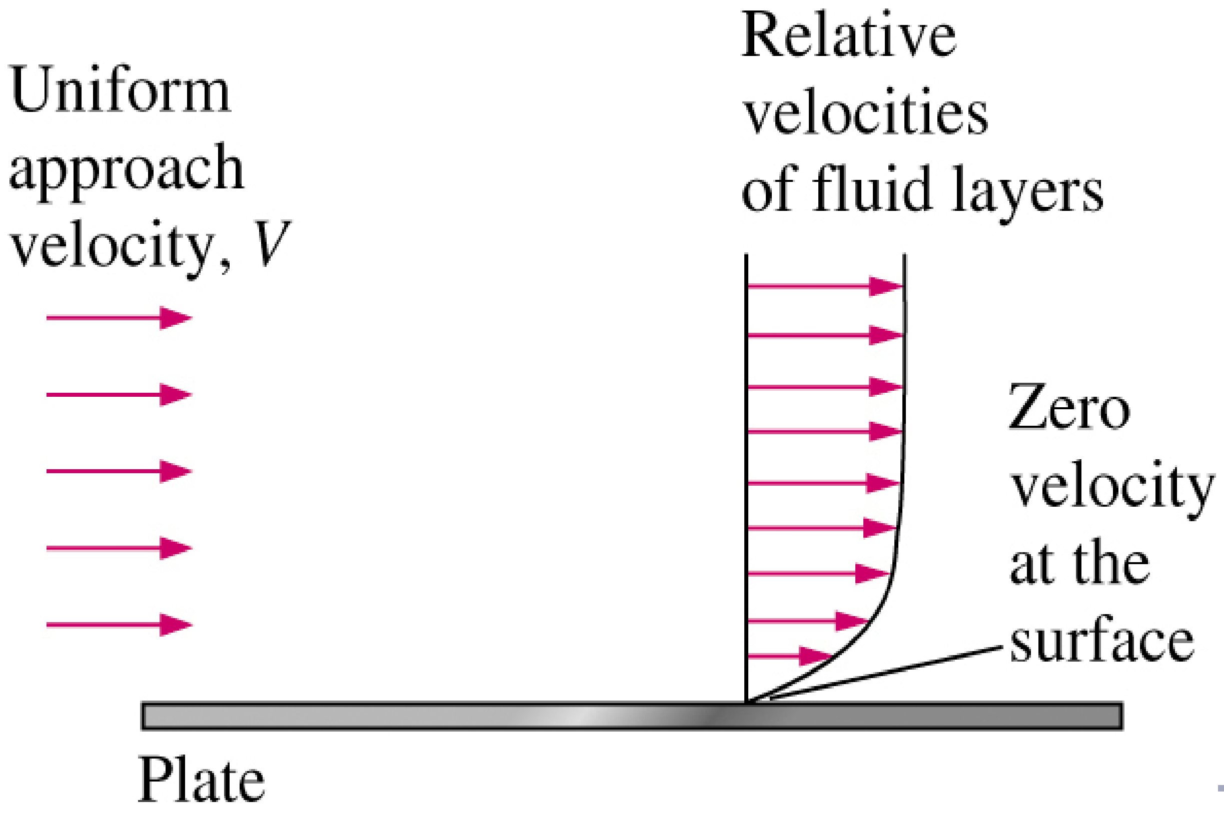
# Difference between viscous flow and perfect flow



## No-slip condition

On body surface, viscous flow must satisfy no-slip condition, *i.e.*, both normal and tangential velocity components of a particle coincides with those of the body at the location.



# Review of Chapter 8

- **Navier-Stokes Equations (N-S Equation)**
  **Governing equation of incompressible viscous flow**

$$\boxed{\begin{gathered}\nabla\cdot\mathbf{V}=0\\ \frac{\partial\mathbf{V}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{V}=-\frac{1}{\rho}\nabla p+\mathbf{g}+\nu\nabla^2\mathbf{V}\end{gathered}}$$

- **Dimensionless N-S Equation**

$$\left(\frac{L}{UT}\right)\frac{\partial\boldsymbol{V}^*}{\partial t^*}+\boldsymbol{V}^*\cdot\nabla^*\boldsymbol{V}^*$$

$$=-\left(\frac{P}{\rho U^2}\right)\nabla^* p^*+\left(\frac{gL}{U^2}\right)\boldsymbol{g}^*+\left(\frac{\nu}{UL}\right)\nabla^{*2}\boldsymbol{V}^*$$

- **Dimensionless Numbers and Their Physical Meaning**

***Strouhal number*** $$St = \frac{L}{UT} = \frac{\text{Local derivative}}{\text{Convective derivative}}$$

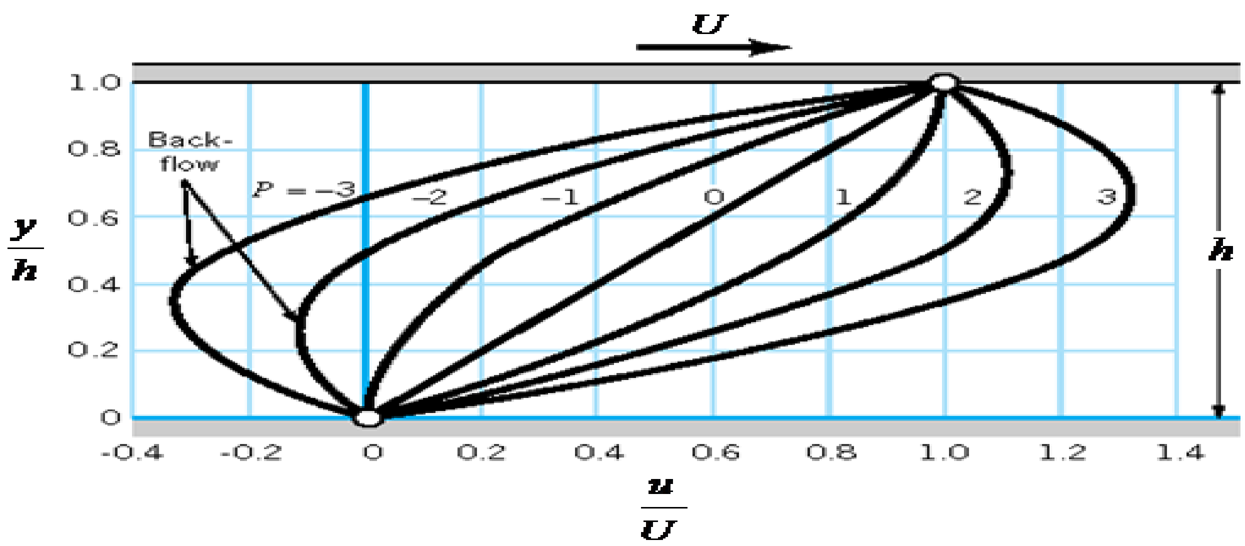
***Euler number*** $$Eu = \frac{P}{\rho U^2} = \frac{\text{Pressure}}{\text{Inertial Force}}$$

***Reynolds number*** $$Re = \frac{UL}{\nu} = \frac{\text{Inertial Force}}{\text{Viscous Force}}$$

***Froude number*** $$Fr^2 = \frac{U^2}{gL} = \frac{\text{Inertial Force}}{\text{Gravity}}$$

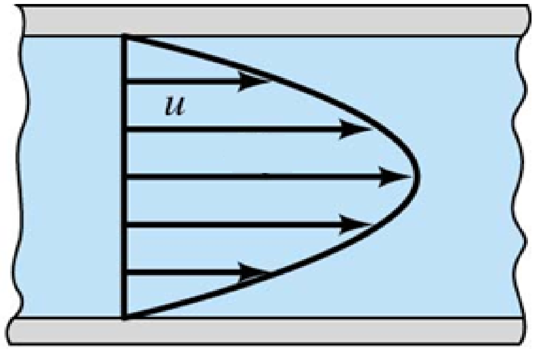
- **2 Dimensional Plane Poiseuille-Couette Flow**

$$u(y)=\underbrace{\left(-\frac{\partial p}{\partial x}+\rho g_x\right)\frac{h^2}{2\mu}\left[\frac{y}{h}-\left(\frac{y}{h}\right)^2\right]}_{\text{Poiseuille Flow}}+\underbrace{U\frac{y}{h}}_{\text{Couette Flow}}$$



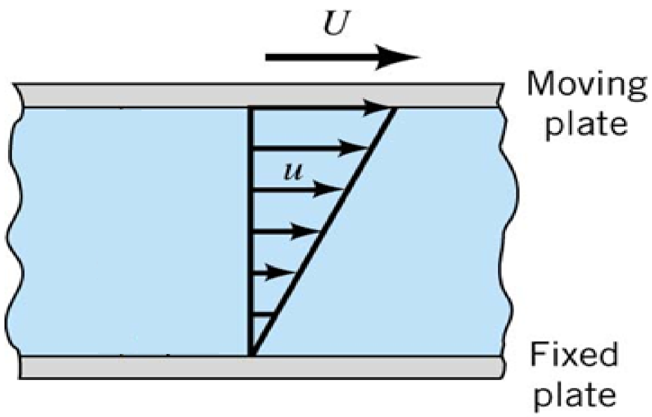
# Review of Chapter 8

- **Poiseuille Flow：** Flow between two fixed parallel plates, driven by pressure gradient.




$$u(y) = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left(y^2 - hy\right)$$

- **Couette Flow：** Flow between two parallel plates, one is fixed, the other moves at constant speed, and without pressure gradient.

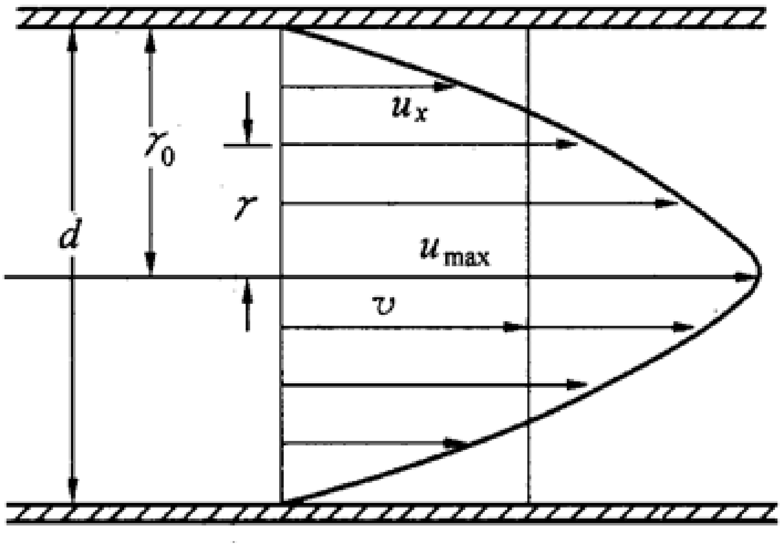



$$u(y) = \frac{y}{h}U$$

- **Circular Pipe Poiseuille Flow**

$$\boxed{u_z(r) = -\frac{dp}{dz}\frac{R^2}{4\mu}\left[1-\frac{r^2}{R^2}\right]}$$

$$u_{\max} = -\frac{R^2}{4\mu}\frac{dp}{dz}$$

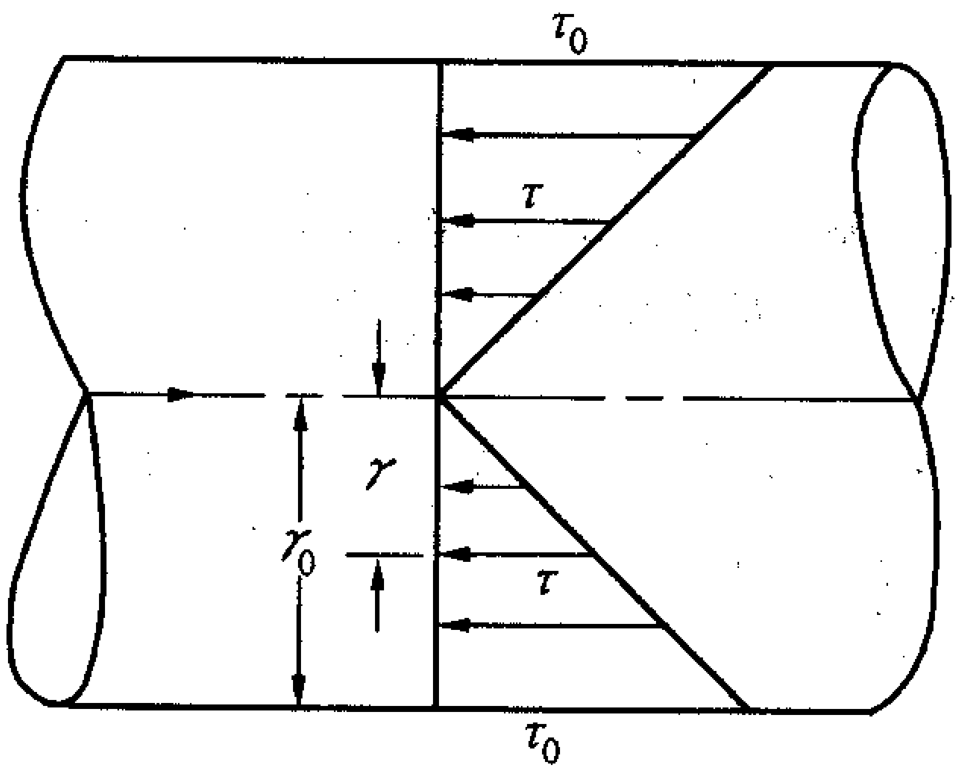



$$\bar{u} = \frac{u_{\max}}{2}$$

$$Q = -\frac{\pi R^4}{8\mu}\frac{dp}{dz}$$

# Review of Chapter 8

$$\tau = \frac{r}{R}\tau_0, \qquad \tau_0 = -\mu \frac{du_z}{dr}\bigg|_{r=R} = -\frac{R}{2}\frac{dp}{dz} = 4\mu\frac{\bar{u}}{R}$$



**Darcy-Weisbach Equation：** **Head loss in circular pipe.**

$$\boxed{h_f = \lambda \frac{L}{D}\frac{\bar{u}^2}{2g}}$$

**Smooth laminar flow：** $\lambda = \dfrac{64}{\mathrm{Re}}$ $\qquad \mathrm{Re} = \dfrac{\rho D \bar{u}}{\mu}$

**Relation of Head Loss to Mean Velocity in Circular Pipe：**

**Laminar flow：** $h_f \propto V^{1.0}$

**Turbulent flow：** $h_f \propto V^{1.75 \sim 2.0}$

- **Flow past Sphere with Small Re ---- Stokes Flow**

**Inertial force is neglected. Drag obeys Stokes law**

$$\boxed{D = 6\pi\mu r_0 V_\infty}$$

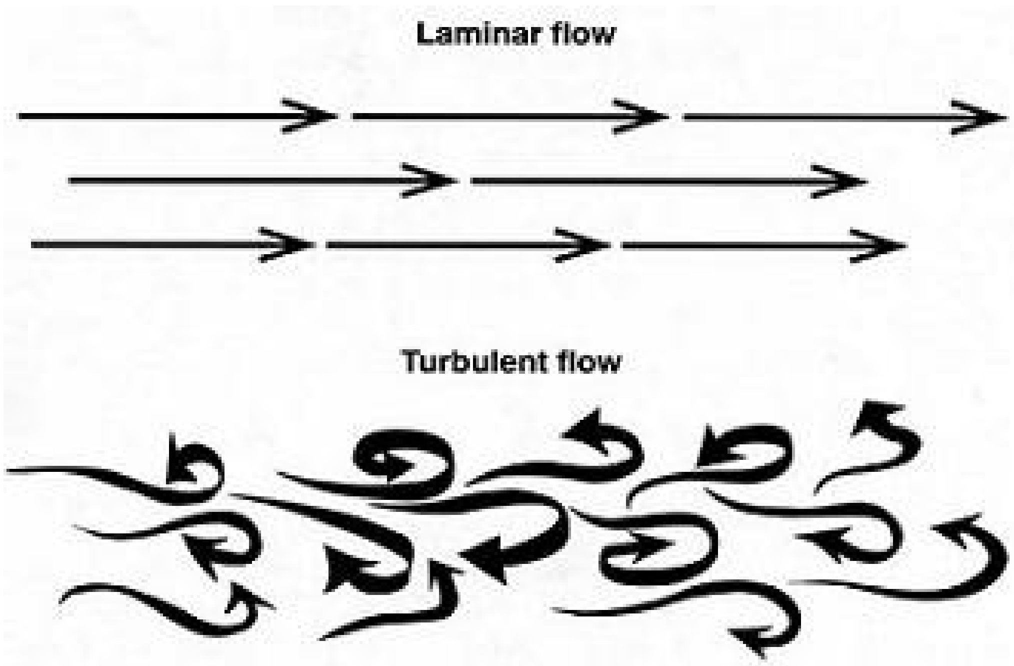
**or in drag coefficient**

$$C_D = \frac{D}{1/2\left(\rho\pi r_0^2 V_\infty^2\right)} = \frac{24}{\mathrm{Re}}, \qquad \mathrm{Re} = \frac{2\rho r_0 V_\infty}{\mu} = \frac{2r_0 V_\infty}{\nu}$$

**States of viscous flow**: ***laminar flow*** **and** ***turbulent flow***

**Laminar Flow:** **The flow is regular and stable. It travels in layers without mixing with other layers. Fluid particle traces smooth curve.**

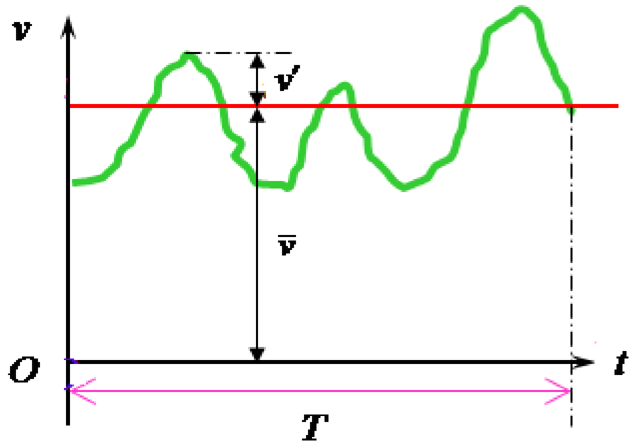
**Turbulent Flow:** **The flow is chaotic, irregular and unstable. It is mixing with each other dramatically.**



## Time Averaging of Flow Quantities

**Instantaneous Quantity = Time Average + Fluctuation**

$$v = \bar{v} + v', \qquad \bar{v} = \frac{1}{T}\int_0^T v\,dt$$




$$\tau = \tau_1 + \tau_2, \qquad \tau_1 = \mu \frac{\mathrm{d}u}{\mathrm{d}y}, \qquad \tau_2 = \rho \ell^2 \left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2$$

# Review of Chapter 8

## Reynolds Average Navier-Stokes Equation (RANSE)

$$\frac{\partial \overline{u}}{\partial x}+\frac{\partial \overline{v}}{\partial y}+\frac{\partial \overline{w}}{\partial z}=0$$

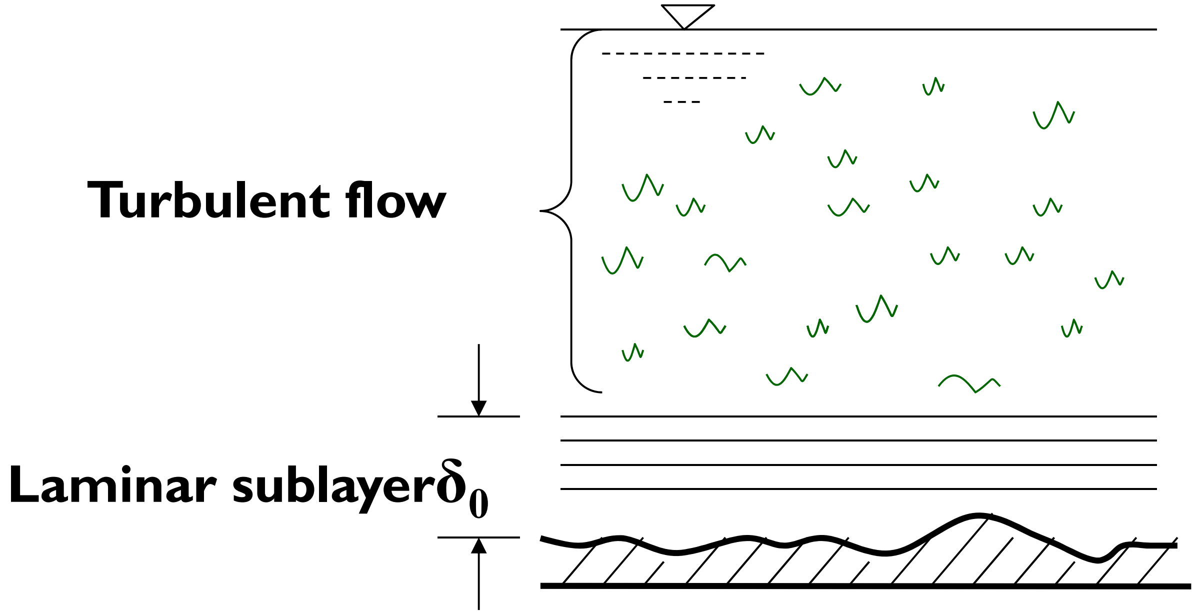
$$\frac{\partial \overline{u_i}}{\partial t}+\overline{u_j}\frac{\partial \overline{u_i}}{\partial x_j}=f_i-\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i}+\frac{1}{\rho}\left(\mu\nabla^2\overline{u_i}-\frac{\partial}{\partial x_j}(\rho\overline{u'_i u'_j})\right)$$

$$R_{ij}=\begin{pmatrix}-\rho\overline{u'u'} & -\rho\overline{u'v'} & -\rho\overline{u'w'}\\ -\rho\overline{v'u'} & -\rho\overline{v'v'} & -\rho\overline{v'w'}\\ -\rho\overline{w'u'} & -\rho\overline{w'v'} & -\rho\overline{w'w'}\end{pmatrix}$$

—— **Turbulent Stress （Reynolds Stress）**

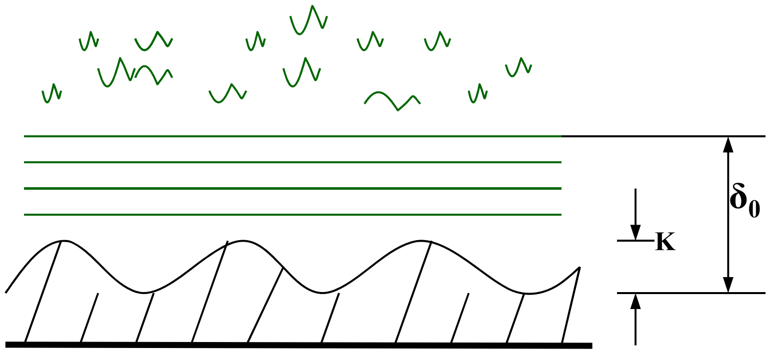
## Laminar Sublayer

**In turbulent flow, in the neighborhood of wall there exists a very thin layer, where viscous shear stress almost does not show fluctuation, and the flow is laminar. This thin laminar flow layer is called laminar sublayer.**

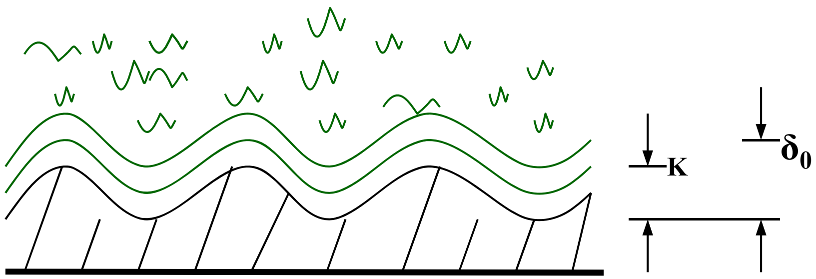



$$\delta_0 = \frac{32.8d}{\mathrm{Re}\sqrt{\lambda}}$$

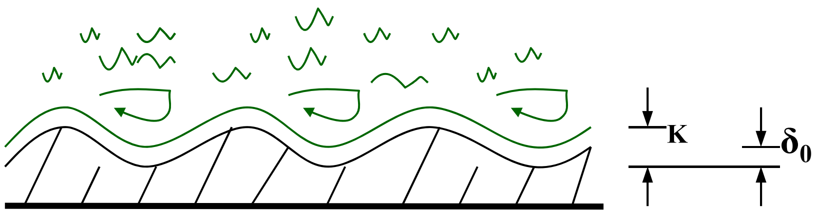
## Wall Roughness




**Smooth wall**



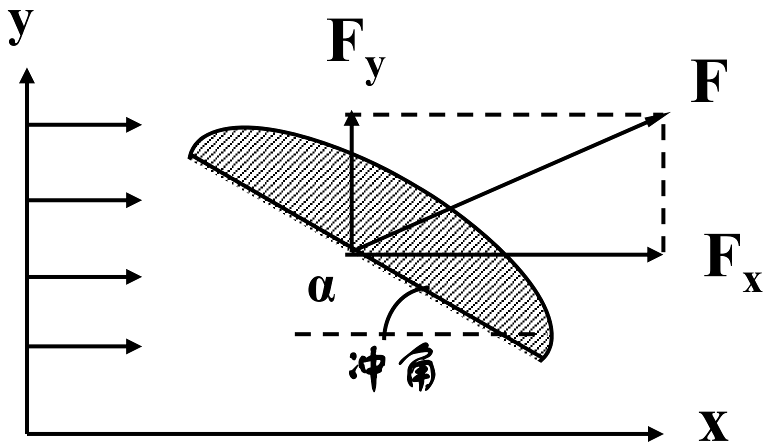

**Transitional rough wall**




**Rough wall**

## Force on Body in Fluid：Lift and Drag

**Depending on velocity, shape and orientation of body.**




**Drag：Frictional Drag + Form Drag**

**Frictional drag is greatly related to surface area of body**

**Form drag is mainly related to shape of body**

***For low Re, mainly frictional drag; for high Re, mainly form drag.***

# Drag coefficient with Re for Circular (Sphere) Flow