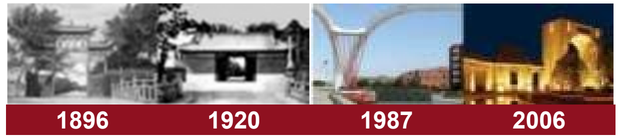
上海交通大學
SHANGHAI JIAO TONG UNIVERSITY



Introduction to Marine Hydrodynamics

(NA235)

Department of Naval Architecture and Ocean Engineering

School of Naval Architecture, Ocean & Civil Engineering

Shanghai Jiao Tong University

Chapter 8

Fundamental Theory of Viscous Incompressible Fluid Flow

8.5 Some Simple Viscous Flows

We noticed that 1/3 of the drag force coming from the difference of fore- and aft-pressures, the remaining 2/3 is due to frictional stresses. That is, *for flow at small Reynolds number, major part of the resultant drag force is due to the frictional viscous stresses.*

But we should understand that with increase of Reynolds number, weight of the viscous friction will decrease, instead, the weight of pressure difference will increase. At high Reynolds number, drag force is almost fully determined by the fore- and aft-pressure difference.

As a result, drag force on a sphere with small Reynolds number is

$$F_D = 6\pi\mu r_0 V_\infty$$

Therefore, *drag force is proportional to the speed of the uniform flow, but it does not explicitly depend on the fluid density.*

8.5 Some Simple Viscous Flows

By introducing a ***drag coefficient*** which is defined as

$$C_D = \frac{F_D}{\frac{1}{2}\rho V_\infty^2 A}$$

for the current flow past a sphere with low Reynolds number, it results

$$C_D = \frac{24}{\mathrm{Re}}$$

$$\mathrm{Re} = \frac{2r_0 V_\infty}{\mu/\rho}$$ is the **Reynolds number**

$$A = \pi r_0^2$$ is **the projected area along the flow**

Therefore, *in low Reynolds number flow, drag force is proportional to the speed* V_∞ . This is called ***Stokes law of resistance***. This result has been confirmed by experiments.

8.5 Some Simple Viscous Flows

It was assumed that *Reynolds number* was very small, that is, *inertia force* was much smaller than *viscous force*. Without that assumption, the above conclusion won't be derived. Let us look at the ratio of inertia force to viscous force.

$$\frac{\text{inertia force}}{\text{viscous force}} = \frac{\rho V_r \dfrac{\partial V_r}{\partial r}}{\mu \dfrac{\partial^2 V_\theta}{\partial r^2}} = \frac{\rho V_\infty^2 \dfrac{1}{r}}{\mu V_\infty \dfrac{1}{r^2}} = \frac{V_\infty r}{\nu} = \frac{r}{2r_0}\mathrm{Re}$$

From this expression, we can find that in the neighborhood of the sphere, where r is comparable with the radius of the sphere, r_0, since Re is assumed to be very small, *inertia force* can always be neglected. But as soon as Re is given, no matter how small it is, this ratio may still be very large, provided distance r from the sphere is large enough, where *inertia force* is now no more negligible. This contradicts the original assumption. This contradiction is called **Stokes paradox**.

8.5 Some Simple Viscous Flows

From above analysis, Stokes' solution of a sphere is not valid at large distances from the body, where inertia force is no more negligible. In 1910, Oseen proposed an improvement to Stokes' solution by partly accounting for the inertia force. Instead of removing them, the inertia terms are linearized and remained. It decomposes flow into a uniform flow and a small disturbance flow due to the existence of the sphere.

$$\mathrm{V} = \mathrm{V}_\infty + \mathrm{V}'$$

Substituting it in **N-S equation**, and removing higher order terms, it results a linearized N-S equation, namely **Oseen's equation**.

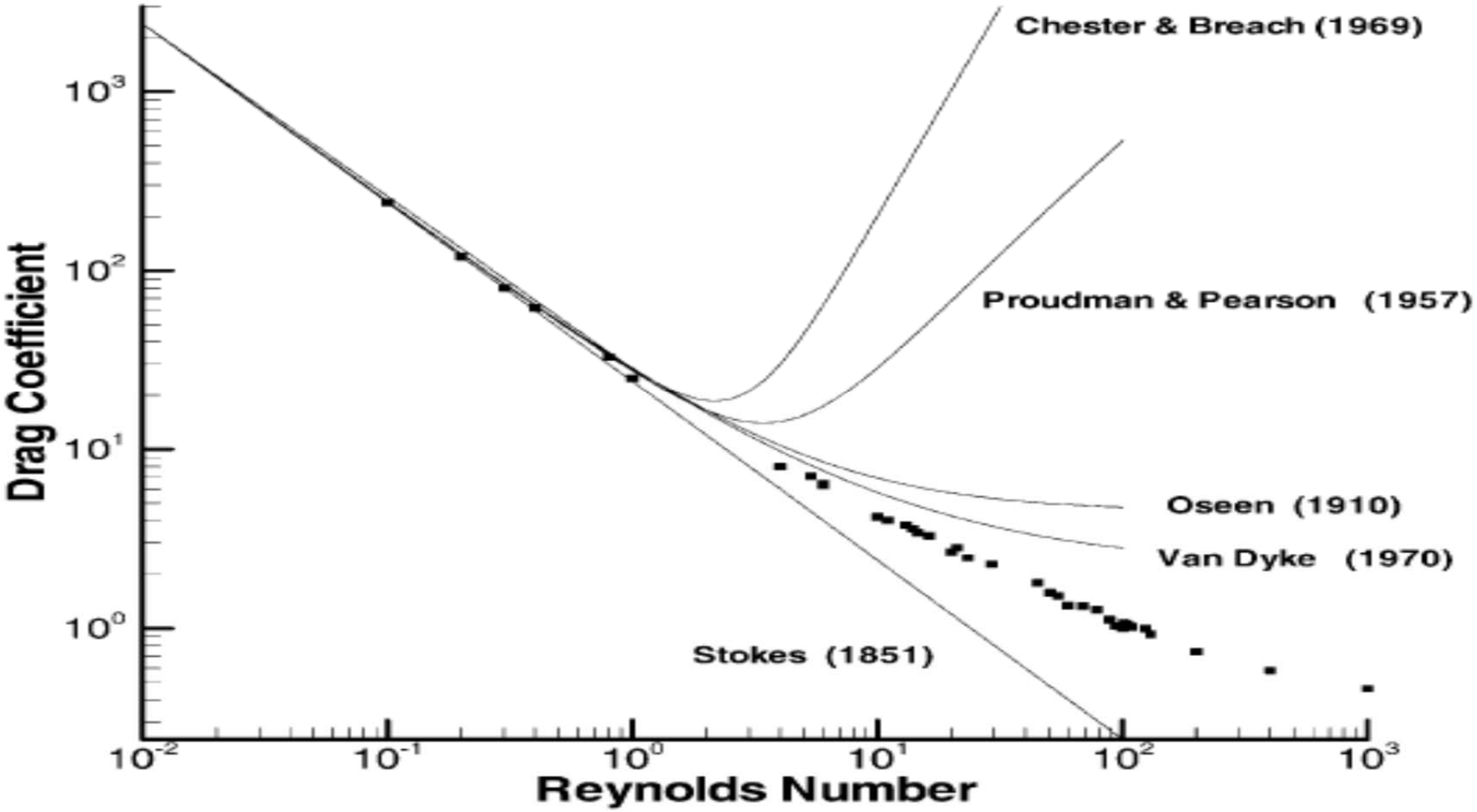
$$\nabla \cdot \mathrm{V}' = 0$$

$$\frac{\partial \mathrm{V}'}{\partial t} + \left(\mathrm{V}_\infty \cdot \nabla\right)\mathrm{V}' = \mathrm{F} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathrm{V}'$$

8.5 Some Simple Viscous Flows

Drag coefficient of a sphere (Oseen's formula)

$$C_D = \frac{24}{\mathrm{Re}}\left(1 + \frac{3}{16}\mathrm{Re}\right)$$



8.5 Some Simple Viscous Flows

PERGAMON

International Journal of Non-Linear Mechanics 37 (2002) 1–18

INTERNATIONAL JOURNAL OF
NON-LINEAR MECHANICS

www.elsevier.com/locate/ijnonlinmec

An analytic approximation of the drag coefficient for the viscous flow past a sphere

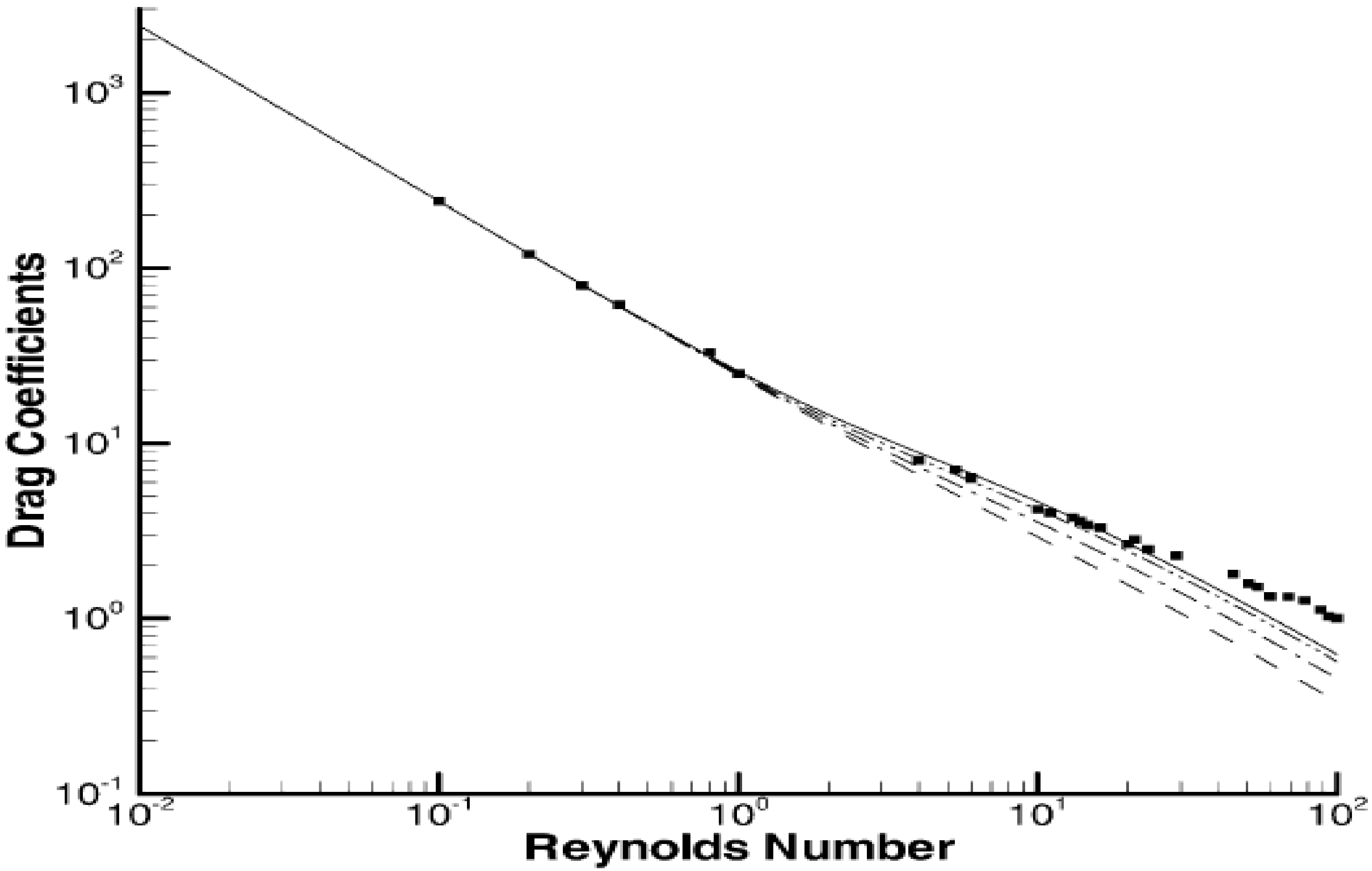
Shi-Jun Liao

School of Naval Architecture and Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200030, People's Republic of China

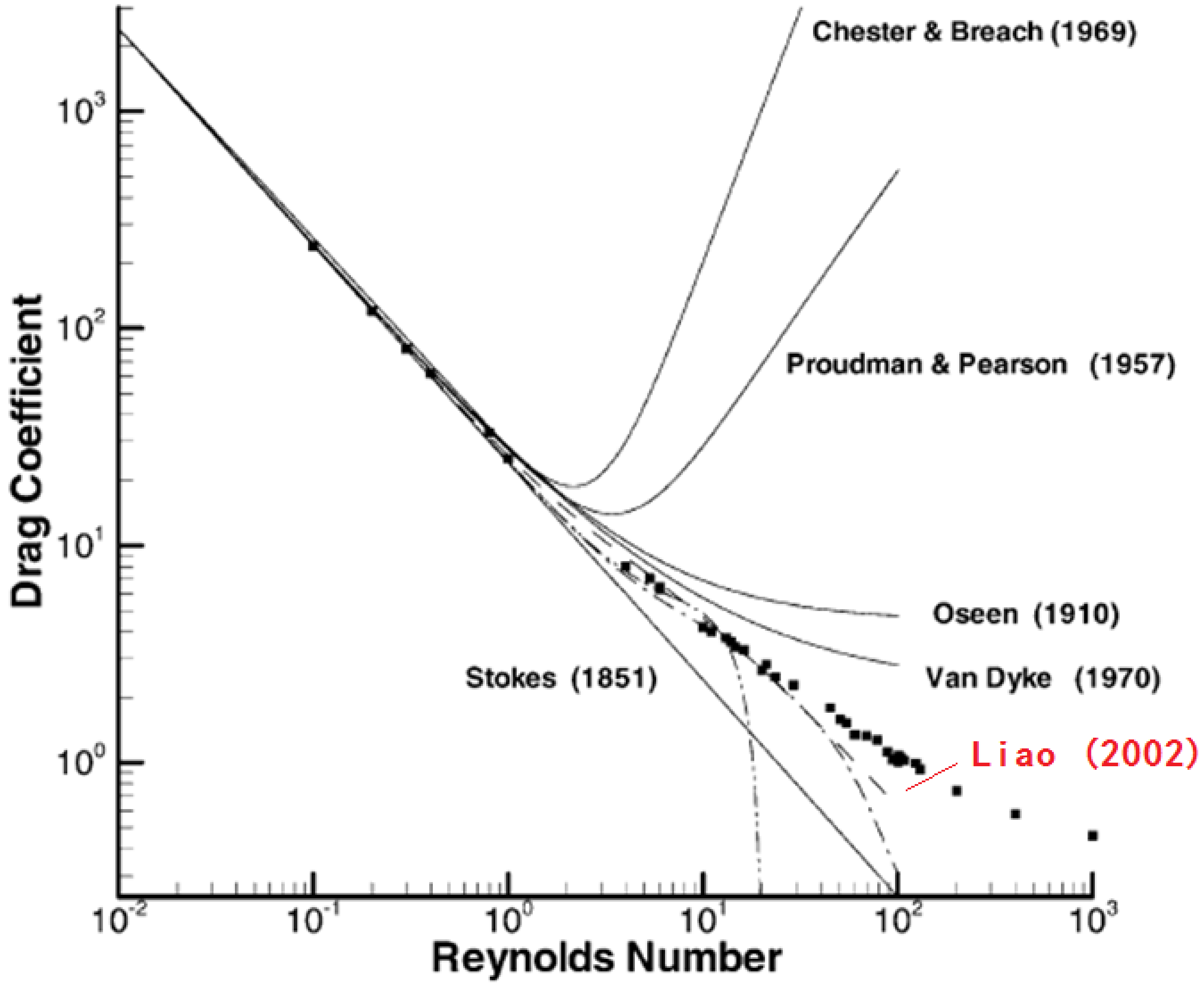
In 2002, Professor Shi-Jun Liao of SJTU directly solved the nonlinear problem of viscous flow past a sphere by means of his Homotopy Analysis Method. The analytical solution he obtained is valid for Reynolds number up to 30.

8.5 Some Simple Viscous Flows

Drag Coefficient of a Sphere (Liao's result)



8.5 Some Simple Viscous Flows



8.6 Laminar and Turbulent Flows

In 1883, a British scientist and mathematician, Osborne Raynolds, carried out a pioneer experiment. He found two types of viscous flows. At low flow rates, fluid moves within parallel layers. This type of flow is called as ***laminar flow***. While at high flow rates, fluid particles are mixed with the surrounding fluid layers macroscopically, motions of them are rather chaotic and present some irregularity. This type of chaotic irregular flows is called as ***turbulent flow***.

8.6 Laminar and Turbulent Flows

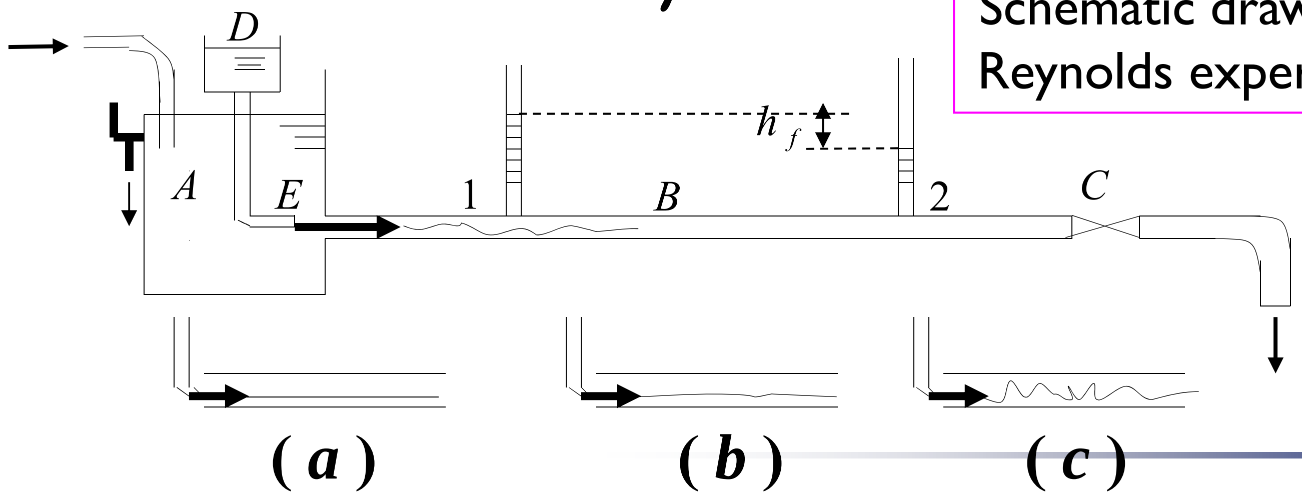
Osborne Reynolds (1842-1916)

Reynolds experiment (1883)

As the figure shows, arrangement of the experiment involves a water tank, *A*, and an outlet through a small glass tube, *B*, at the end of which equipped with a stopcock to change the speed of water through the tube.

Inside the horizontal small glass tube, flow is steady. As mentioned in the previous section, ***head loss due to friction*** is evaluated by the difference of pressure heads at two ends.

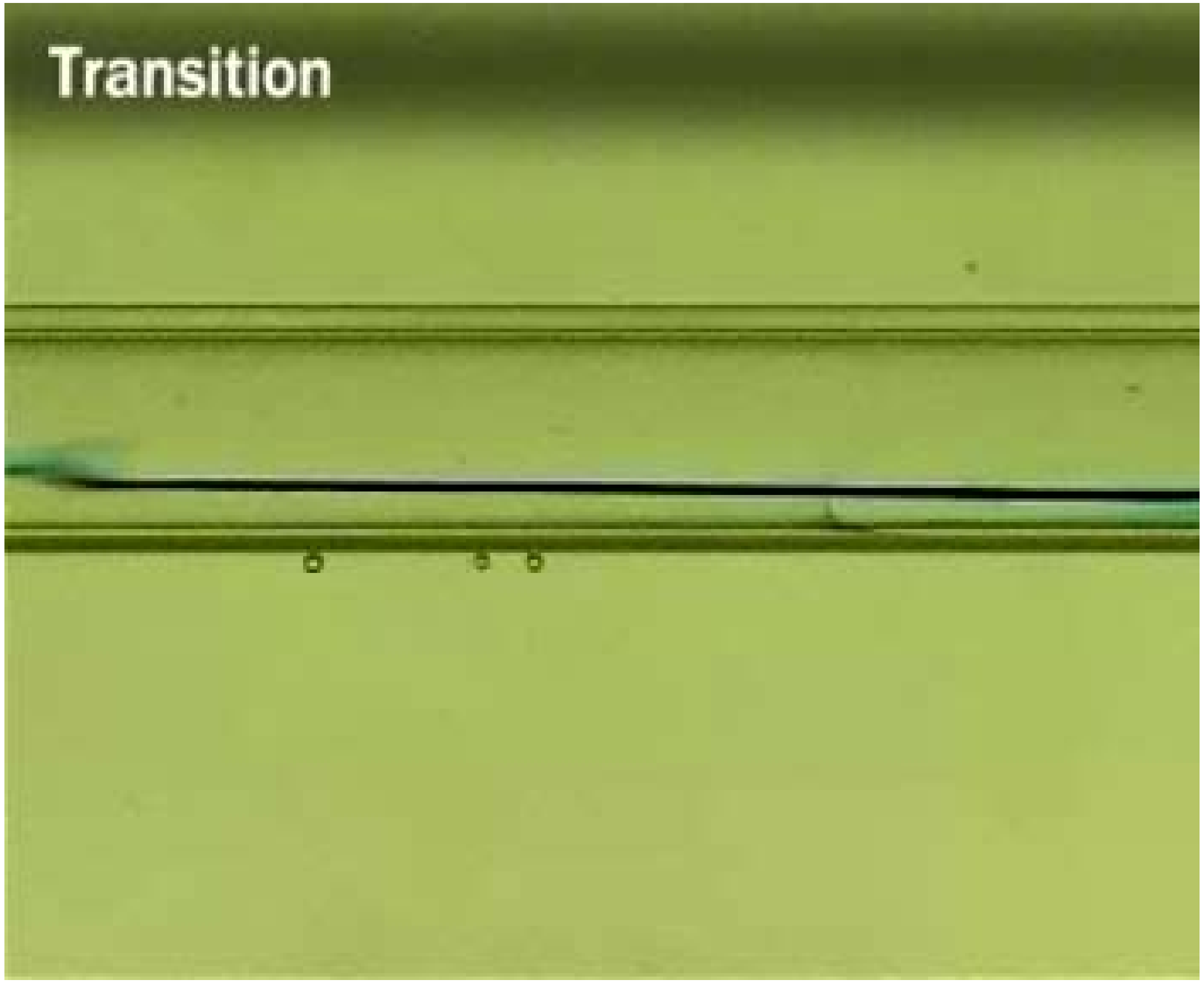
$$h_f = \frac{p_1 - p_2}{\gamma}$$



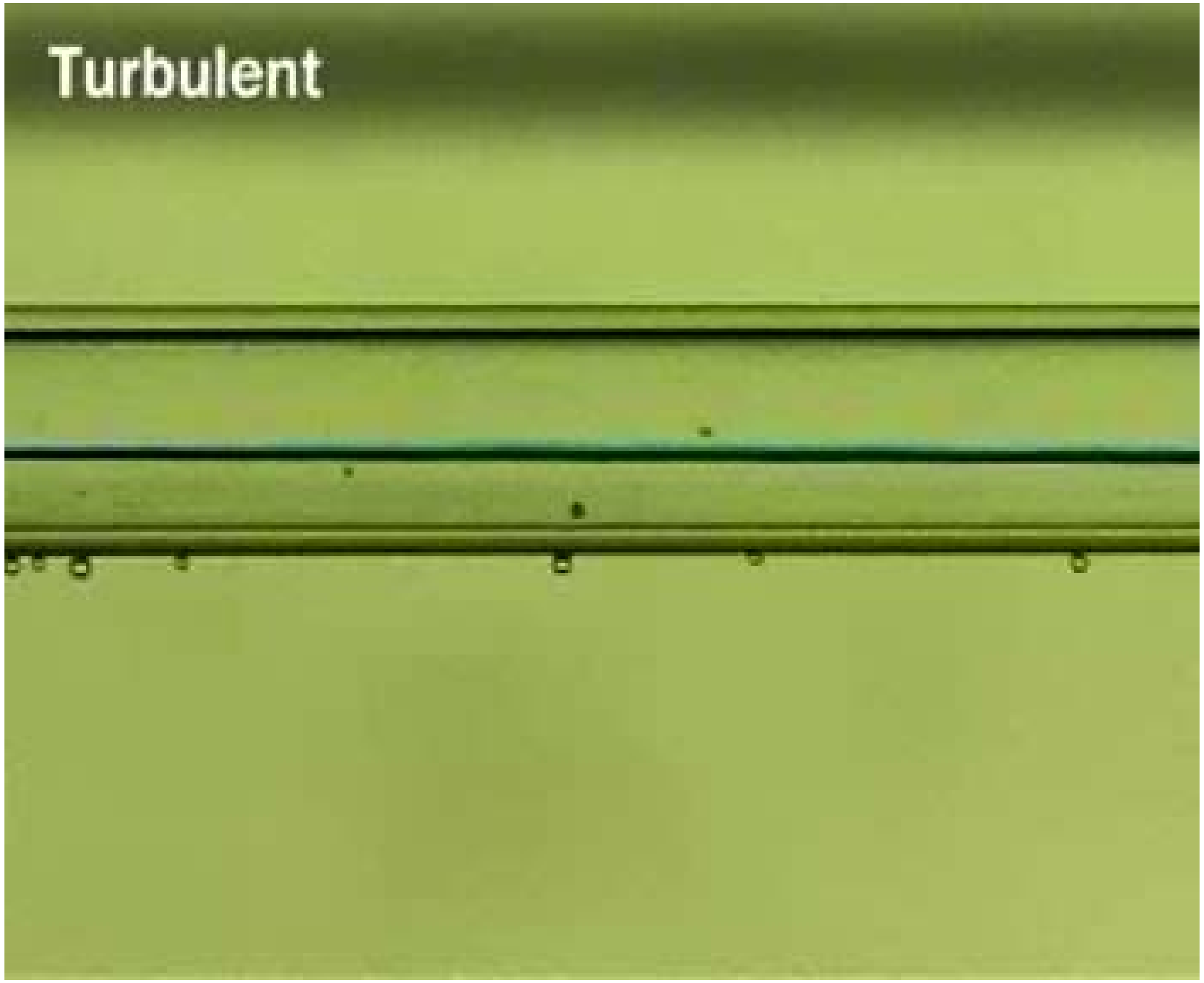
Schematic drawing of Reynolds experiment

8.6 Laminar and Turbulent Flows

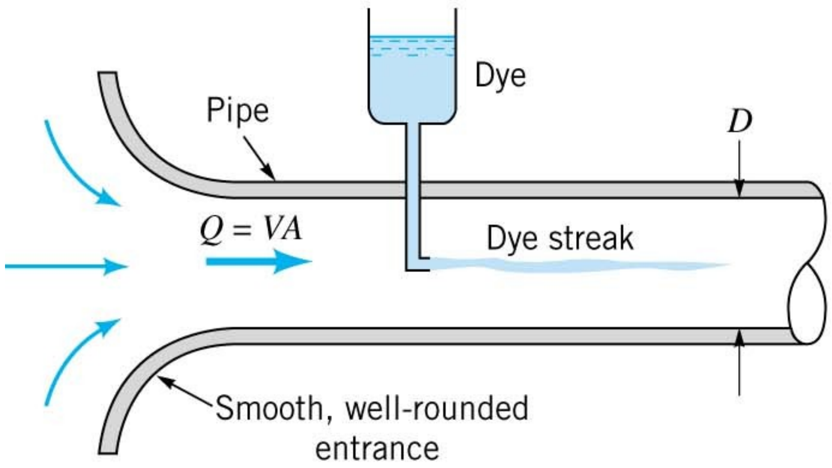
8.6 Laminar and Turbulent Flows



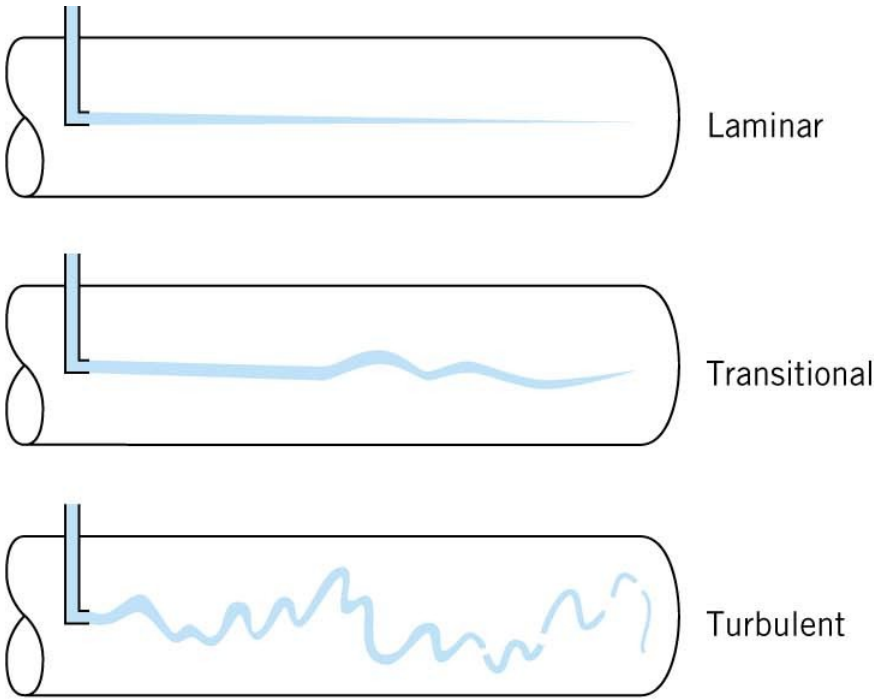
8.6 Laminar and Turbulent Flows



8.6 Laminar and Turbulent Flows

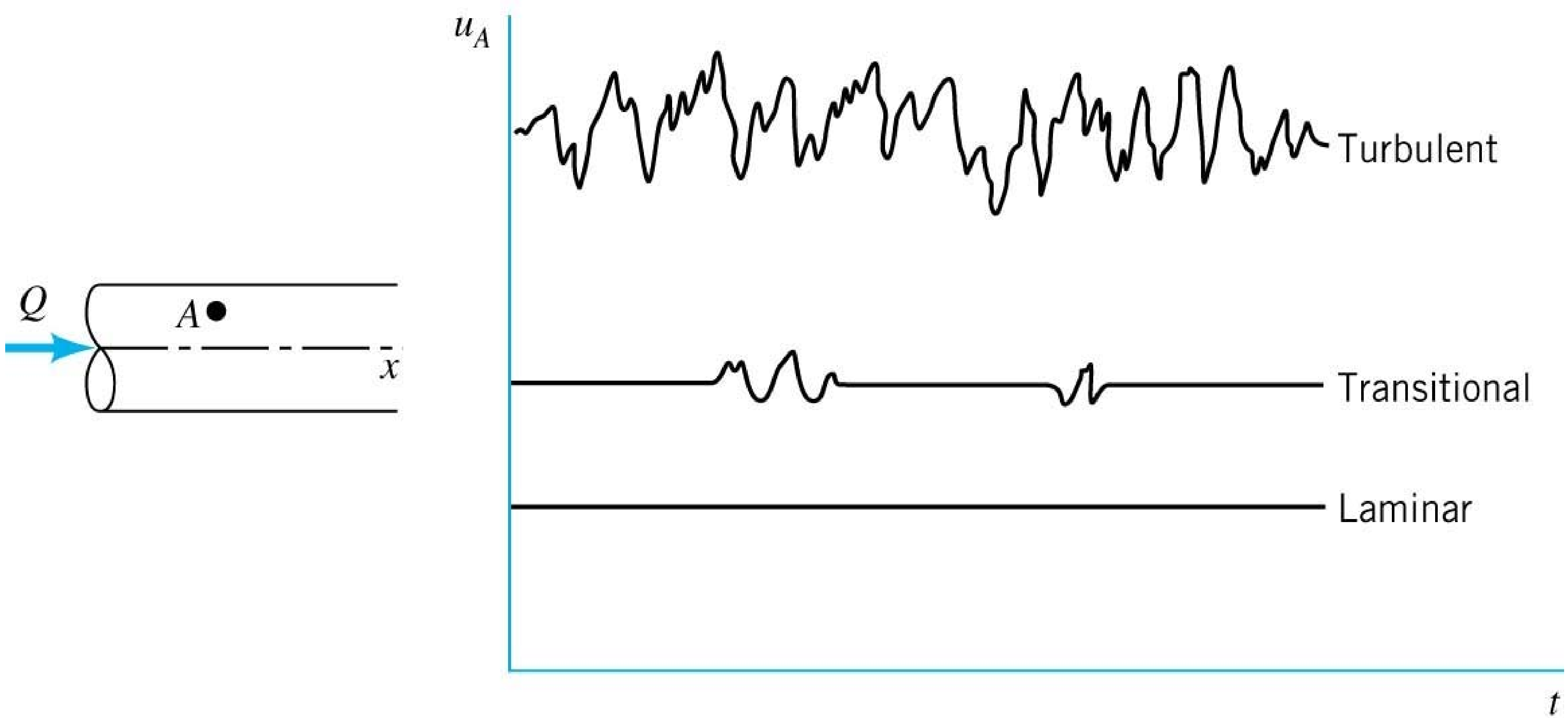


(a)



(b)

8.6 Laminar and Turbulent Flows



8.6 Laminar and Turbulent Flows

At very small flowrates, velocity of the flow in the glass tube will be relatively small. The observed streakline of the dye in the tube was nearly a straight line and very stable. It demonstrates that water particles move straight forward in a line parallel to the centerline of the tube, without apparent transverse motions, and do not mixed with the surrounding water. This sort of flow is known as ***laminar flow***.

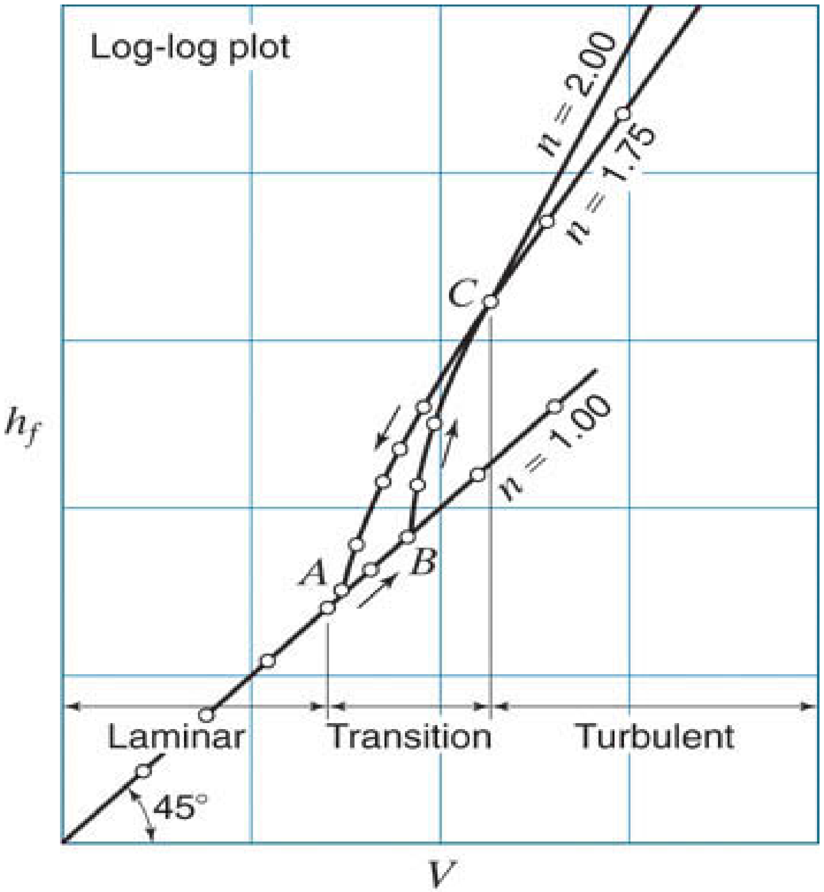
With gradually increase of the flowrate, dye streak will start to fluctuate and be not a straight line any more. This intermediate flow state is called as ***transitional flow***.

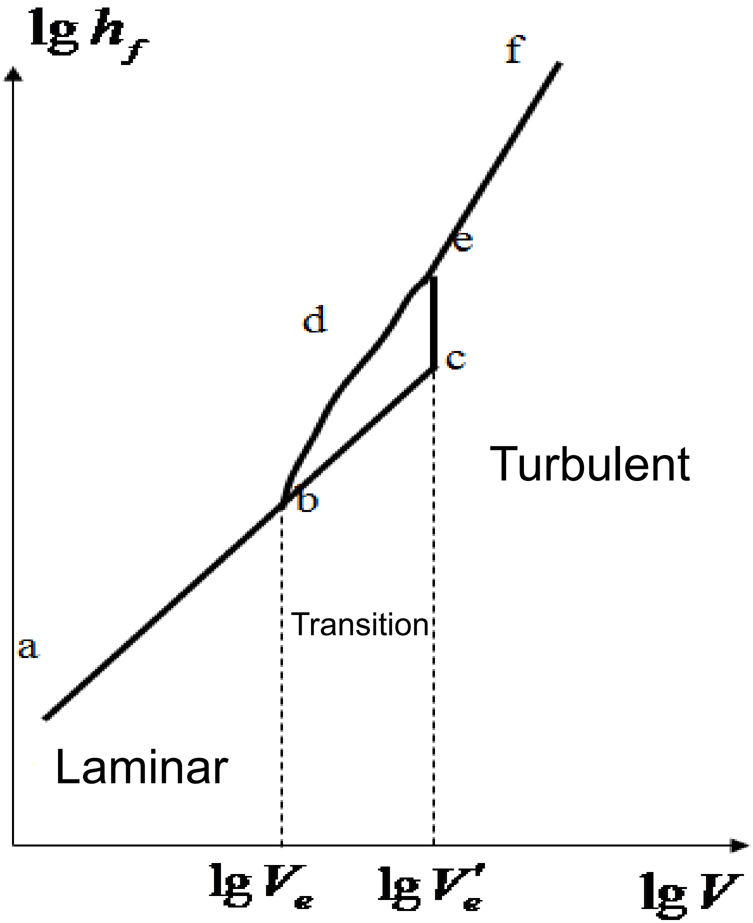
At very high flowrates, fluid velocity will also become very large, and the dye streak will fluctuate both temporarily and spatially, and intermittent bursts of irregular behaviour along the streak. When flowrate becomes high enough, over a threshold value, the dye streaks will be immediately getting blurred and spreads across the entire tube in a chaotic and random fashion. Not only the major axial velocity component, but also transverse components, though possibly small, accompany with irregular fluctuations. This chaotic irregular flow is known as ***turbulent flow***.

Now, reversely, if the flowrate is managed to be gradually reduced from higher rate to lower rate, the flow style will be gradually changed from a chaotic turbulent flow to intermediate transitional flow and finally back to stable laminar flow.

8.6 Laminar Flow and Turbulent Flow

Below are diagrams of the correspondence between average flow velocity, *V* , and water head loss, h_f. For clarity, both scales are logarithmic.





8.6 Laminar Flow and Turbulent Flow

From the diagram, we can see that points of the measured data lie on straight lines, though different lines for laminar flow and for turbulent flow respectively. Formally, this kind of straight line relationship can be expressed in equation

$$\lg h_f = \lg k + m \lg V$$

where **lg** ***k*** is the vertical intercept,

m is the slope, $m = \tan\theta$ (θ: angle of the line inclination).

8.6 Laminar Flow and Turbulent Flow

Statistically, parameters can be obtained from regression of the measured data. Following are the results.

For laminar flow,

$$\theta_1 = 45°, \qquad m = 1$$

$$\lg h_f = \lg k_1 + \lg V, \quad \text{or,} \quad h_f = k_1 V$$

That is, head loss be proportional to the average velocity.

$$h_f \propto V^{1.0}$$

For turbulent flow,

$$\theta_2 > 45°, \qquad m = 1.75 \sim 2.0$$

$$\lg h_f = \lg k_2 + m \lg V, \quad \text{or,} \quad h_f = k_2 V^m$$

So, head loss is proportional to the 1.75～2-*th* power of the average velocity.

$$h_f \propto V^{1.75\sim2.0}$$

8.6 Laminar Flow and Turbulent Flow

Classification of Flow State – Reynolds number

Reynolds number: Ratio of the inertial force to viscous force

Inertial force ma with dimension $\rho L^3 \frac{V^2}{L}$

Viscous force $\mu \frac{\mathrm{d}u}{\mathrm{d}y} A$ with dimension $\mu \frac{V}{L} L^2$

$$\mathrm{Re} = \frac{\rho L^3 \frac{V^2}{L}}{\mu \frac{V}{L} L^2} = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$

8.6 Laminar Flow and Turbulent Flow

Reynolds number, Re, is a major dimensionless parameter as a criterion to differentiate flow states, a *laminar flow*, a *turbulent flow* or in between a *transitional flow*. For circular pipe flow, Reynolds number, Re, is conventionally defined as

$$\mathrm{Re} \equiv \frac{\rho \bar{V} d}{\mu}$$

ρ : fluid density

μ : fluid dynamic viscosity

$\bar{V}$: average velocity over the cross-section

d : diameter of the circular pipe

Circular Pipe Flow

$$\begin{cases} \mathrm{Re} < 2,000 & \text{Laminar flow} \\ 2,000 < \mathrm{Re} < 4,000 & \text{Transitional flow} \\ \mathrm{Re} > 4,000 & \text{Turbulent flow} \end{cases}$$

8.6 Laminar Flow and Turbulent Flow

There are two very different flow states ---- *laminar flow* and *turbulent flow*. Another one is an intermediate state in between them, *transitional flow*, partly laminar and partly viscous.

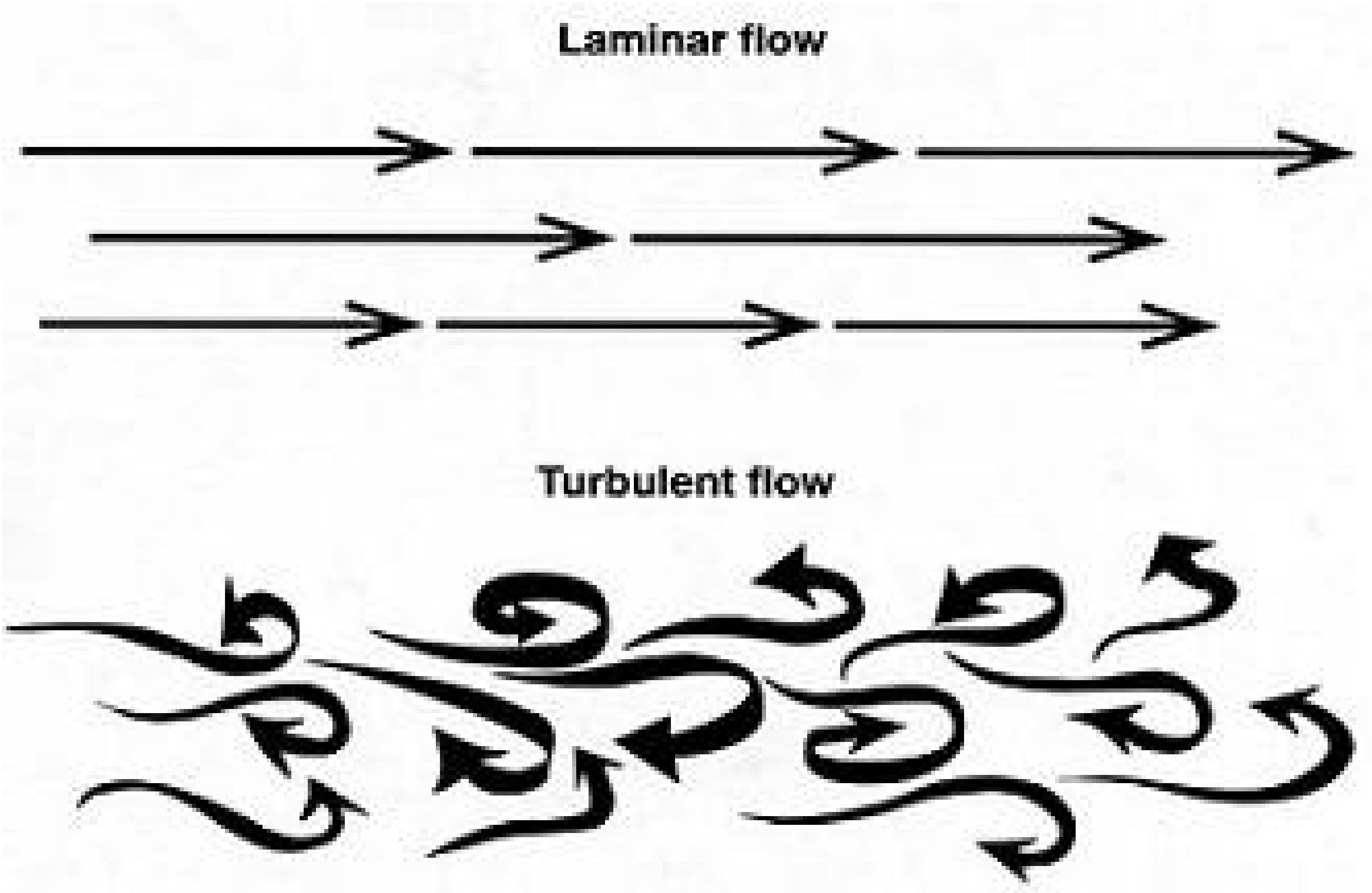
Laminar flow: The flow field looks regular and stable. Fluid particles travel along different smooth layers without mixture.

Turbulent Flow: The flow field looks irregular and unstable. Fluid particles travel chaotically and randomly, mixed with surrounding particles and layers heavily.

Laminar flow and turbulent flow are essentially different flow states.

Reynolds number, Re, is a major criterion to judge the flow state, whether it is a laminar flow or a turbulent flow, for a specific flow.

8.6 Laminar Flow and Turbulent Flow

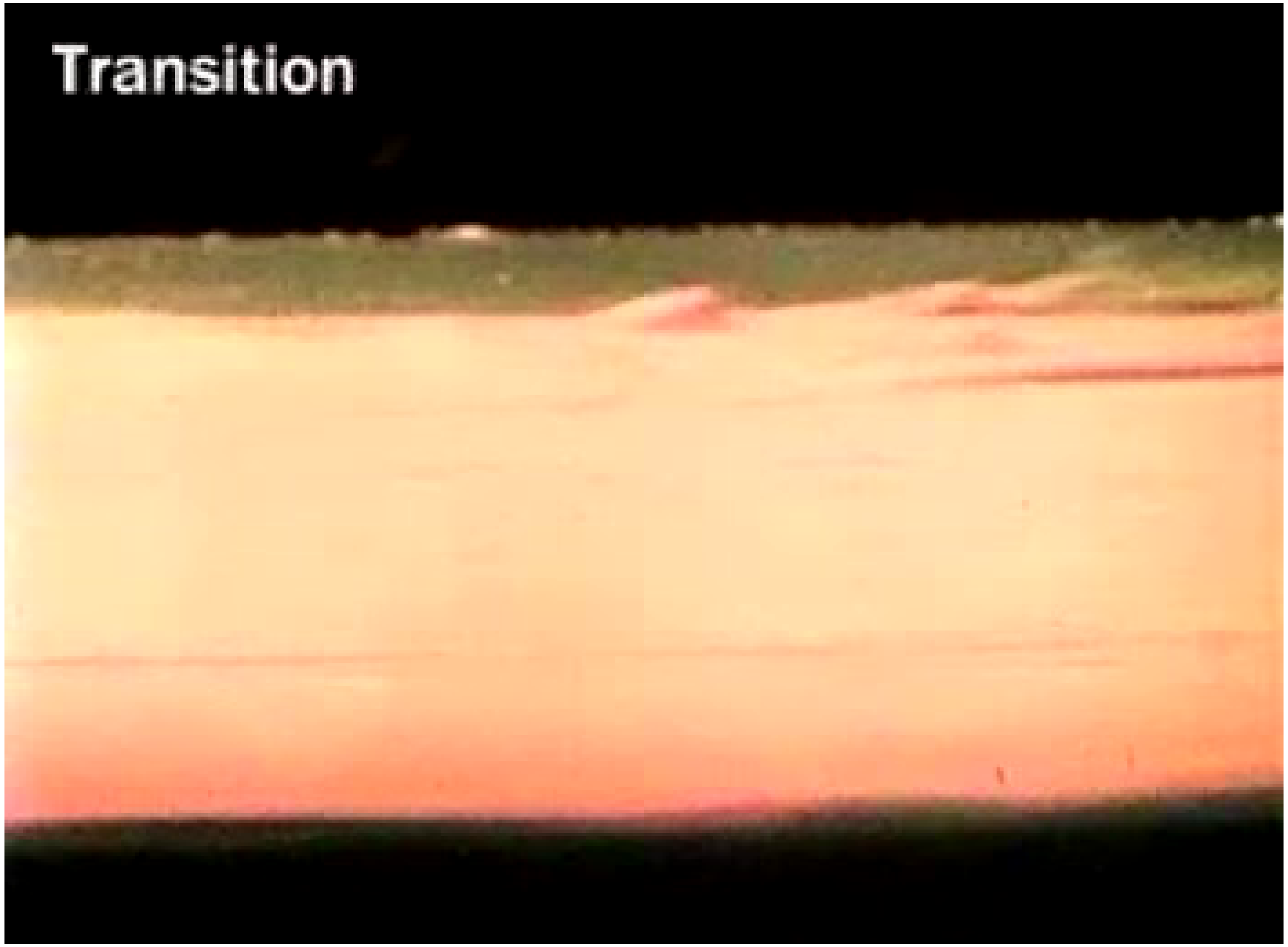


8.6 Laminar Flow and Turbulent Flow

Laminar flow

8.6 Laminar Flow and Turbulent Flow

Transitional flow

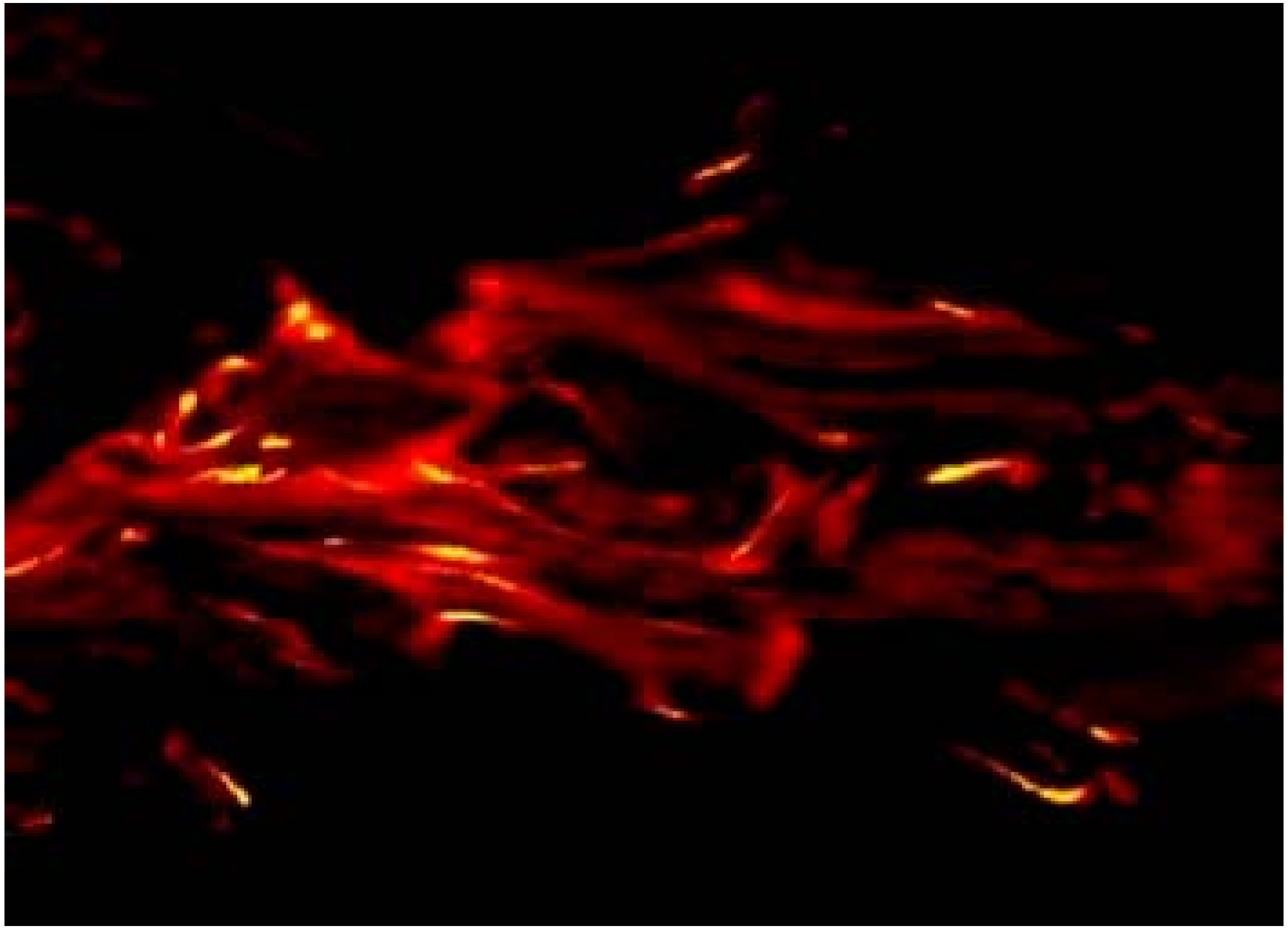


8.6 Laminar Flow and Turbulent Flow

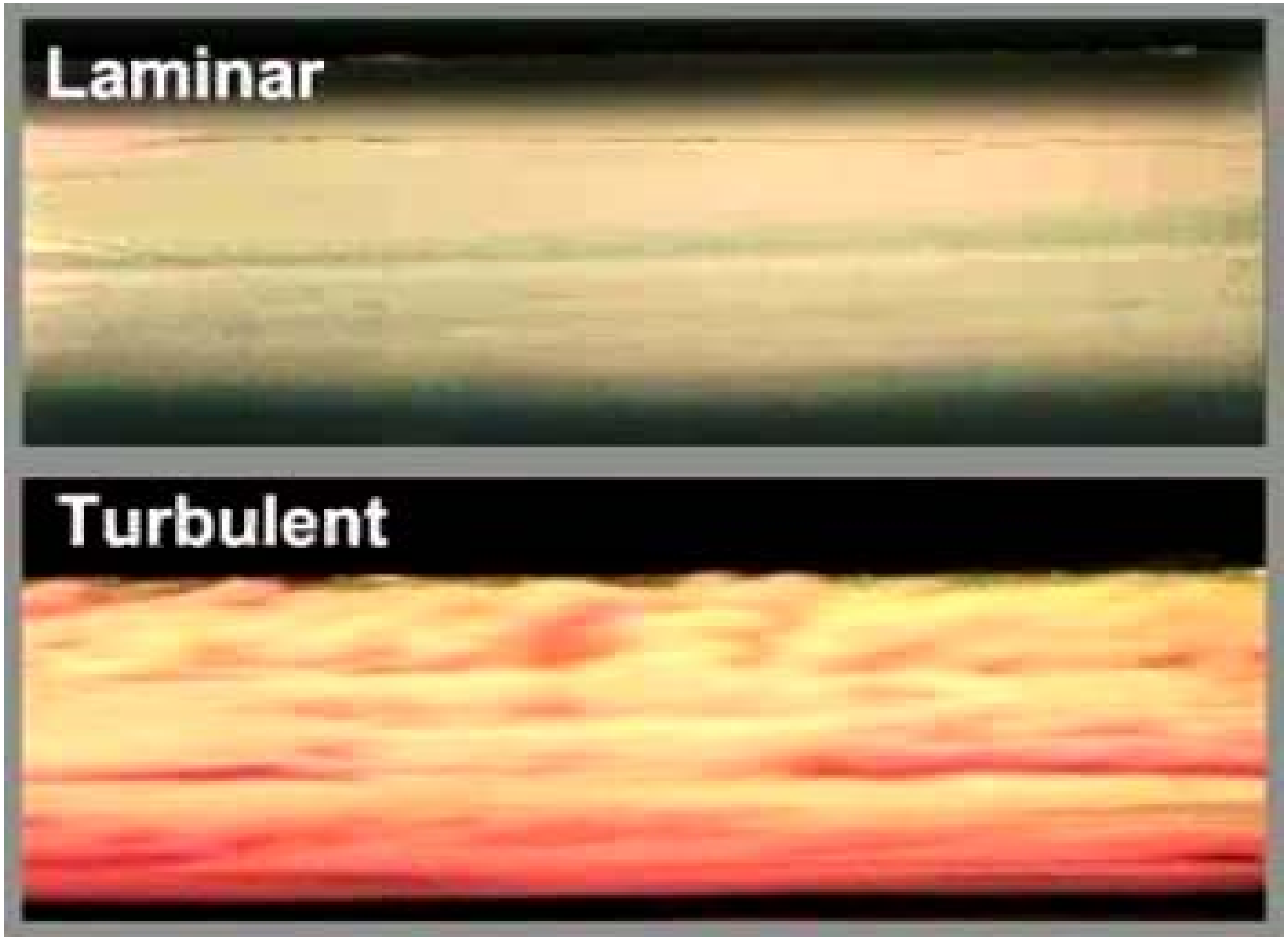
Turbulent flow

8.6 Laminar Flow and Turbulent Flow

Turbulent flow



8.6 Laminar Flow and Turbulent Flow

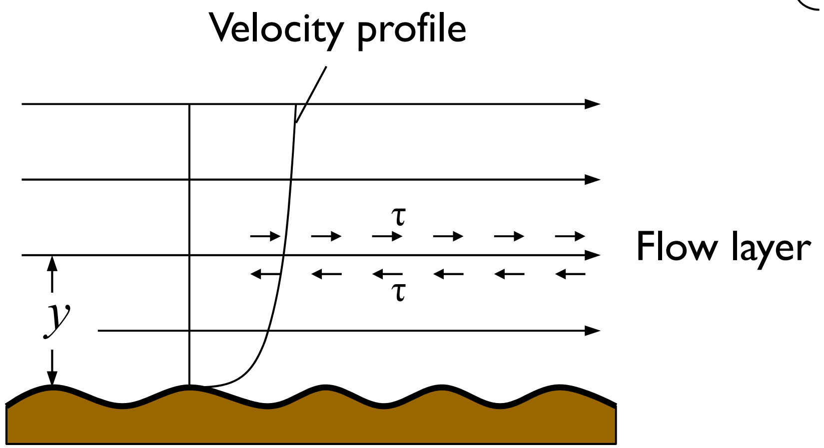


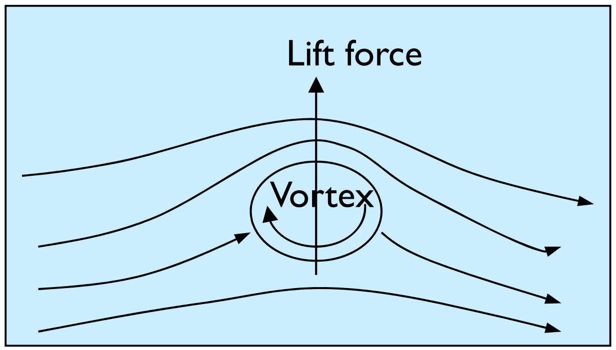
8.7 Turbulent Flow

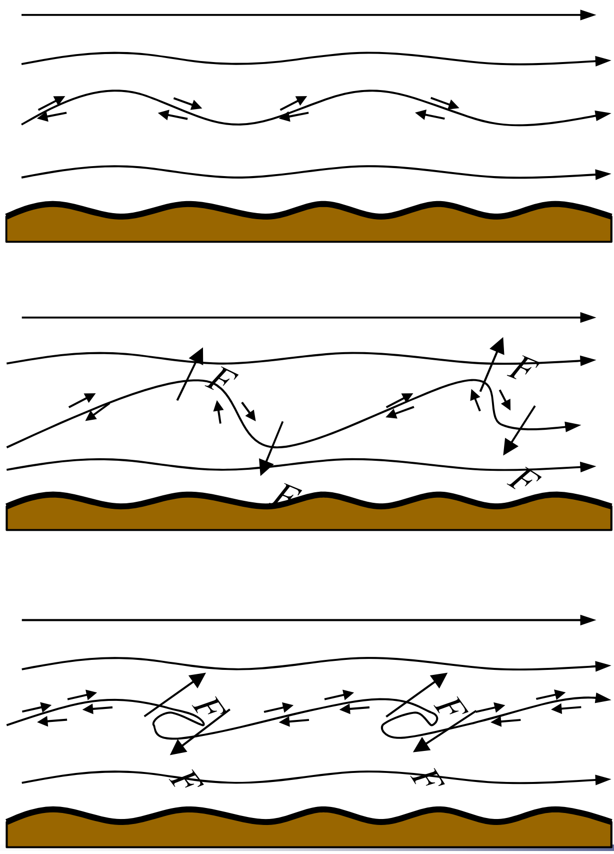
Mechanism of Turbulent Flow

Essential factors

- Vortex generation (fluid viscosity, obstacles)
- Reynolds number is large enough



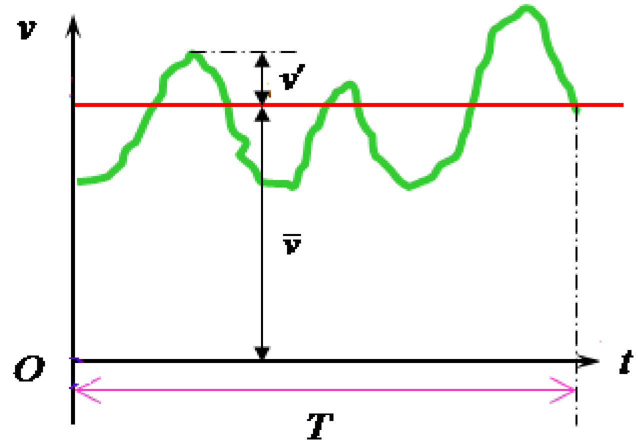




Time Averaging of Flow Quantities

For turbulent flow, physical quantities, such as velocity, pressure and so on, always vary with time. But its average value is relatively stable. The variation around the mean is called fluctuation. Time averaging is effective and useful.

$$\overline{v} = \frac{1}{T}\int_0^T v dt$$



Instantaneous velocity

$$v = \overline{v} + v'$$

where v' is the velocity fluctuation. That is, instantaneous velocity is decomposed into the time averaged velocity and a velocity fluctuation. For other quantities, time averaging procedure is the same. *Turbulent flow is always an unsteady, but after time averaging, the average flow may be steady.*

Reynolds-averaged Navier-Stokes Equation (RANSE)

Following Reynolds, ***turbulent flow is assumed still obeying Navier-Stokes equation instantaneously***.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial u_i}{\partial t}+\frac{\partial (u_i u_j)}{\partial x_j}=f_i-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\nu\nabla^2 u_i$$

$$u=\overline{u}+u'$$
$$v=\overline{v}+v'$$
$$w=\overline{w}+w'$$
$$p=\overline{p}+p'$$

Time averaging

Time averaging

$$\frac{\partial \overline{u}}{\partial x}+\frac{\partial \overline{v}}{\partial y}+\frac{\partial \overline{w}}{\partial z}=0$$

$$\frac{\partial \overline{u_i}}{\partial t}+\overline{u_j}\frac{\partial \overline{u_i}}{\partial x_j}=f_i-\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i}+\frac{1}{\rho}\left(\mu\nabla^2\overline{u_i}-\frac{\partial}{\partial x_j}\left(\rho\,\overline{u'_i u'_j}\right)\right)$$

RANSE (1895)

8.7 Turbulent Flow

$$\frac{\partial \overline{u}_i}{\partial t}+\overline{u}_j\frac{\partial \overline{u}_i}{\partial x_j}=f_i-\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i}+\frac{1}{\rho}\left(\mu\nabla^2\overline{u}_i-\frac{\partial}{\partial x_j}(\rho\,\overline{u'_i u'_j})\right)$$

$\mu\nabla^2\overline{u}_i$ —— *average viscous stress*

$$R_{ij}=\begin{pmatrix}-\rho\overline{u'u'} & -\rho\overline{u'v'} & -\rho\overline{u'w'}\\ -\rho\overline{v'u'} & -\rho\overline{v'v'} & -\rho\overline{v'w'}\\ -\rho\overline{w'u'} & -\rho\overline{w'v'} & -\rho\overline{w'w'}\end{pmatrix}$$

Turbulent stress （Reynolds stress）

This is a symmetric tensor with 6 independent element. It is due to momentum transportation due to turbulent fluctuation. Turbulent stress, $-\rho\overline{u'_i u'_j}$, might be closely related to the average flow quantities.

Well-determined turbulent flow → Equations: 4 Unknowns: 10 → **A turbulence model** : relation of turbulent stress to average velocities.

Turbulent shear stress

$$\tau = \tau_1 + \tau_2$$

Viscous shear stress due to average velocity difference between neighboring layers

Additional shear stress due to velocity fluctuation

$$\tau_1 = \mu \frac{\mathrm{d}u}{\mathrm{d}y}, \quad \tau_2 = \rho \ell^2 \left(\frac{\mathrm{d}u}{\mathrm{d}y} \right)^2$$

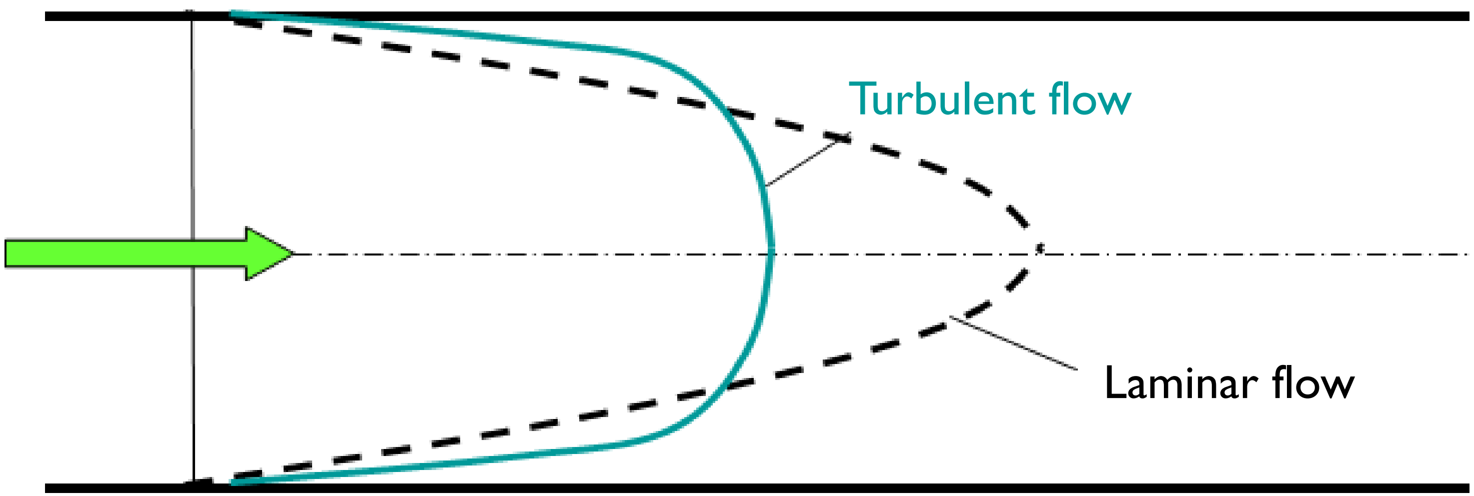
where ℓ is namely the mixing length, evaluated by one of the two empirical formulas below.

$$\ell = \kappa y \quad \text{or} \quad \ell = \kappa y \sqrt{1 - \frac{y}{r_0}}$$

8.7 Turbulent Flow

Due to mixture and collision, momentums are exchanged among neighboring fluid particles. Momentums are transferred from particles with larger value to the particles with less value. As a result, distribution of momentum, *i.e.* velocity, over a cross-section tends to be more uniform.

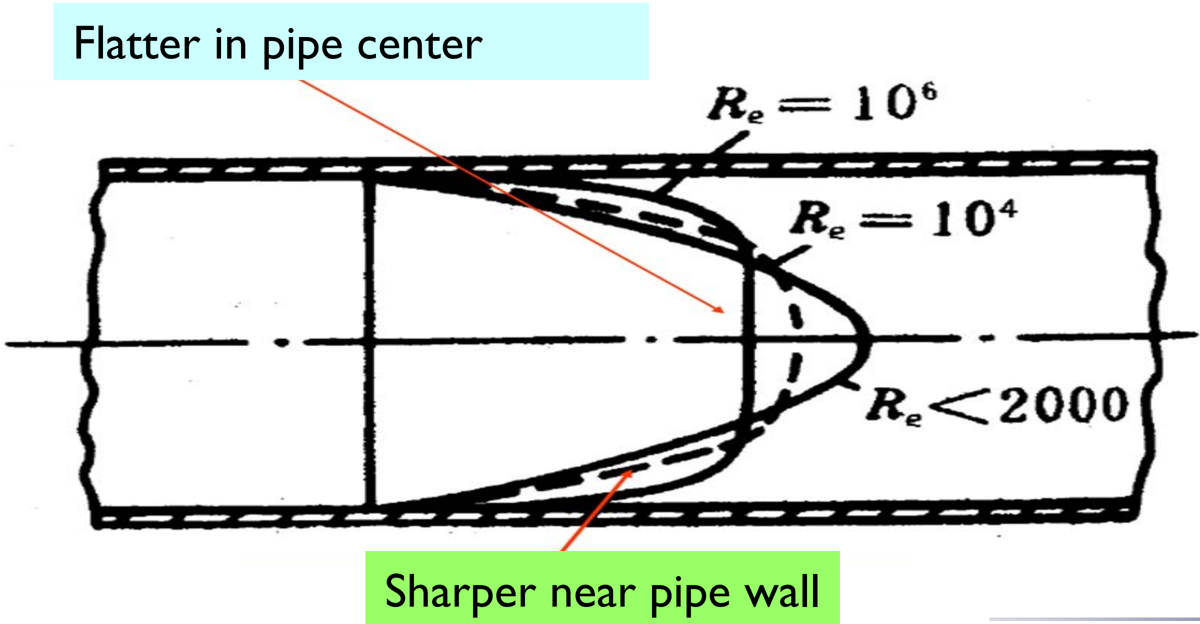
Cross-sectional velocity distribution



8.7 Turbulent Flow

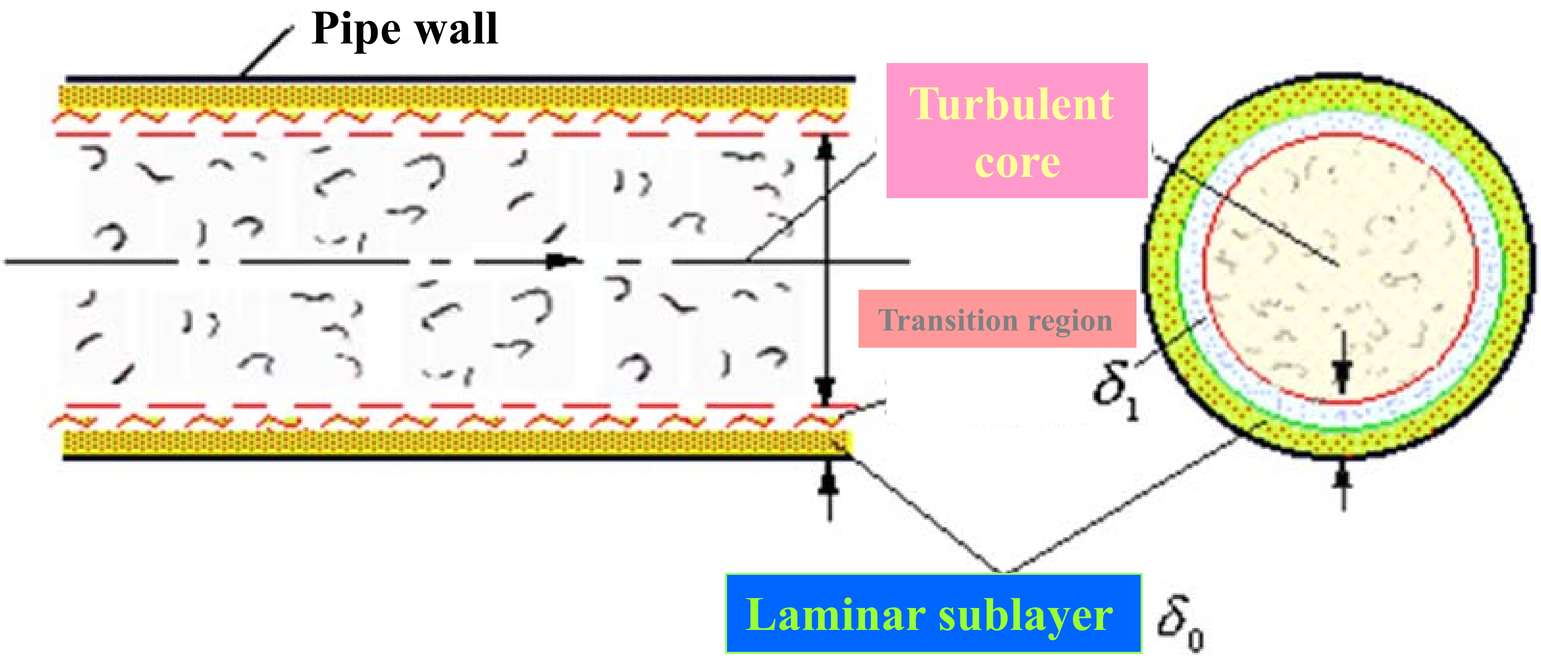
Measurements demonstrated that velocity distribution inside a pipe with smooth wall obeys ***power laws***.

$$\frac{u_x}{u_m} = \left(\frac{y}{r_0}\right)^n \quad \begin{cases} \text{If } Re < 10^5, & n = \dfrac{1}{7} \\ \text{If } Re > 10^5, & n = \dfrac{1}{8}, \dfrac{1}{9}, \text{ or, } \dfrac{1}{10} \end{cases}$$



Laminar sublayer and turbulent core

—— In turbulent flow, there exists a thin ***laminar sublayer***, where flow is dominated by viscous shear force, rather than by an additional shear force due to fluctuations. The latter is very small in that layer comparing to the former viscous shear force.



8.7 Turbulent Flow

In general, thickness, δ_0, of the laminar sublayer is only several 10th millimeters, but largely affects the drag.

$$\delta_0 = \frac{32.8d}{\mathrm{Re}\sqrt{\lambda}}$$

Though thickness of the ***laminar sublayer*** is very thin, it greatly affects the state of the turbulent flow. It is meaningful to clarify the effect of laminar sublayer on *turbulent head loss*.

1. Laminar sublayer $(y \le \delta_0)$:

$$\tau = \mu \frac{du}{dy} = \tau_0$$ (y: distance normal to the wall)

$$\Rightarrow \quad u = \frac{\tau_0}{\mu} y + C_1 \quad \xrightarrow{y=0,\ u=0} \quad u = \frac{\tau_0}{\rho \nu} y$$

$$\Rightarrow \quad \frac{u}{V_*} = \frac{y V_*}{\nu}$$ ($V_* = \sqrt{\frac{\tau_0}{\rho}}$ — ***friction velocity***)

Denote $u^+ = \frac{u}{V_*}$, $y^+ = \frac{y V_*}{\nu}$, it becomes

$$u^+ = y^+ \qquad (y^+ \le 5)$$

The law of dimensionless velocity distribution in laminar sublayer.

2. Transition region $(5 < y^+ < 30)$

$$u^+ = 11\arctan\left(\frac{y^+}{11}\right)$$

3. Hydraulic smooth turbulent core $(y^+ \geq 30)$

Logrithmic law $$\frac{u}{V_*} = \frac{1}{k}\ln\frac{yV_*}{\nu} + c$$

Based on *Nikuradse* tests

$$k = 0.4\,, c = 5.5 \quad \Longrightarrow \quad u^+ = 2.5\ln y^+ + 5.5$$

The maximum velocity $(y = a)$

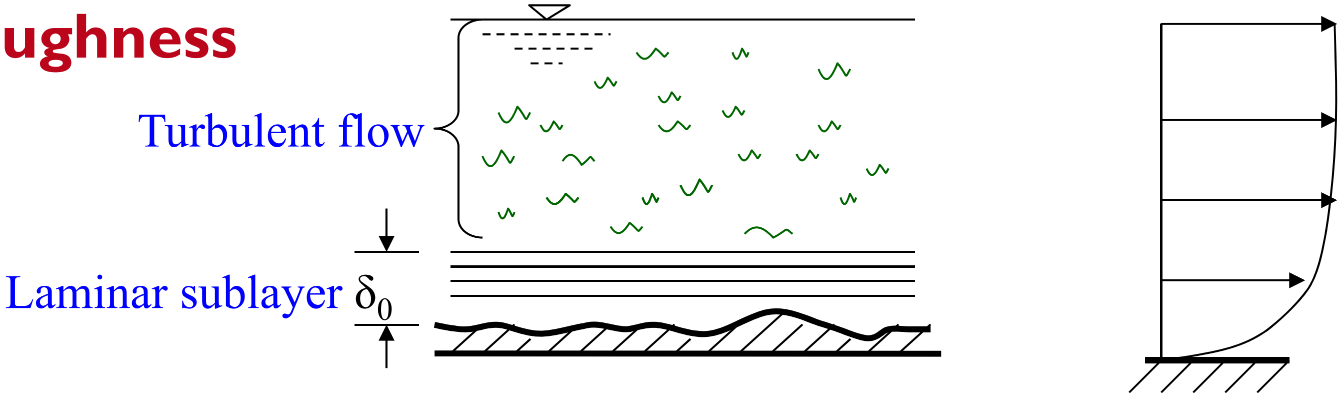
$$\frac{u_{\max}}{u_\tau} = 2.5\ln\frac{au_\tau}{\nu} + 5.5$$

Mean velocity

$$u_m = u_{\max} - 3.75u_\tau$$

8.7 Turbulent Flow

Wall roughness

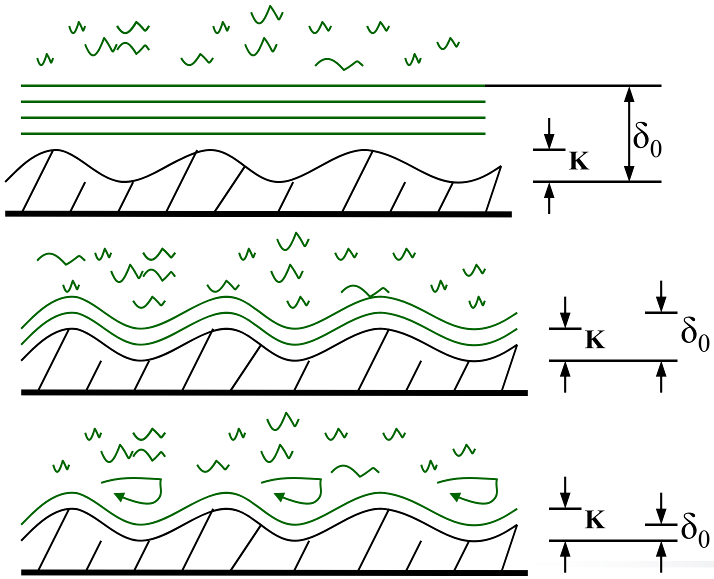


$$\delta_0 = \frac{32.8d}{\mathrm{Re}\sqrt{\lambda}}$$

Thickness of laminar sublayer is inversely proportional to *Re*.

For small *Re*

For large *Re*



Hydraulic smooth wall

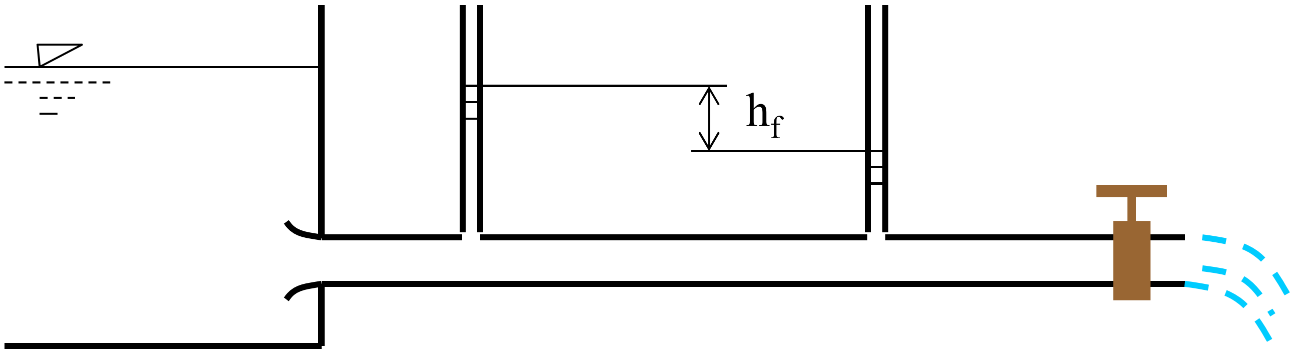
Rough wall for transitional flow

Hydraulic rough wall

8.7 Turbulent Flow

Nikuradse tests

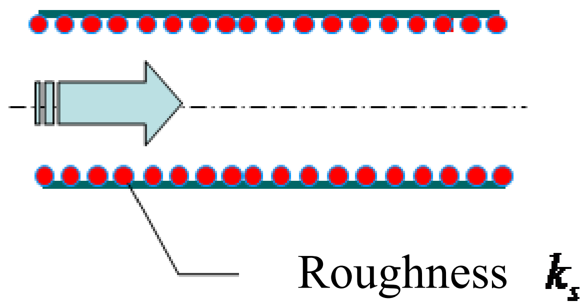
$$h_f = \lambda \frac{L}{d}\frac{V^2}{2g}$$



Reynolds number $\mathrm{Re} = \frac{Vd}{\nu}$

Relative roughness k_s/d

Relative smoothness d/k_s

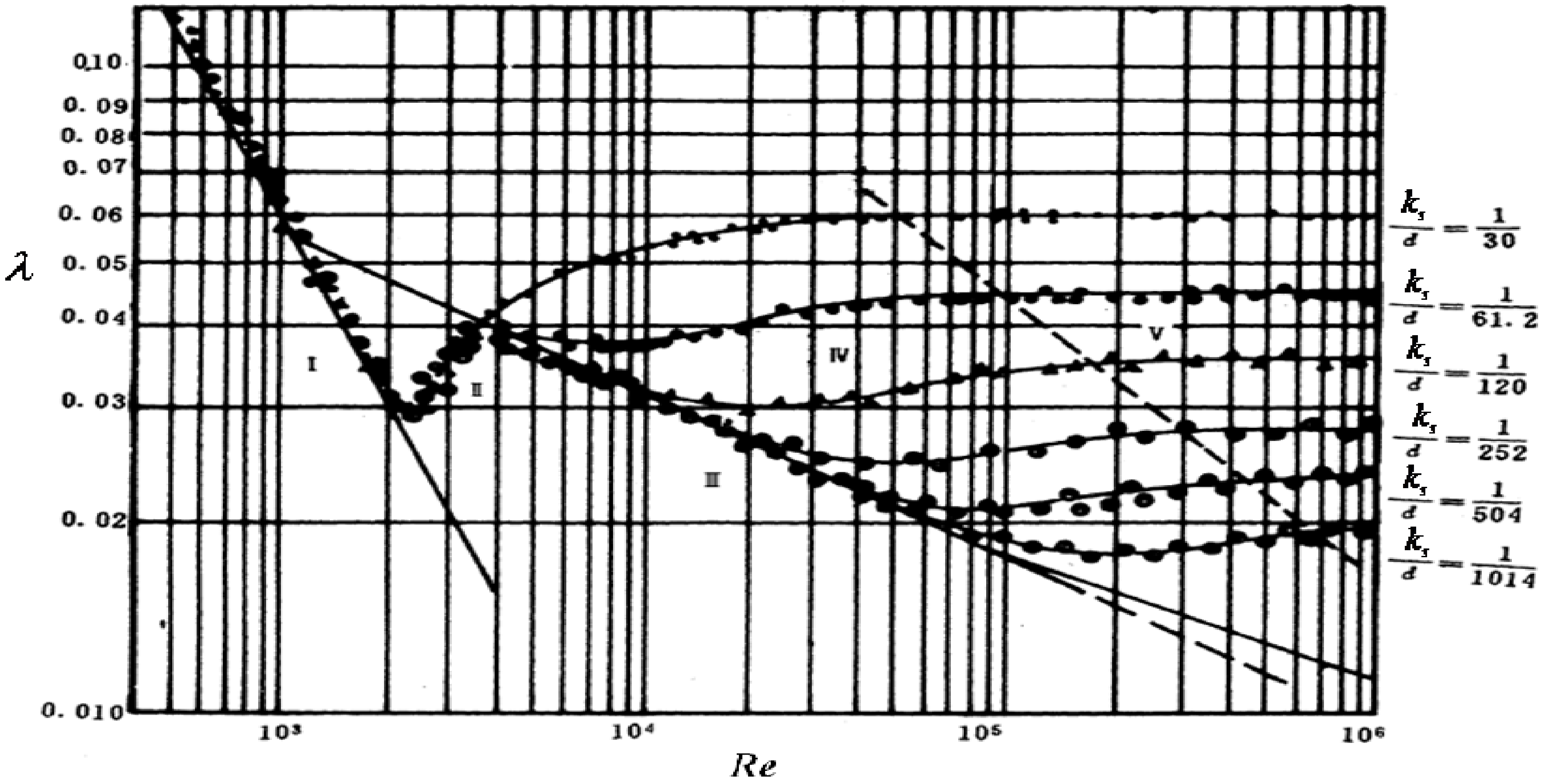


8.7 Turbulent Flow

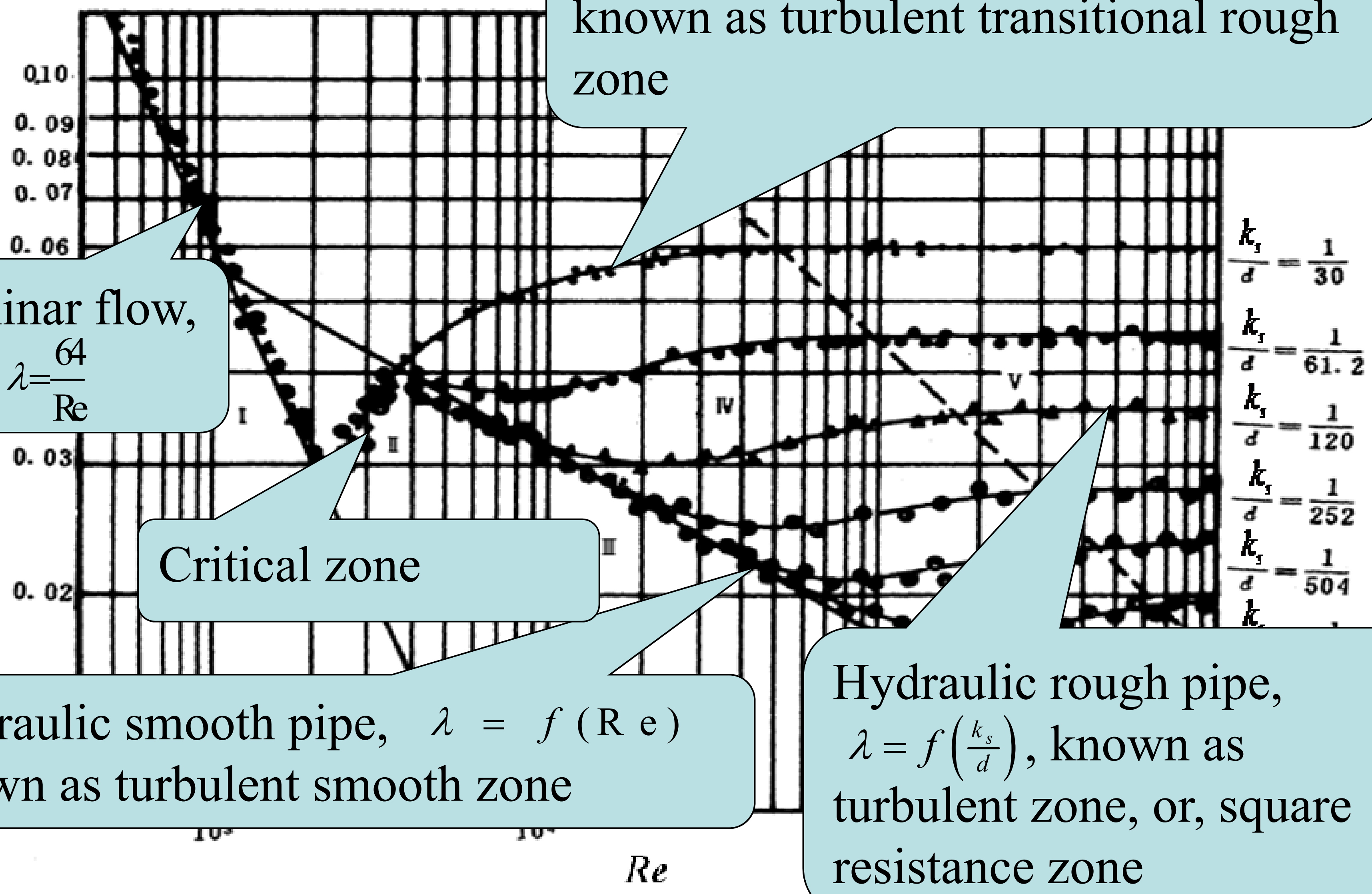
Johann Nikuradse

Results of *Nikuradse* tests

Nikuradse glues sand grains of known size onto pipe walls to produce pipes with sandpaper-type surfaces with different roughness.



8.7 Turbulent Flow



Transitional rough pipe, $\lambda = f\left(\mathrm{Re}, \frac{k_s}{d}\right)$ known as turbulent transitional rough zone

Laminar flow, $\lambda = \frac{64}{\mathrm{Re}}$

Critical zone

Hydraulic smooth pipe, $\lambda = f(\mathrm{Re})$ known as turbulent smooth zone

Hydraulic rough pipe, $\lambda = f\left(\frac{k_s}{d}\right)$, known as turbulent zone, or, square resistance zone

8.7 Turbulent Flow

- **Laminar zone（I）：** $\lambda = f(\mathrm{Re}) = \dfrac{64}{\mathrm{Re}}$
- **Transitional zone（II）：** $\lambda = f(\mathrm{Re})$
- **Hydraulic smooth zone（III）：** $\lambda = f(\mathrm{Re}) \quad (\delta_0 > k_s)$
- **Turbulent transition zone （IV）：** $\lambda = f(\mathrm{Re}, {}^{k_s}\!/\!_{d})$
- **Complete turbulence zone（V）：** $\lambda = f({}^{k_s}\!/\!_{d}) \quad (\delta_0 < k_s)$

Moody chart： dependence of friction factor on Reynolds number

Nikuradse investigated pipes of different *artificial* wall roughness. The situation is different from the reality of industrial pipes, and the result is not directly applicable. Moody systematically summarized the results, gives out formulas for different zones, such as smooth pipes, transitional rough pipes, rough pipes, and represented in a visible drawing, *Moody chart*, which shows the dependence of friction factor on Reynolds number, where *relative roughness* is a parameter. Moody chart is classified in 5 zones – *laminar zone*, *critical zone*, *hydraulic smooth zone*, *transitional zone*, and the *complete turbulence zone (rough pipe)*.

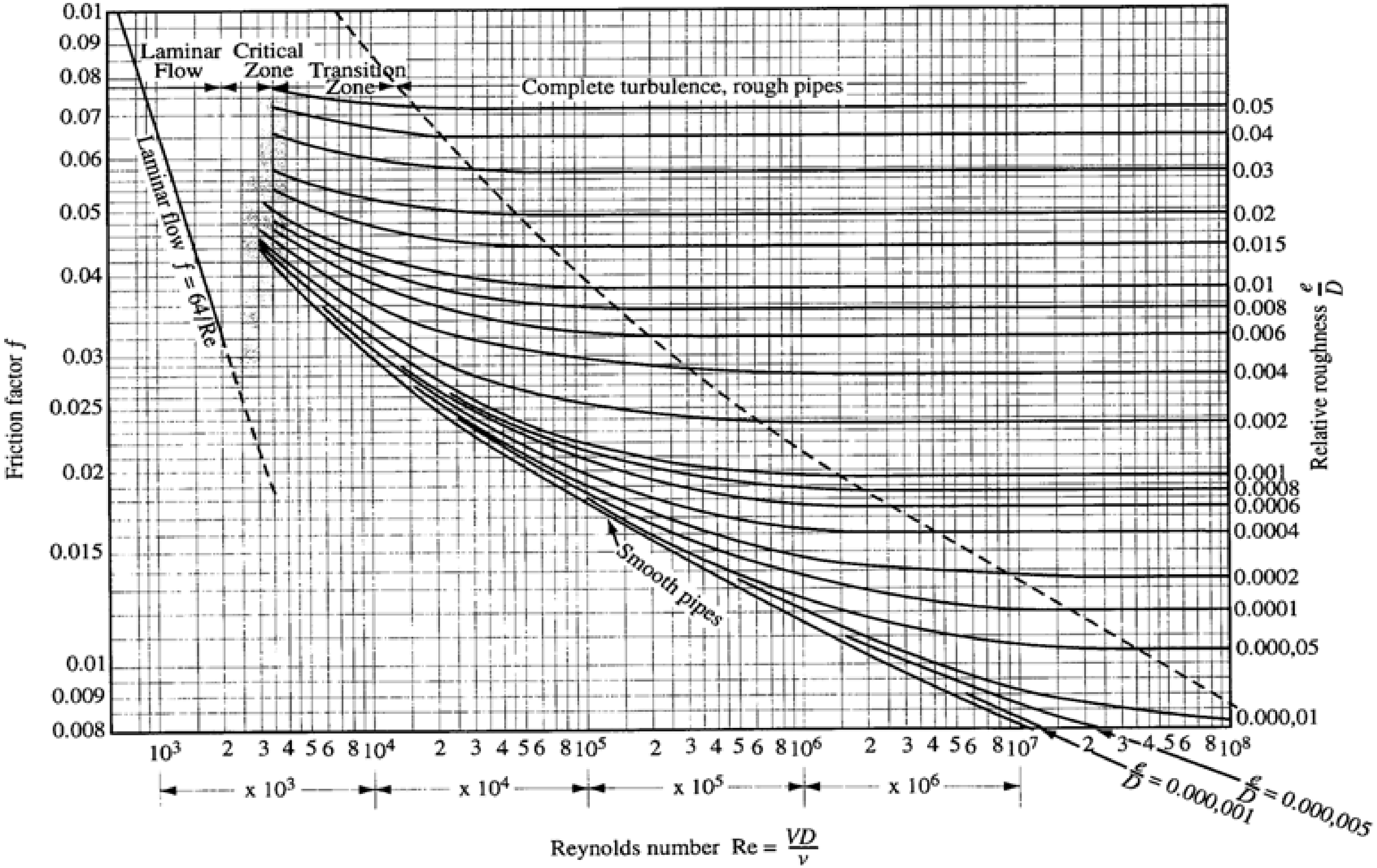
If Reynolds number and relative roughness are given, head loss of an industrial pipe can be evaluated in terms of the Moody chart.

8.7 Turbulent Flow

Lewis Moody

8.7 Turbulent Flow

Moody chart： dependence of friction factor on Reynolds number



8.7 Turbulent Flow

- **Laminar zone（I）**

$$\lambda = \frac{64}{\mathrm{Re}}$$

Theoretical formula,
agrees well with the measured data

$$h_f \propto V^{1.0}$$

- **Critical zone（II）**

Transition from laminar to turbulence.

8.7 Turbulent Flow

- **Turbulent smooth zone（III）**

$$\lambda = \frac{0.316}{\mathrm{Re}^{0.25}}$$

***Blasius* formula**

$$h_f \propto V^{1.75}$$

- **Transition zone（IV）**

$$\lambda = \frac{0.0179}{d^{0.3}}(1 + \frac{0.867}{V})^{0.3}$$

8.7 Turbulent Flow

- **Complete turbulence zone（V）：**

$$\lambda = \left(2\lg \frac{r_0}{k_s} + 1.74 \right)^{-2}$$

***Nikuradse* formula**

$$\lambda = 0.11 \left(\frac{k_s}{d} \right)^{0.25}$$

$$\lambda = \frac{0.0210}{d^{0.3}}$$

$$h_f \propto V^2$$

8.7 Turbulent Flow

- **Other formulas for complete turbulence zone**

$$\sqrt{\lambda} = -2\lg\left(\frac{k_s}{3.7d} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$$

$$\lambda = 0.11\left(\frac{k_s}{d} + \frac{68}{\mathrm{Re}}\right)^{0.25}$$