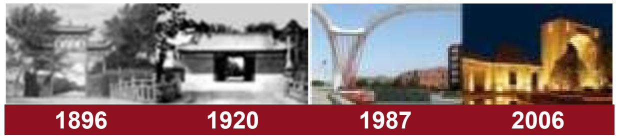


Introduction to Marine Hydrodynamics

(NA235)

Department of Naval Architecture and Ocean Engineering

School of Naval Architecture, Ocean & Civil Engineering

Shanghai Jiao Tong University

Chapter 8

Fundamental Theory of Viscous Incompressible Fluid Flow

8.1 Viscous Flow Phenomena

It can be concluded that viscous flows have characters as follows.

1) **Rotational:** vorticity may be non-zero

2) **Dissipation:** mechanical energy may be changed to other energy

3) **Diffusion:** physical quantity may diffuse due to its gradient

4) **Unsteady:** physical quantity may vary with time

5) **Unstable:** physical quantity may change essentially due to some small disturbance

6) **Random:** physical quantity may change in a random fashion and become indeterminable

8.2 Difference of Viscous Flow from Ideal Flow

1) Fluid Viscosity:

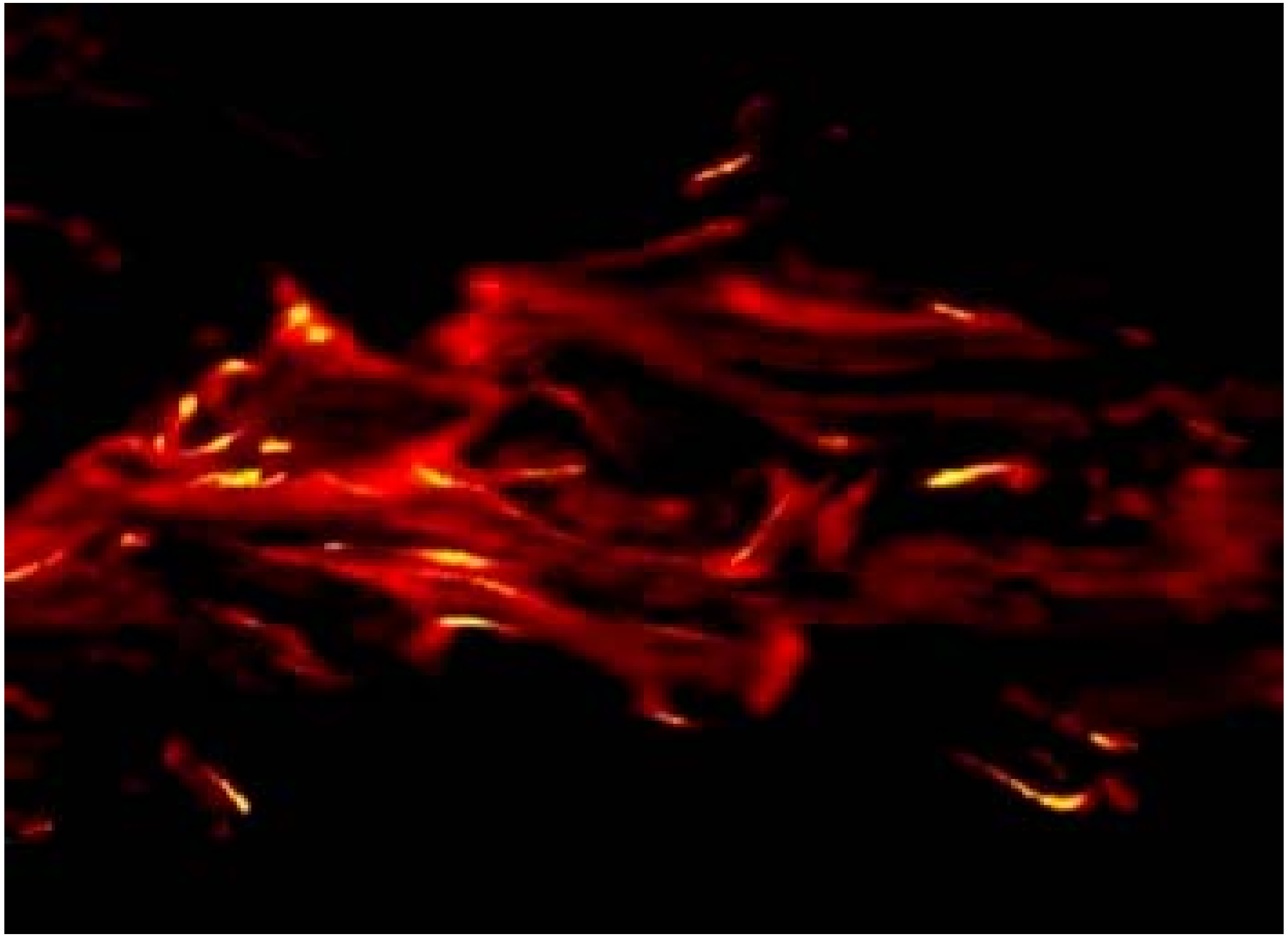
Viscous flow – with viscosity

Ideal flow (*perfect flow*) – without viscosity

During the course of fluid motion, fluid particles will accompany with deformation, and cause viscous forces, i.e., inner frictions, between adjacent fluid layers, which transfer mechanical energy to irreversible energies, such as thermal energy, and as a result *(mechanical) energy dissipated*.

8.2 Difference of Viscous Flow from Ideal Flow

Complexity of viscous flows



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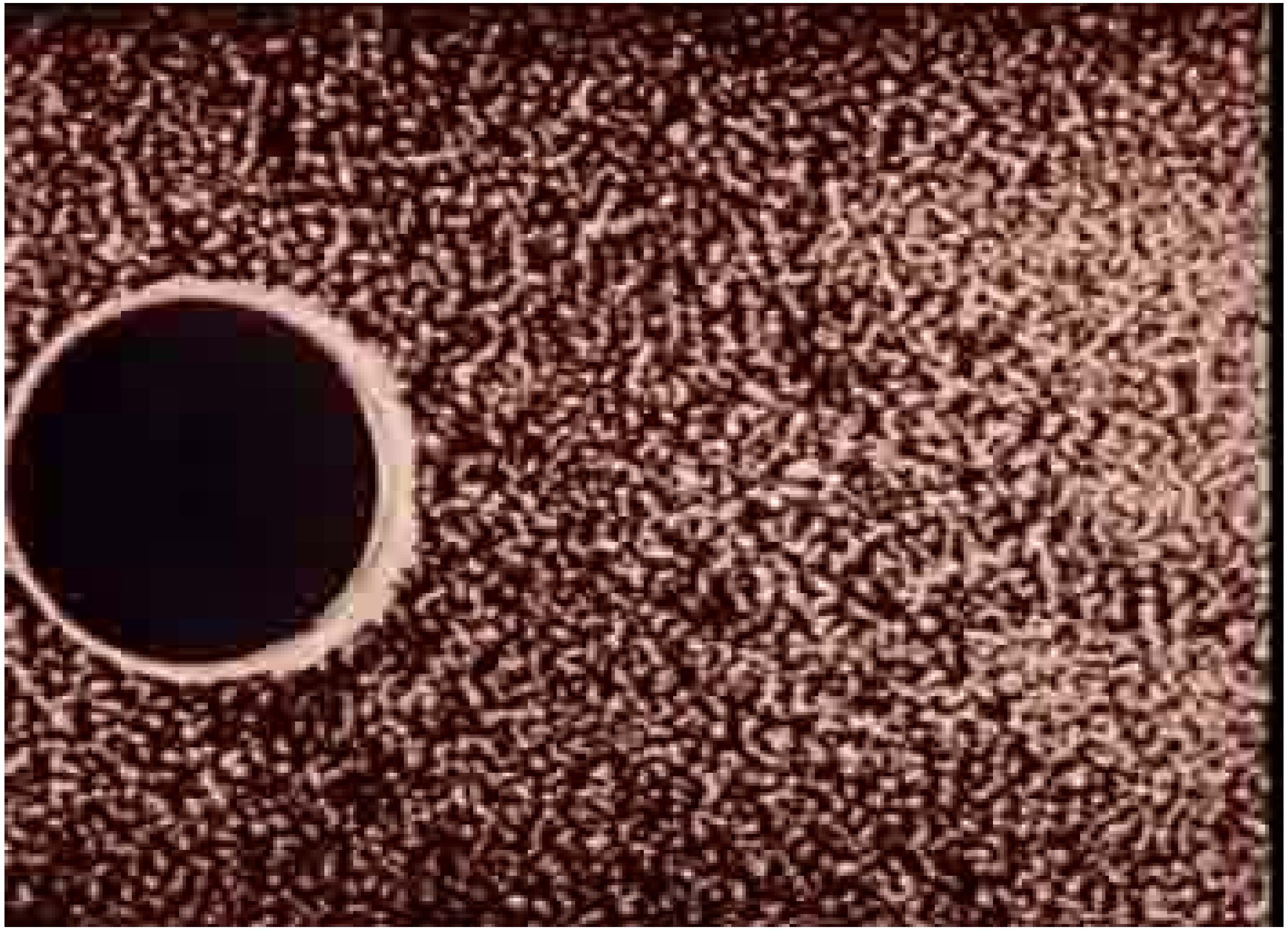

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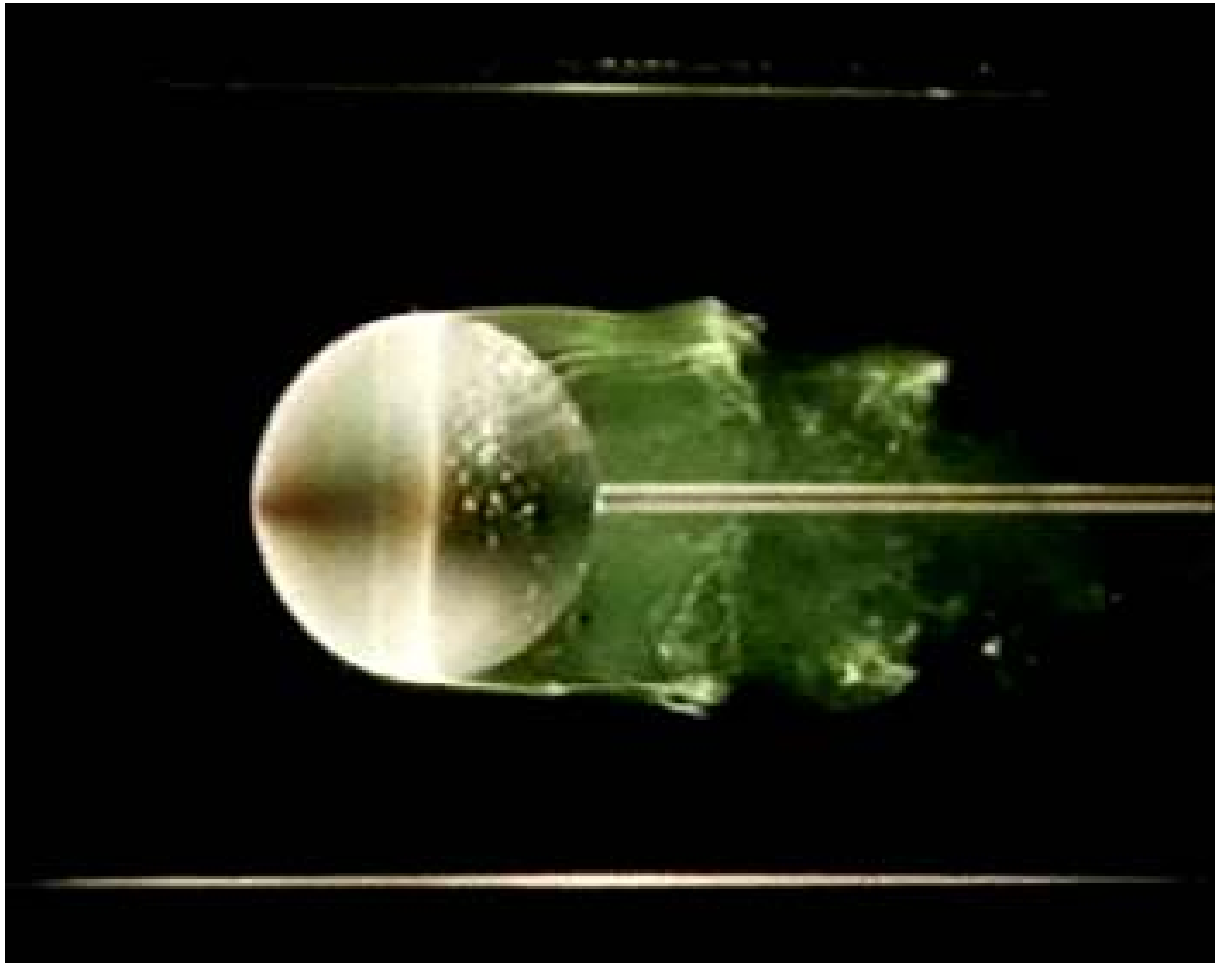
Complexity of viscous flows



8.2 Difference of Viscous Flow from Ideal Flow

2) Body surface condition:

for viscous flow, ***no-slip*** condition (***sticky***)

for ideal (perfect) flow, ***impermeable*** (but ***slipable***)

Slipable (impermeable): Coincidence of the normal velocity component on body surface.

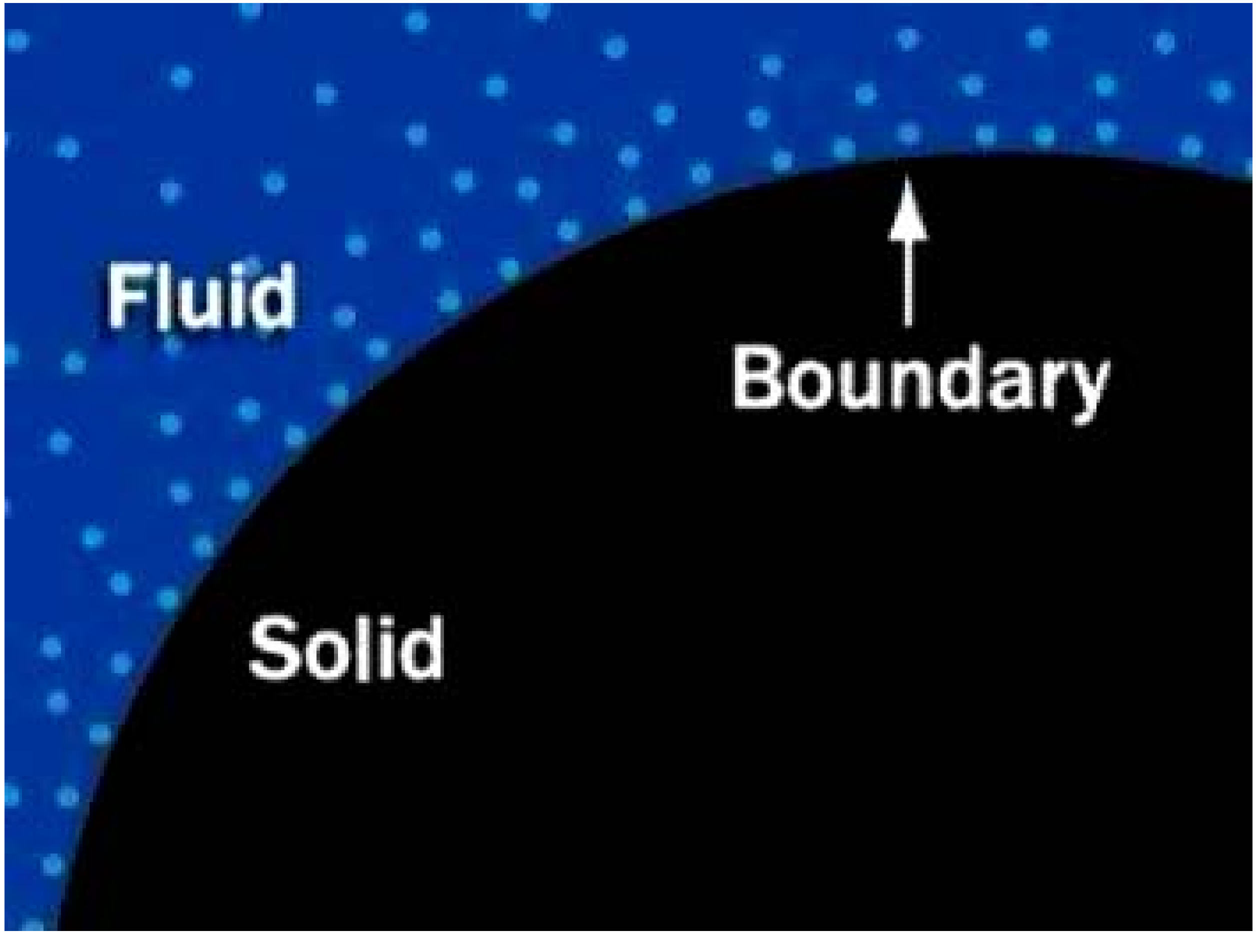
$$V_n = U_n$$

No-slip: Coincidence of both the normal and the tangential velocity components.

$$V_n = U_n, \quad V_\tau = U_\tau$$

8.2 Difference of Viscous Flow from Ideal Flow

Slipable (impermeable) condition:



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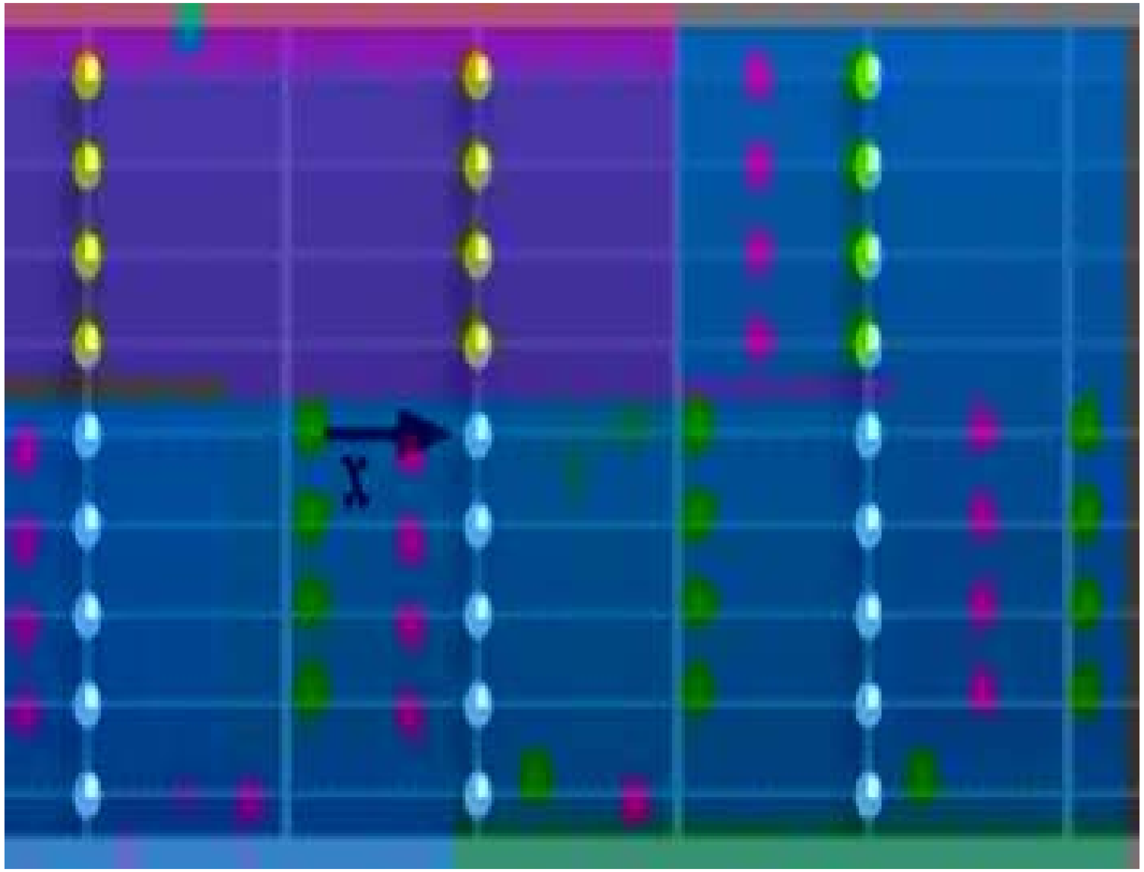
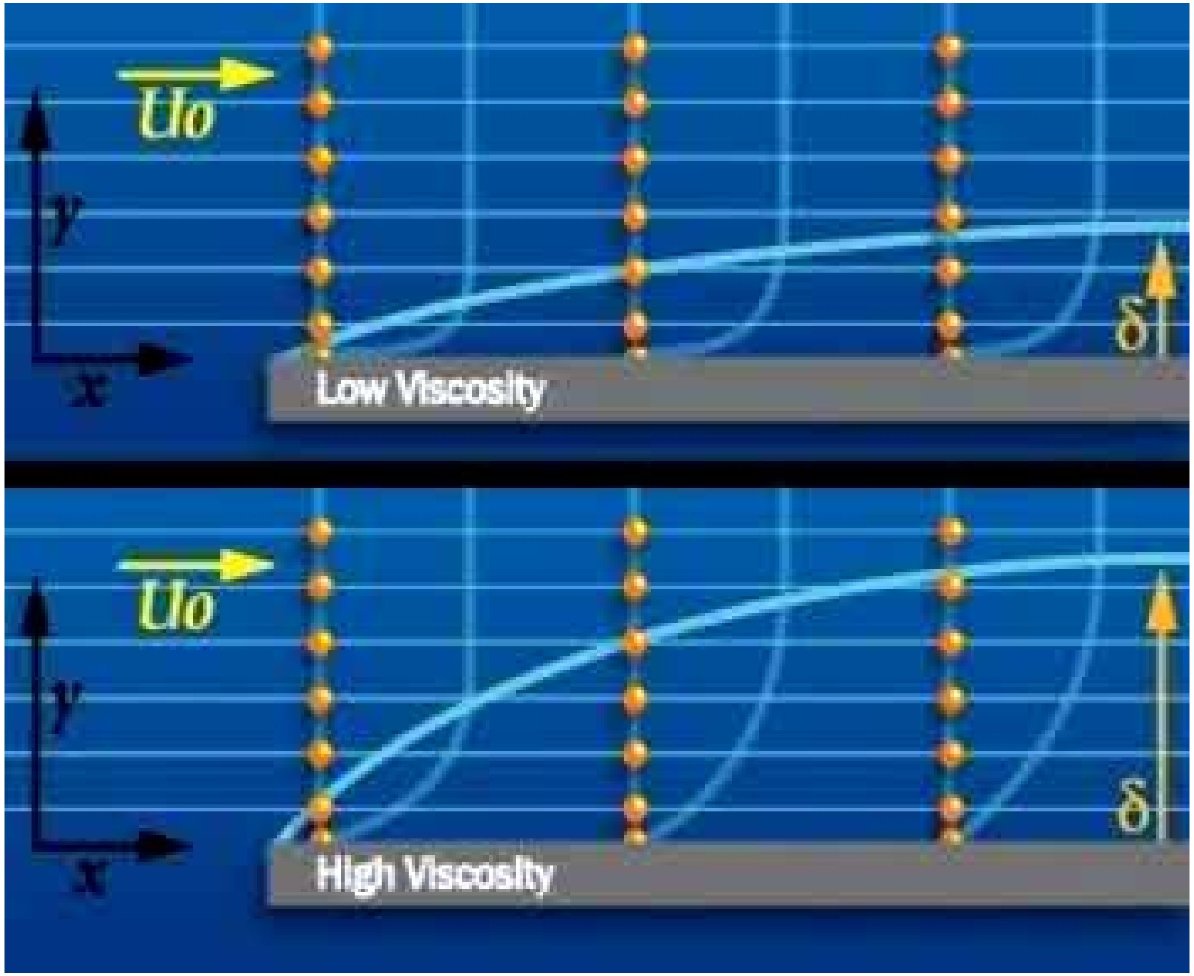


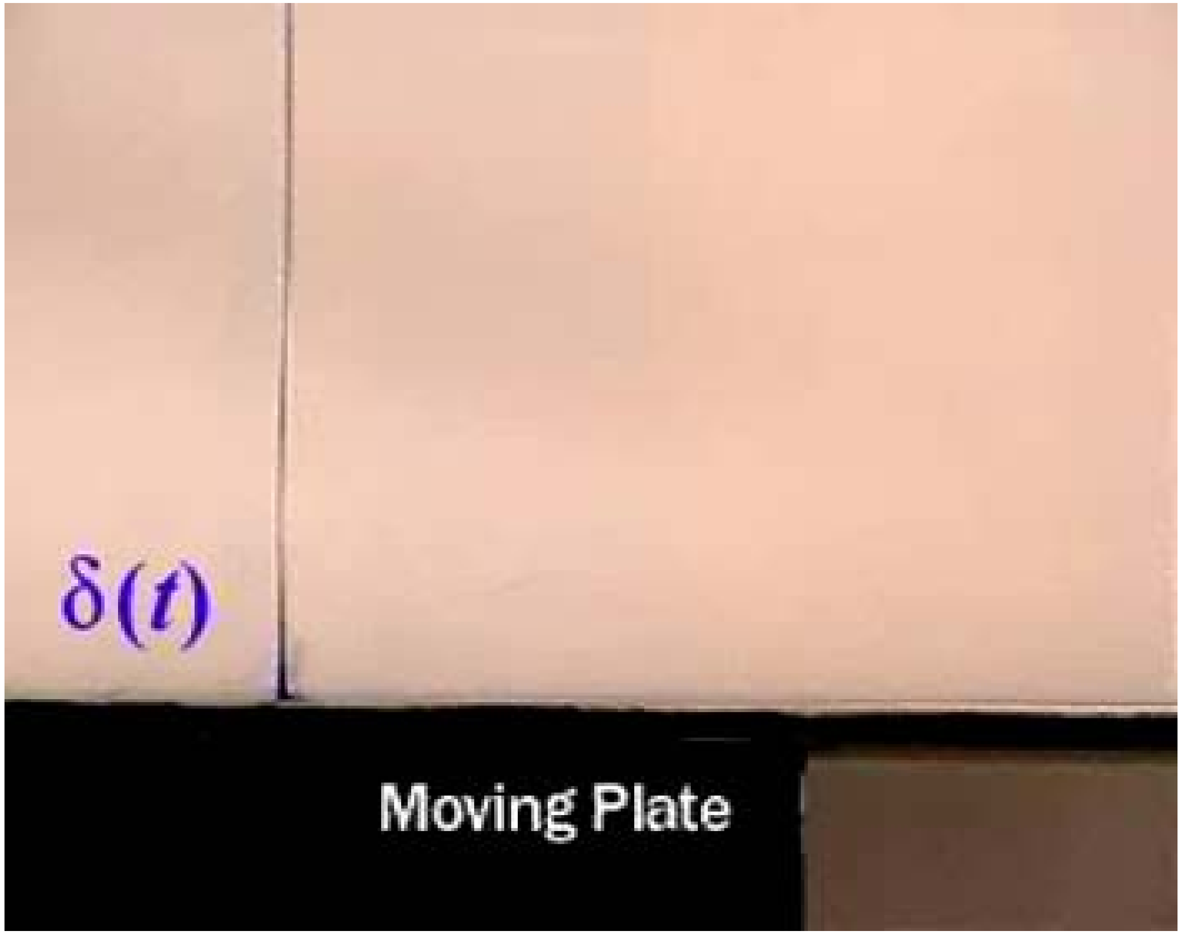
8.2 Difference of Viscous Flow from Ideal Flow

No-slip (sticky) condition:



8.2 Difference of Viscous Flow from Ideal Flow

In reality, all flows are *no-slip* (*sticky*) on body surface.



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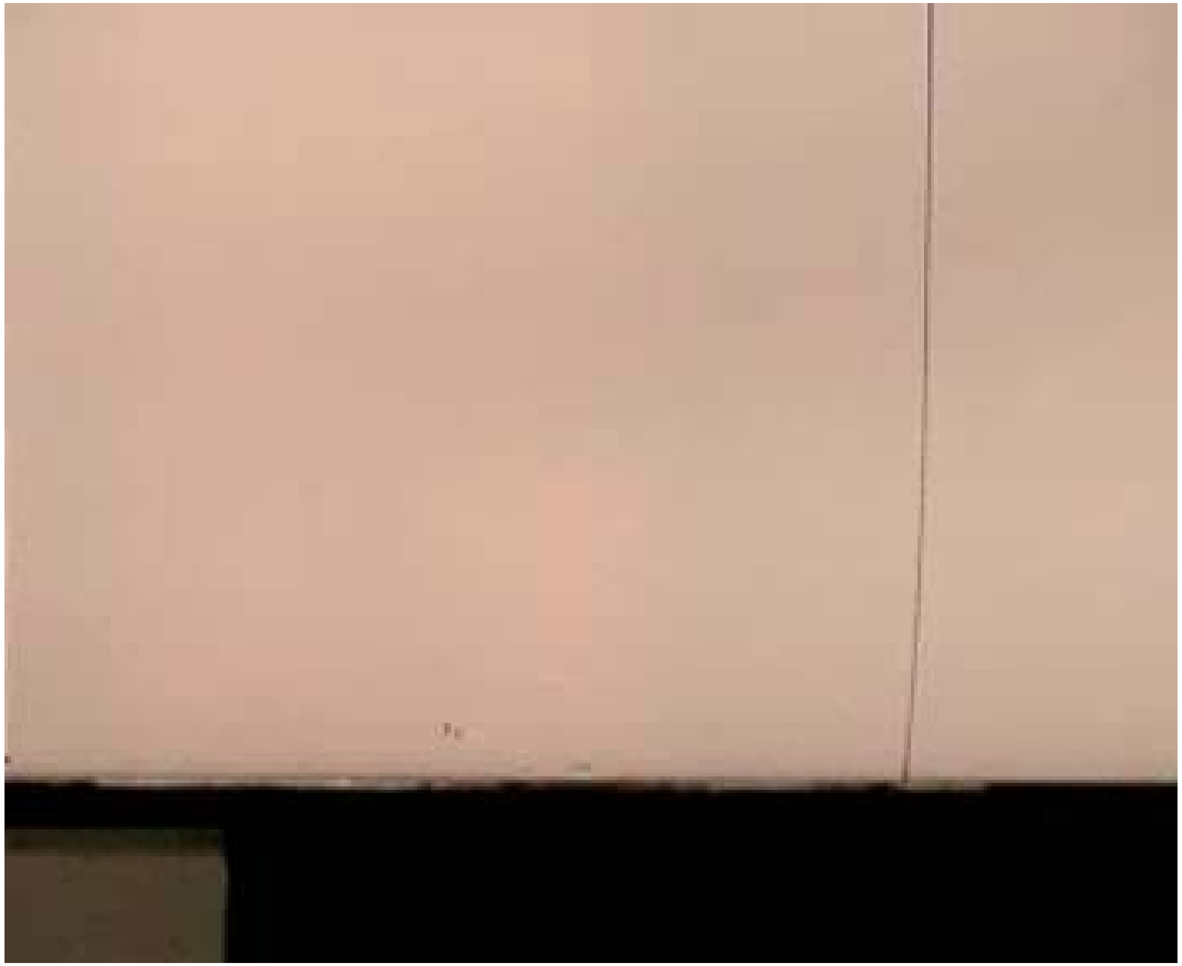

8.2 Difference of Viscous Flow from Ideal Flow

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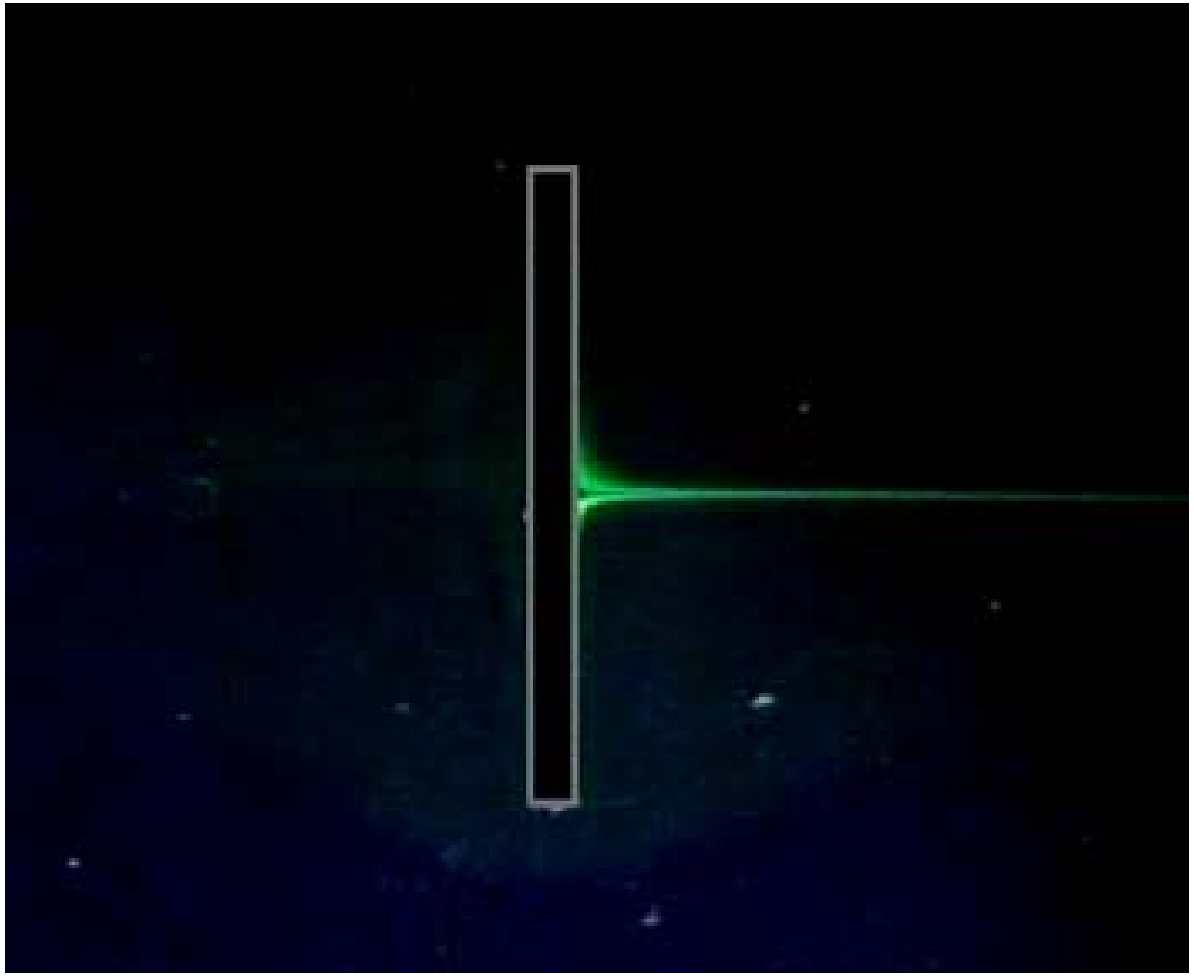
8.2 Difference of Viscous Flow from Ideal Flow

A comparison of viscous flow with ideal flow.



8.2 Difference of Viscous Flow from Ideal Flow

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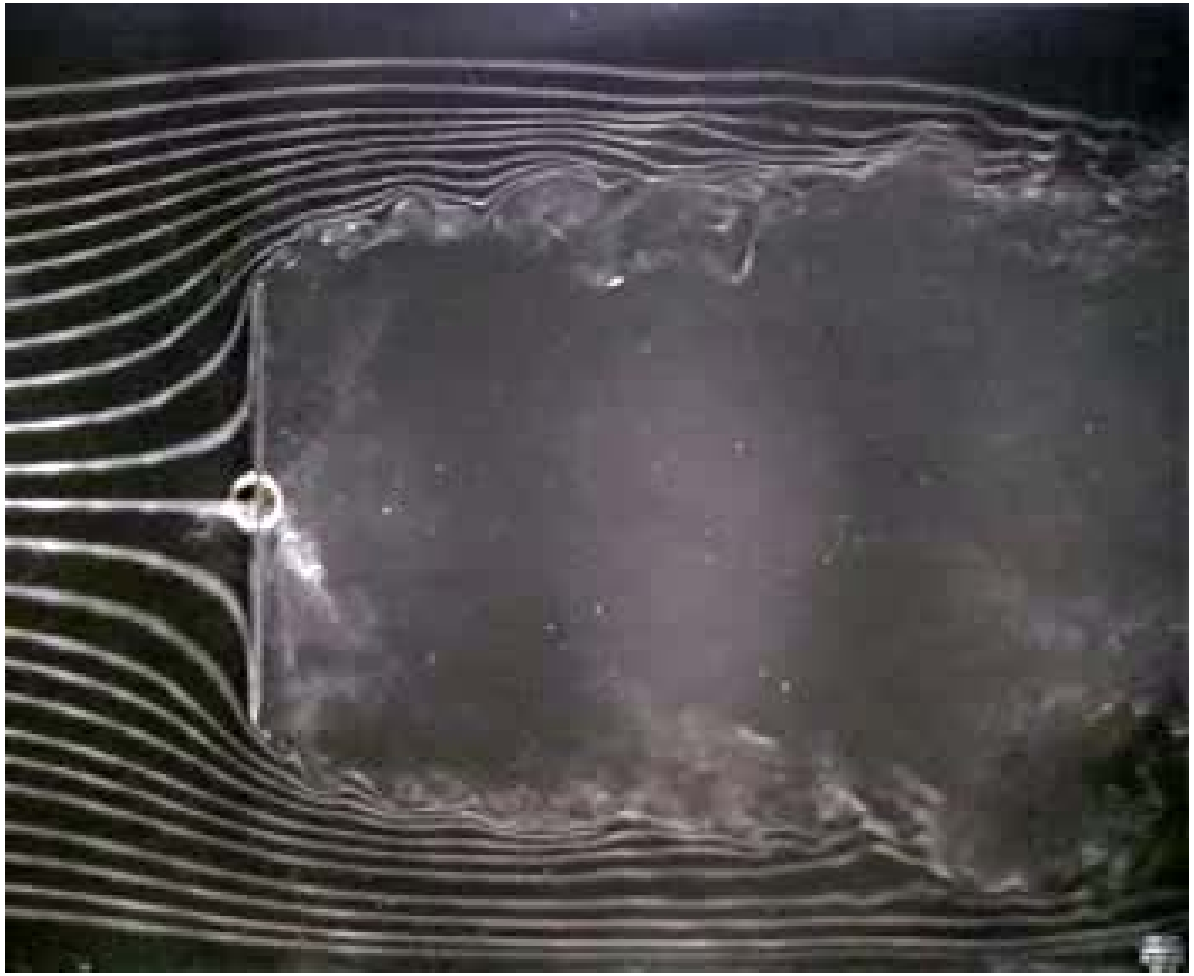
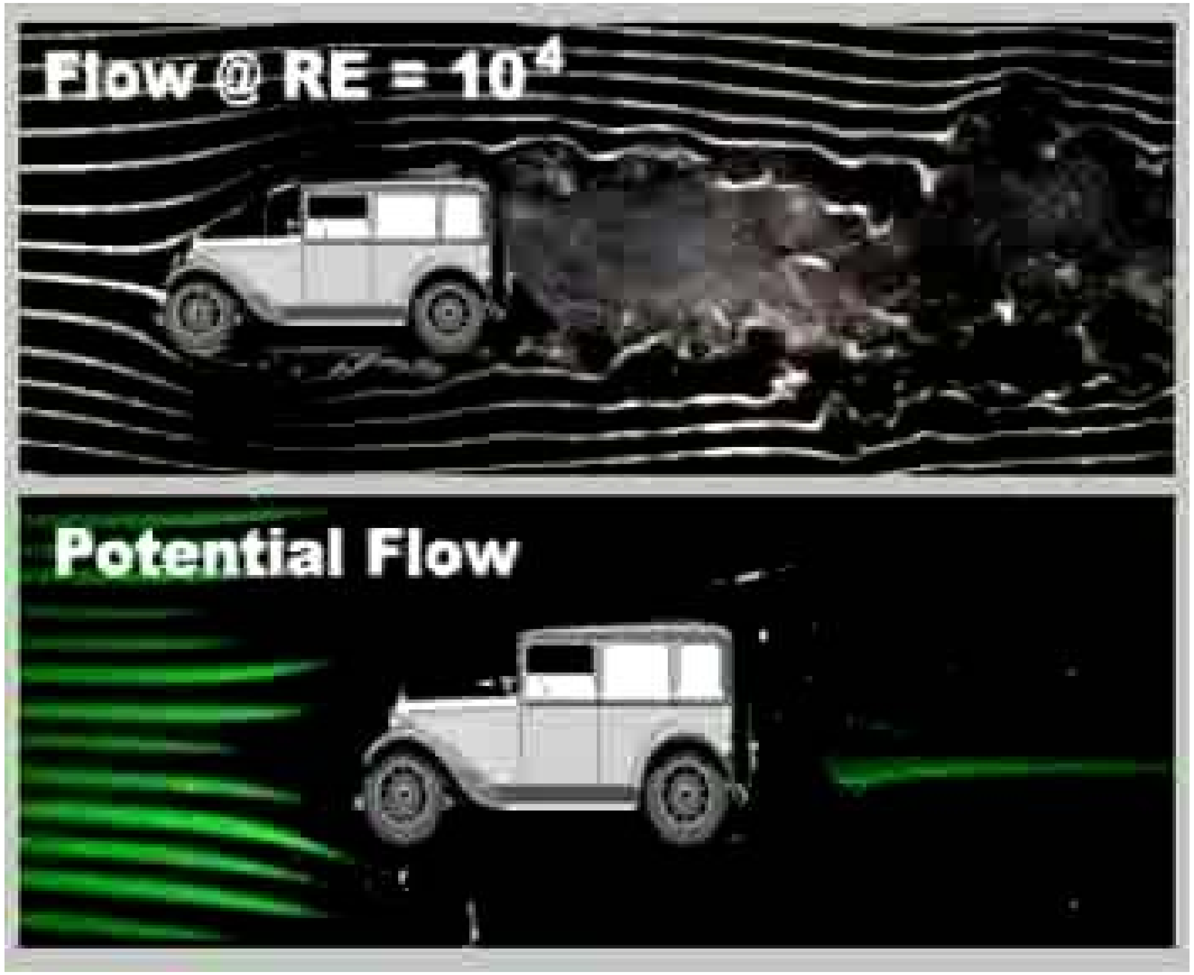


8.2 Difference of Viscous Flow from Ideal Flow

A comparison of *viscous flow* with ideal *potential flow*.



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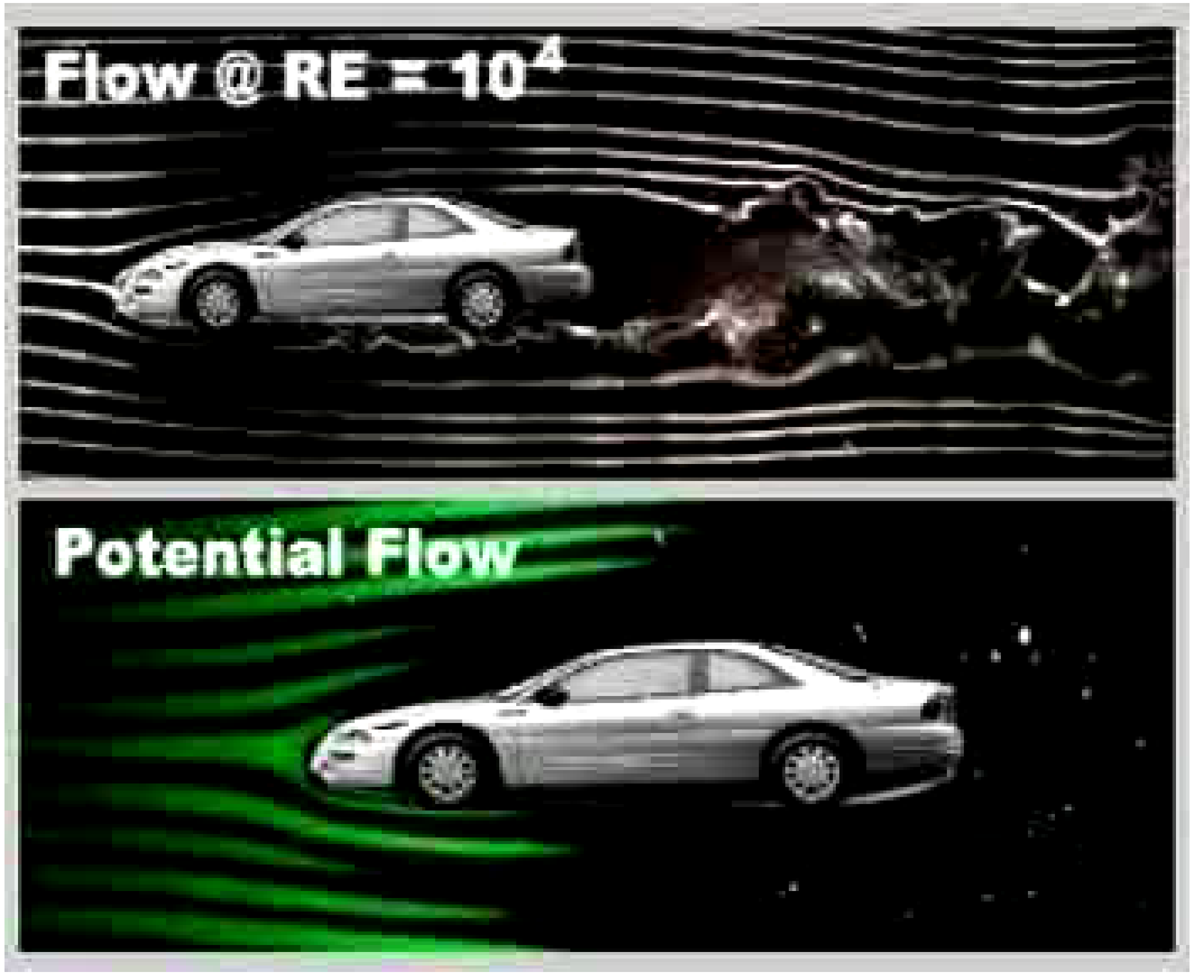


8.3 Governing Equations of Viscous Flow

In hydrodynamics, generally we only focused on ***incompressible Newtonian fluid***. As discussed in Chapter 3, flows of this sort of fluid are governed by the ***continuity equation*** (*dilatation* vanishment) and the dynamic equation of ***Navier-Stokes equation***, or simply ***N-S equation***.

Continuity equation

$$\nabla \cdot \mathbf{V} = 0$$

Navier-Stokes equation

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{1}{\rho}\nabla p + \mathbf{g} + \nu \nabla^2 \mathbf{V}$$

In addition, ***boundary conditions*** and ***initial conditions*** should be also satisfied. All of these equations form a set of governing equations for viscous flows.

8.3 Governing Equations of Viscous Flow

Applying Einstein's summation convention, they can be written as

$$\frac{\partial u_i}{\partial x_i} = 0$$

$$\rho\left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \rho g_i + \mu \frac{\partial^2 u_i}{\partial x_j^2}$$

(I) (II) (III) (IV) (V)

or

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + g_i + \nu \frac{\partial^2 u_i}{\partial x_j^2}$$

where $\nu = \mu/\rho$ is designated for the fluid ***kinematic viscosity***.

Physical meaning of the terms in *N-S momentum equation*

(I) *local acceleration*

(II) *convective acceleration*

inertia, convection

nonlinear terms

(III) *pressure gradient*

(IV) *volume force* or body force (gravity is a general case)

(V) *viscous diffusion of momentum* owing to molecular viscosity of the fluid

8.3 Governing Equations of Viscous Flow

Expressions of ***Continuity*** and ***N-S*** equations in rectangular coordinate systems (x, y, z)

Continuity:
$$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}=0$$

x-component:
$$\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)+g_x$$

y-component:
$$\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)+g_y$$

z-component:
$$\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right)+g_z$$

Expressions of ***Continuity*** and ***N-S*** equations in cylindrical coordinate systems (r, θ, z)

Continuity: $$\frac{1}{r}\frac{\partial(ru_r)}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0$$

r-component: $$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta^2}{r}+u_z\frac{\partial u_r}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}$$
$$+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_r)\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right]+g_r$$

θ-component: $$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r u_\theta}{r}+u_z\frac{\partial u_\theta}{\partial z}=-\frac{1}{\rho r}\frac{\partial p}{\partial \theta}$$
$$+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right]+g_\theta$$

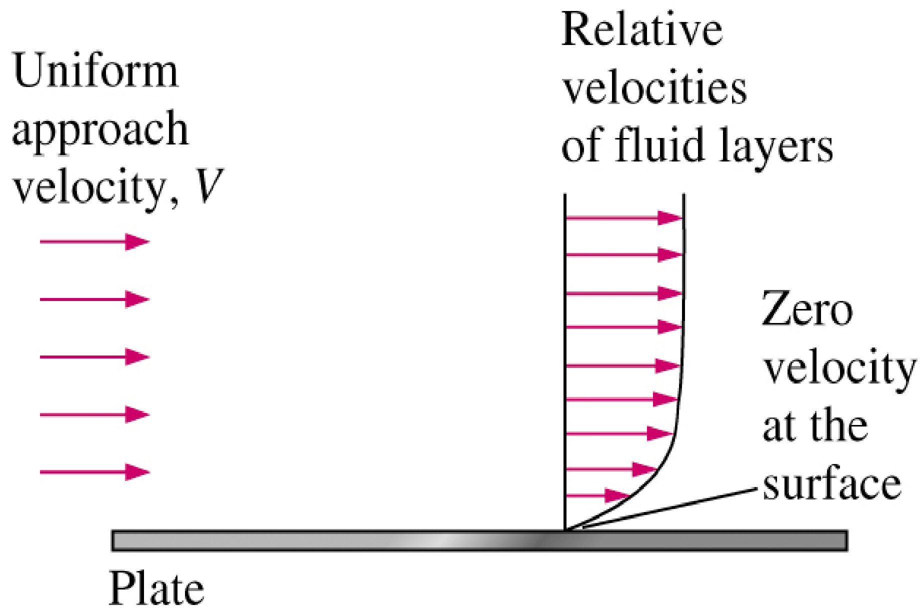
z-component: $$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}$$
$$+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+g_z$$

Boundary condition

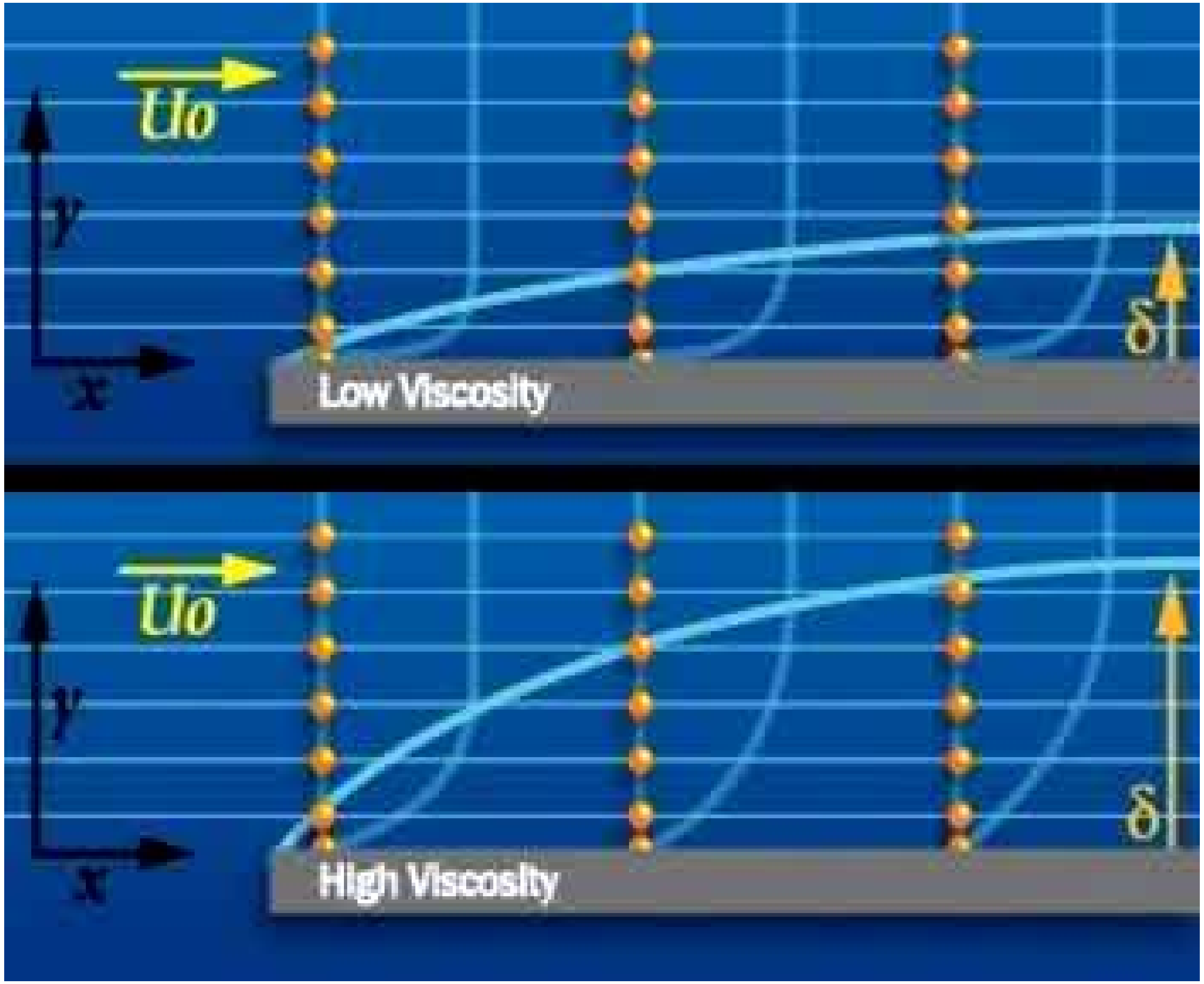
1) Fluid-body interface

On fluid-body interface, fluid particle velocity should be equal to the velocity of the body at that point, that is, ***no-slip*** condition.

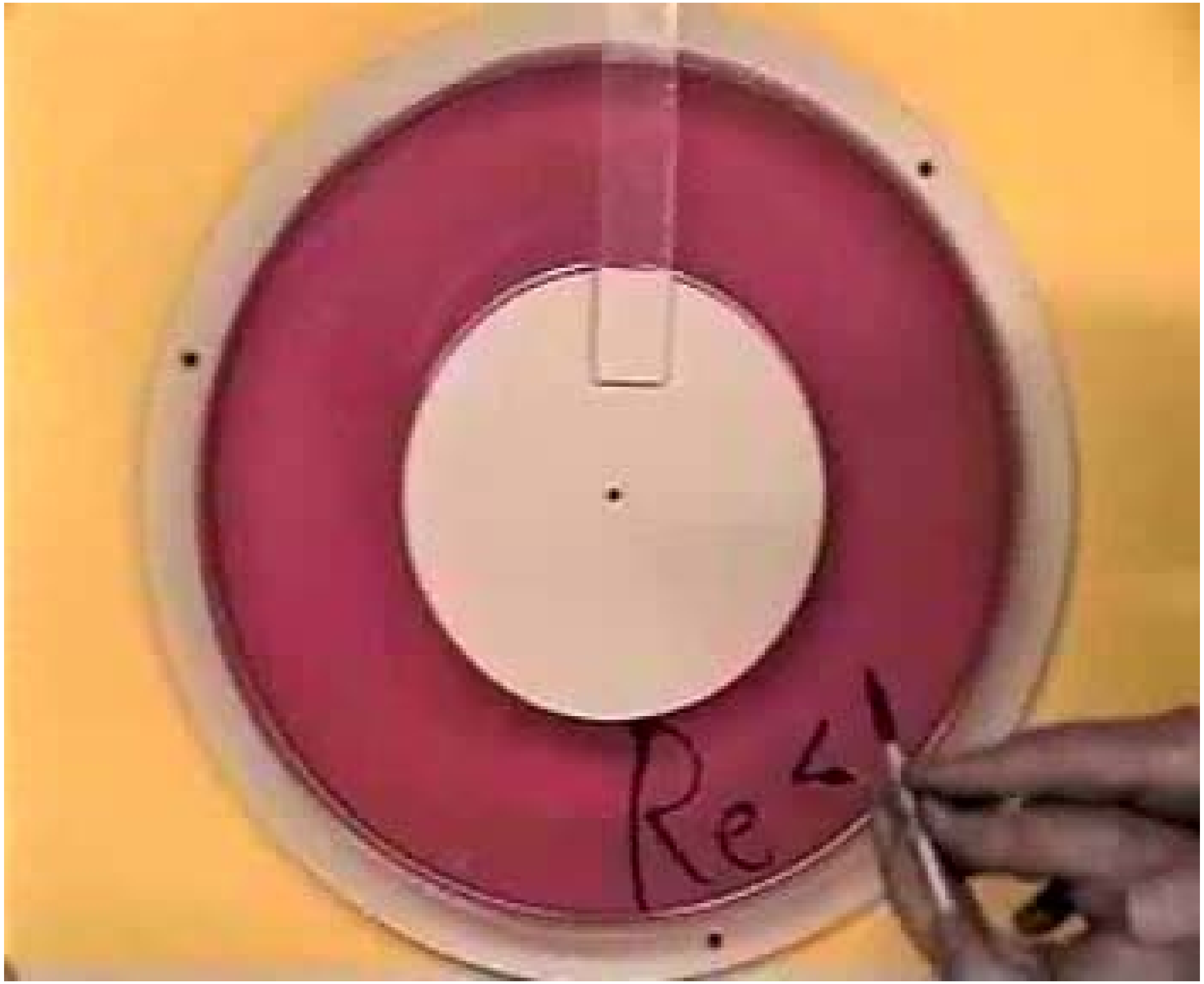
$$\boldsymbol{V}_{\text{fluid}} = \boldsymbol{V}_{\text{solid}} \qquad \text{on fluid-body interface}$$



8.3 Governing Equations of Viscous Flow



8.3 Governing Equations of Viscous Flow



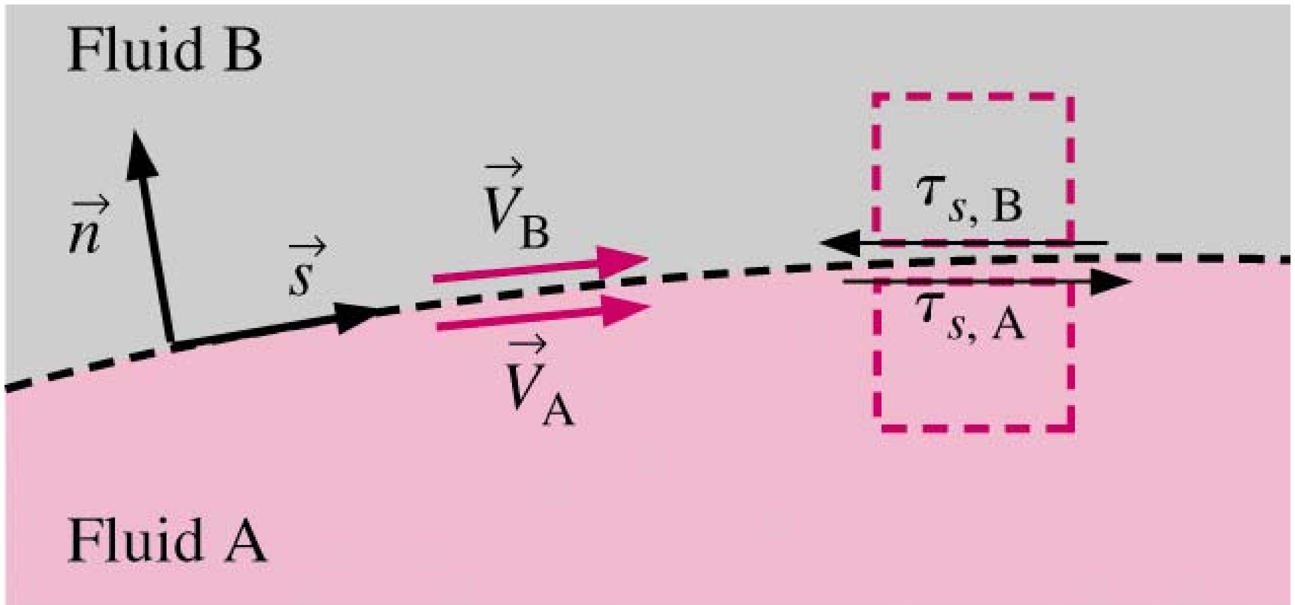
8.3 Governing Equations of Viscous Flow

2) Fluid-fluid interface

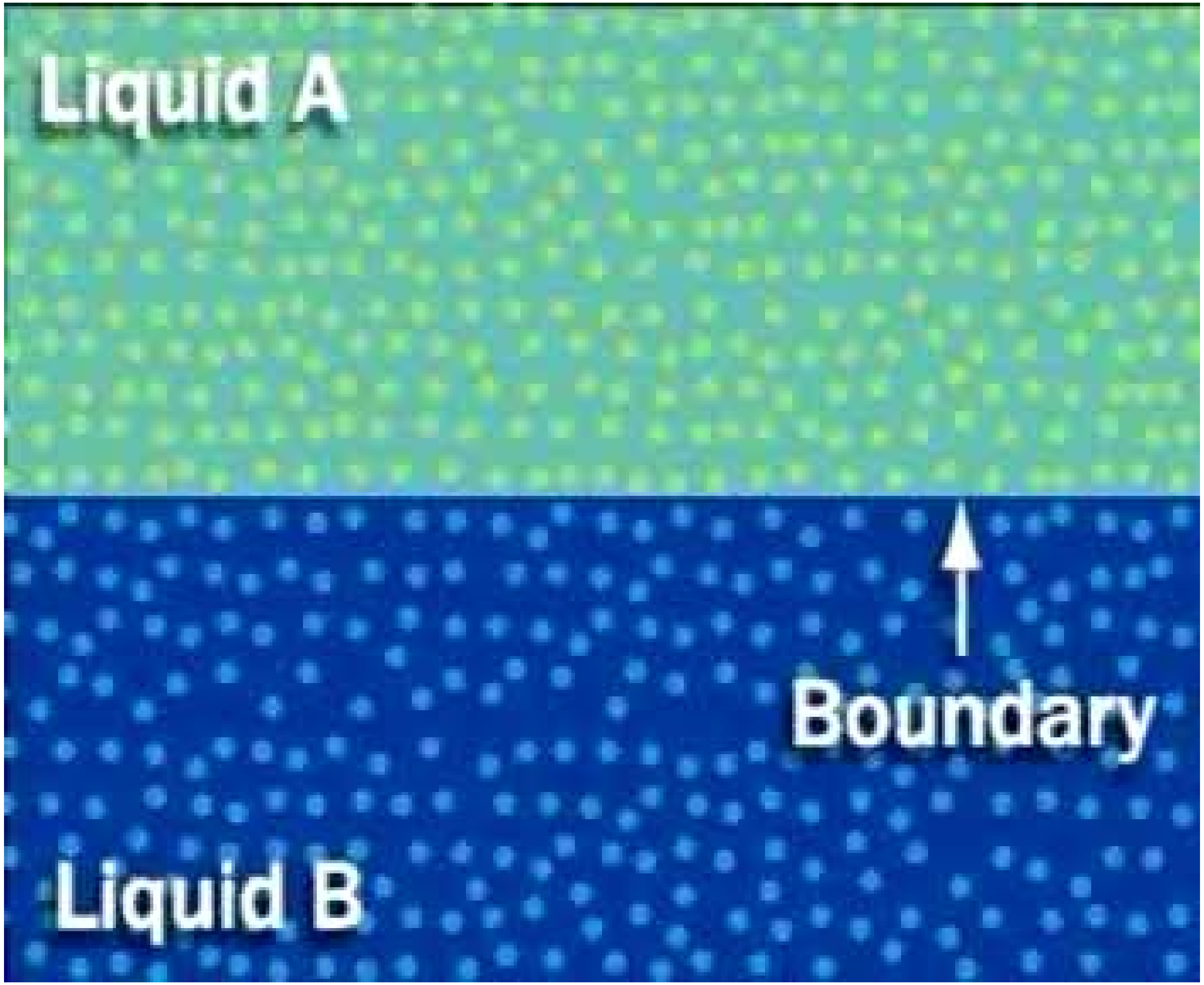
On fluid-fluid interface, both velocity and stress are continuous.

$$\mathbf{V}_A = \mathbf{V}_B, \qquad \boldsymbol{\tau}_A = \boldsymbol{\tau}_B \qquad \text{on fluid-fluid interface}$$

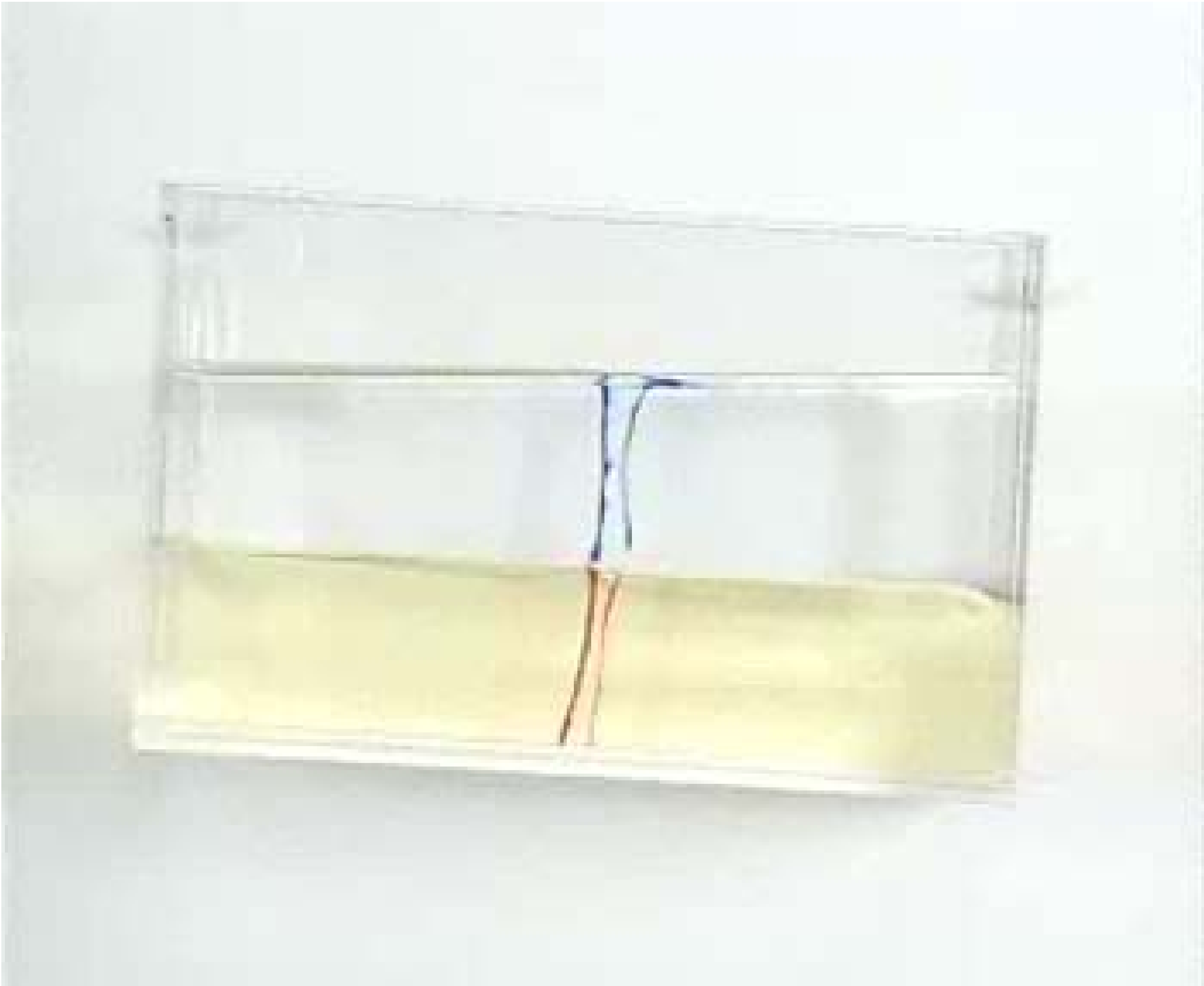
$$p_A = p_B, \qquad \mu_A \left.\frac{du}{dy}\right|_A = \mu_B \left.\frac{du}{dy}\right|_B$$



8.3 Governing Equations of Viscous Flow



8.3 Governing Equations of Viscous Flow

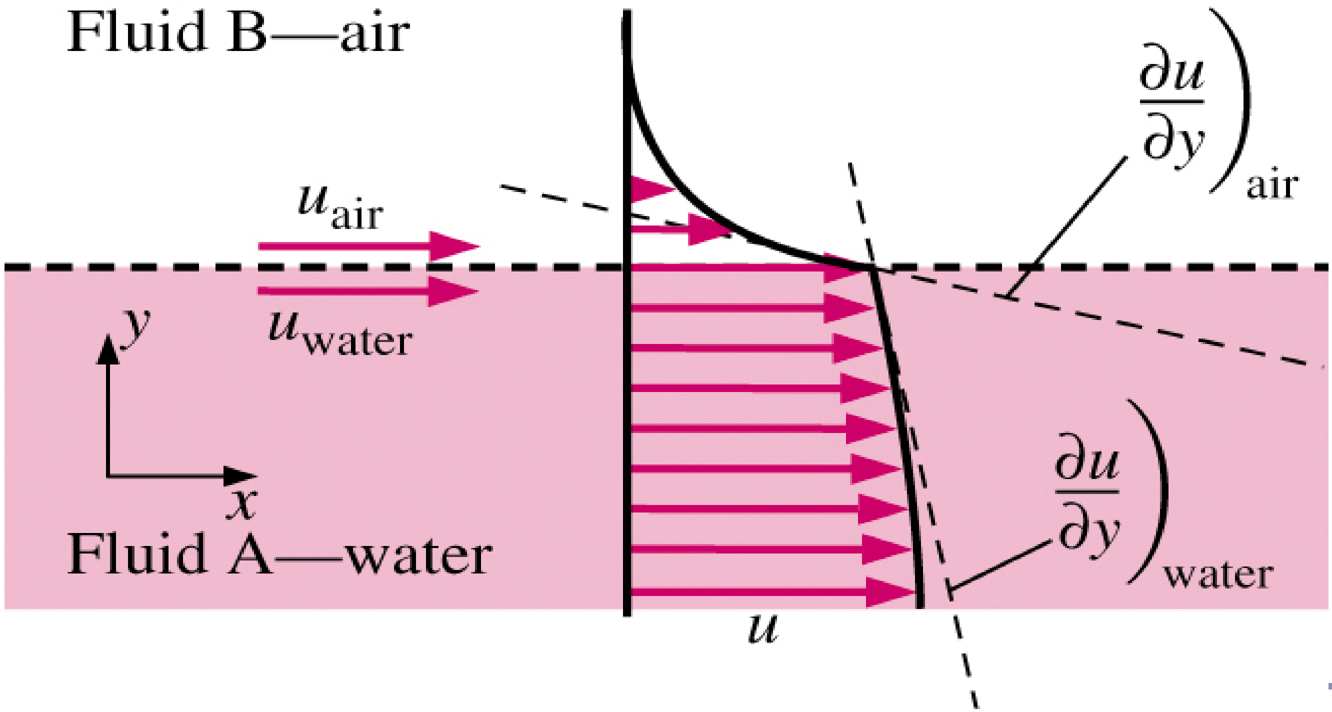


8.3 Governing Equations of Viscous Flow

Free surface condition (air-water interface $\mu_{\text{air}} << \mu_{\text{water}}$)

Usually, tangential stress is negligible, then

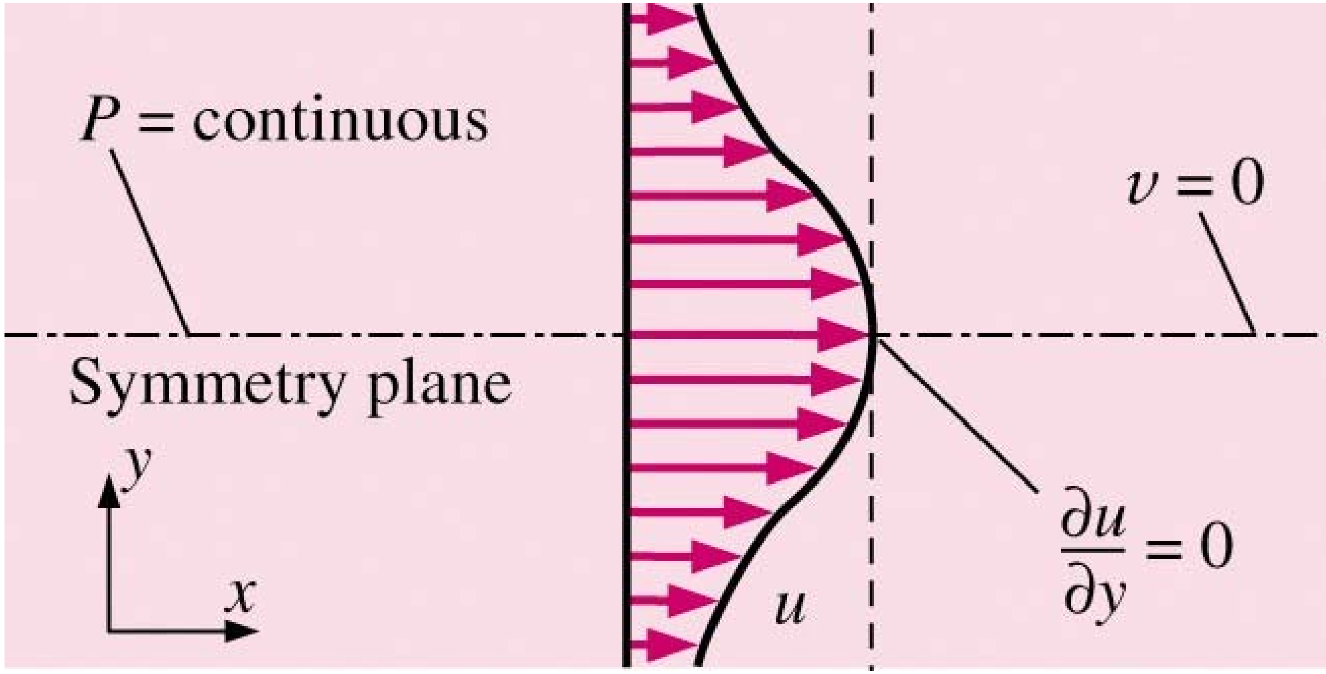
$$V_{\text{air}} = V_{\text{water}}, \quad p_{\text{air}} = p_{\text{atmosphere}}, \quad \mu_{\text{water}} \left. \frac{du}{dy} \right|_{\text{water}} = 0$$



3) Other boundary conditions

Inlet condition, outlet condition, periodic condition, symmetry, etc.

For different physical problem, different boundary conditions should be satisfied.



Initial Conditions

For unsteady flow, initial conditions should be given as well.

8.4 Simplification of N-S Equation

Generally, direct solution of the full ***N-S equation*** show some difficulty. It is mainly due to the following three factors.

(1) ***Nonlinearity:*** The convective terms are nonlinear.

(2) Both ***convective*** and ***diffusive:*** N-S equation not only have convective terms and have diffusive terms as well. The viscous term is diffusive. The corresponding physical phenomena and mathematical characteristics are extremely complicate.

(3) ***Coupling*** of hydrodynamic pressure with velocity. In N-S equation, time derivative and the Laplacian of velocity are very different from the pressure gradient. The former corresponds convection and diffusion. They are explicitly time dependent. The latter is independent on time, at least not explicitly.

8.4 Simplification of N-S Equation

***N-S* equation can also be written as a matrix form**

$$\begin{bmatrix} \mathbf{0} & \mathbf{B} \\ -\mathbf{B}^T & \mathbf{D} \end{bmatrix} \cdot \begin{bmatrix} p/\rho \\ \mathbf{V} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{g} \end{bmatrix}$$

where

$$\mathbf{B} = \frac{\partial(\ )_i}{\partial x_i} \leftarrow$$ ***Divergence operator***

$$\mathbf{B}^T = \frac{\partial(\ )}{\partial x_i} \leftarrow$$ ***Gradient operator***

$$\mathbf{D} = \mathbf{M} + \mathbf{N} + \mathbf{L}$$

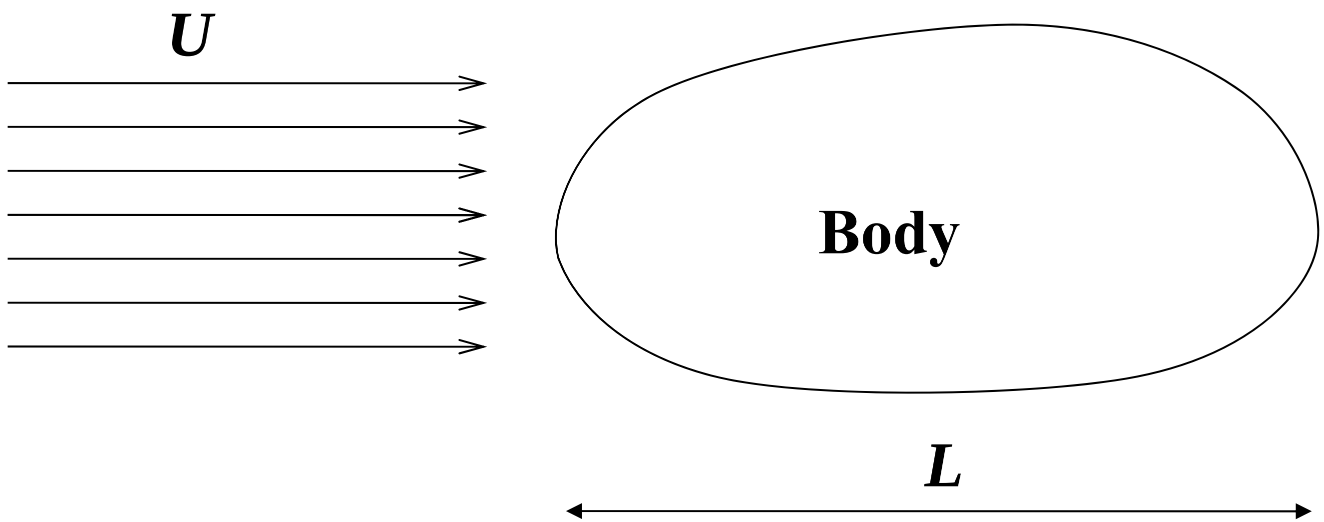
$$\mathbf{M} = \frac{\partial(\ )}{\partial t} \leftarrow$$ ***Time derivative operator***

$$\mathbf{N} = u_j \frac{\partial(\ )_i}{\partial x_j} \leftarrow$$ ***Convection operator***

$$\mathbf{L} = -\nu \frac{\partial^2(\ )_i}{\partial x_j \partial x_j} \leftarrow$$ ***Diffusion operator***

8.4 Simplification of N-S Equation

In solution of *N-S equation*, for special cases, some terms may be of very small value relative to other terms, and less important, and become negligible. While N-S equation is written in a dimensionless form, as will be given later, we can simply determine whether a term is negligible or not. As an example, we look at an *unsteady flow past a body*.



We choose four ***Characteristic quantities*** below.

L – length of the body; U – uniform speed;

T – time; P – pressure at infinity.

8.4 Simplification of N-S Equation

In terms of these characteristic scales, physical quantities can be ***non-dimensionalized***.

$$\mathbf{V}^* = \mathbf{V}/U, \quad t^* = t/T, \quad \mathbf{x}^* = \mathbf{x}/L, \quad p^* = p/P, \quad \mathbf{g}^* = \mathbf{g}/g$$

And ***N-S equation*** is rewritten in these dimensionless quantities.

$$\left(\frac{L}{UT}\right)\frac{\partial \mathbf{V}^*}{\partial t^*} + \mathbf{V}^* \cdot \nabla^* \mathbf{V}^* = -\left(\frac{P}{\rho U^2}\right)\nabla^* p^* + \left(\frac{gL}{U^2}\right)\mathbf{g}^* + \left(\frac{\nu}{UL}\right)\nabla^{*2}\mathbf{V}^*$$