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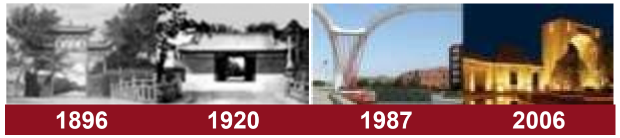


Introduction to Marine Hydrodynamics

(NA235)

Department of Naval Architecture and Ocean Engineering

School of Naval Architecture, Ocean & Civil Engineering

Shanghai Jiao Tong University

Chapter 7
Water Waves

Review of Chapter 7

A: wave amplitude

H = 2A: wave height

λ: wave length

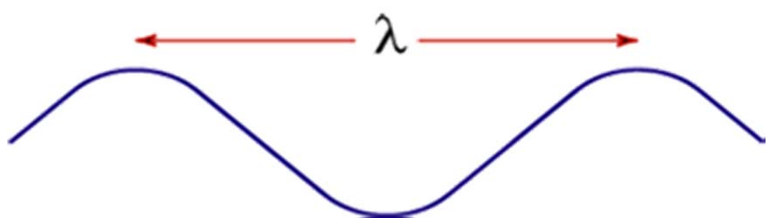
crest

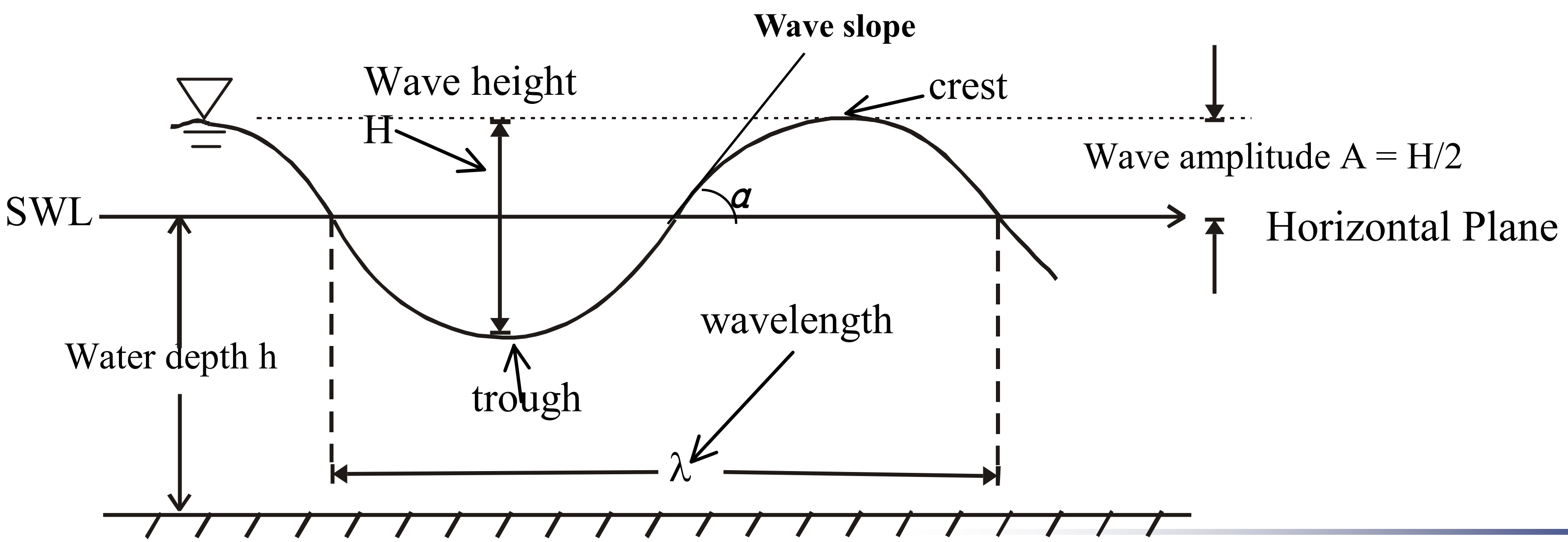
trough

h: water depth

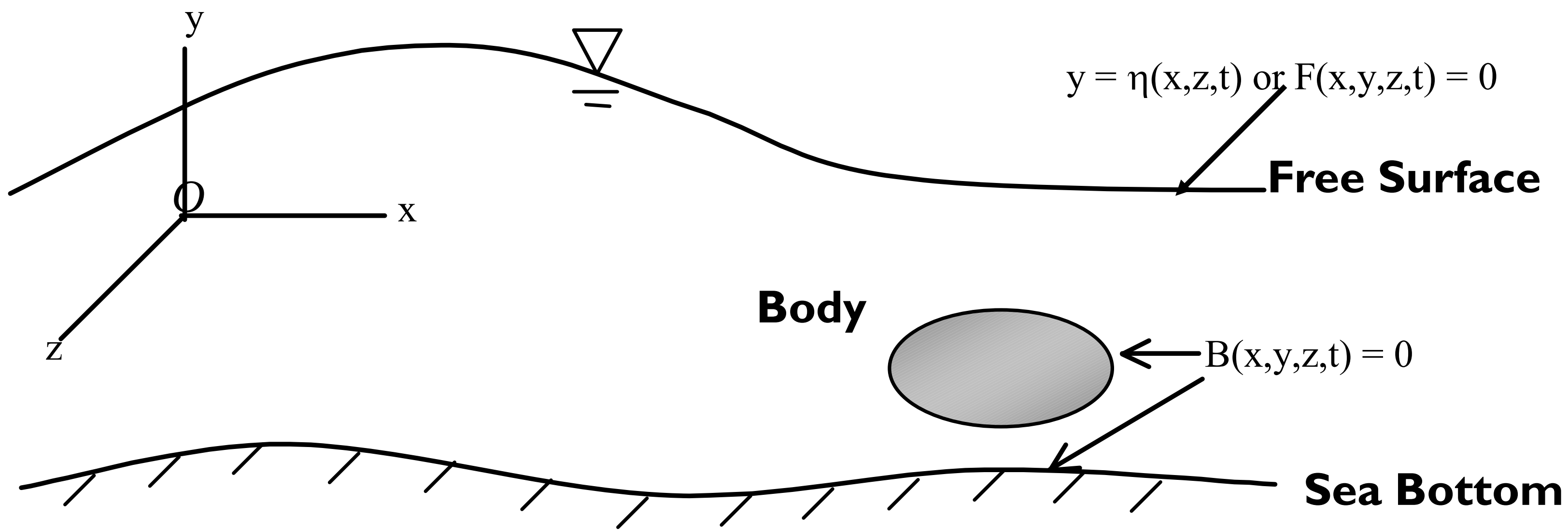
α : wave slope

$k = 2\pi/\lambda$: wave number





Governing equation of water waves



Review of Chapter 7

A summary of governing equations of water waves.

Field equations: in $y \le \eta(x, z, t)$

Laplace's Eq. $\nabla^2 \phi = 0$

Dynamic cond.
$$\frac{\partial \phi}{\partial t} + \frac{|\nabla \phi|^2}{2} + \frac{p - p_a}{\rho} + gy = 0$$

Far field cond.
$$\frac{\partial \phi}{\partial t} \to 0, \quad \mathbf{V} = \nabla \phi \to 0, \quad p = p_a - \rho g y$$

Initial cond.
$$\begin{cases} \phi(x, \eta, z, 0) = f(x, z) & \textbf{on } y = \eta \\ \eta(x, z, 0) = g(x, z) \end{cases}$$

Review of Chapter 7

Boundary conditions:

On body surf., B

$$\frac{\partial \phi}{\partial \mathbf{n}} = \mathbf{U}_n \quad \textbf{or} \quad \frac{\partial B}{\partial t} + (\nabla \phi \cdot \nabla) B = 0$$

On the bottom, $y = -h$

$$\frac{\partial \phi}{\partial y} = 0$$

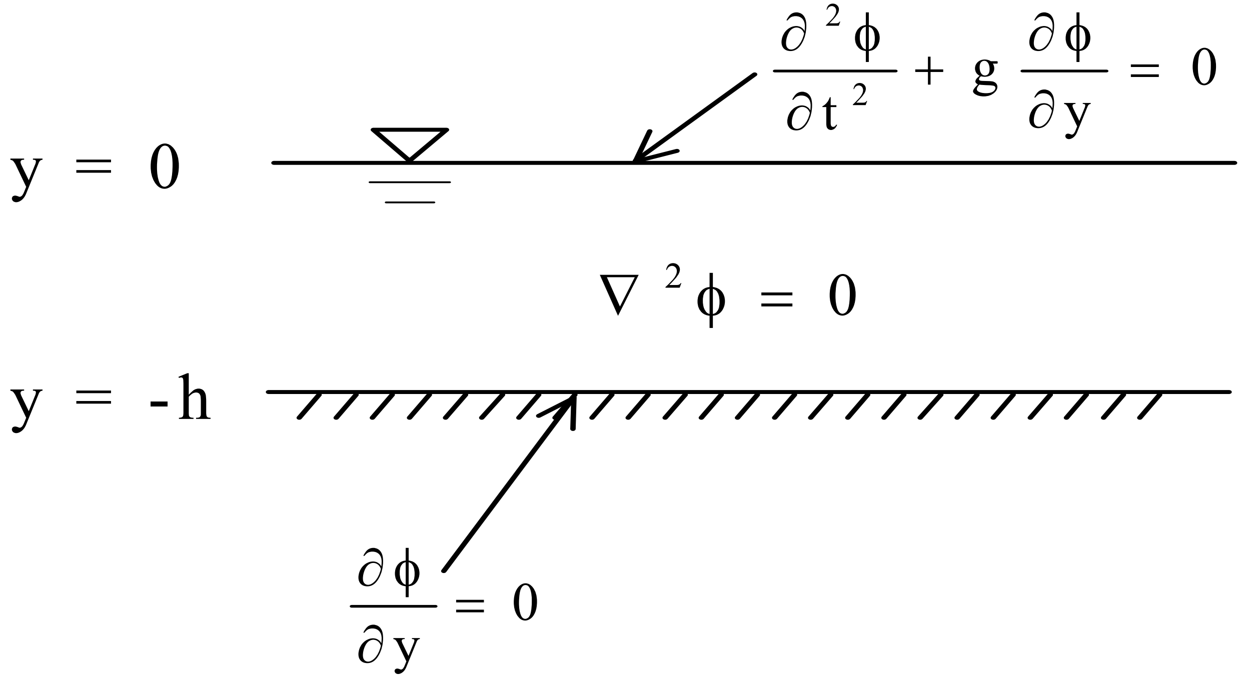
On free surface, $y = \eta$
(kinematic cond.)

$$\frac{\partial \phi}{\partial y} = \frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x}\frac{\partial \eta}{\partial x} + \frac{\partial \phi}{\partial z}\frac{\partial \eta}{\partial z}$$

On free surface, $y = \eta$
(dynamic cond.)

$$\frac{\partial \phi}{\partial t} + \frac{|\nabla \phi|^2}{2} + g\eta = 0$$

Governing equations of Airy wave



Review of Chapter 7

Linearized governing equations of Airy waves are as follows.

Field equations: in $y \le \eta(x, z, t)$

1. Laplace's Eq. $\nabla^2 \phi = 0$

2. Dynamic cond. $p - p_a = -\rho \dfrac{\partial \phi}{\partial t} - \rho g y$

3. Far field cond. $\dfrac{\partial \phi}{\partial t} \to 0, \quad \mathbf{V} = \nabla \phi \to 0, \quad p = p_a - \rho g y$

4. Initial cond. $\begin{cases} \phi(x, 0, z, 0) = f(x, z) \\ \eta(x, z, 0) = g(x, z) \end{cases}$

Review of Chapter 7

Boundary conditions:

5. On body surface, B

$$\frac{\partial \phi}{\partial \mathbf{n}} = \mathbf{U}_n \text{ or } \frac{\partial B}{\partial t} + \left(\nabla \phi \cdot \nabla\right) B = 0$$

6. On the bottom, $y = -h$

$$\frac{\partial \phi}{\partial y} = 0$$

7. On the mean free surface, $y = 0$

$$g\frac{\partial \phi}{\partial y} + \frac{\partial^2 \phi}{\partial t^2} = 0$$

$$\eta = -\frac{1}{g}\frac{\partial \phi}{\partial t}$$

Review of Chapter 7

Velocity potential function of Airy waves can be derived.

$$\phi = \frac{gA}{\omega}\sin(kx - \omega t)\frac{\cosh k(y+h)}{\cosh kh}$$

As the depth of water tends to infinity, $h \to \infty$, it becomes

$$\phi = \frac{gA}{\omega}\sin(kx - \omega t)e^{ky}$$

Review of Chapter 7

Wave elevation of Airy wave

$$\eta = -\frac{1}{g}\frac{\partial \phi}{\partial t}\bigg|_{y=0} = A\cos(kx-\omega t)$$

From the dynamic condition, pressure distribution of Airy wave is

$$p - p_a = -\rho\frac{\partial \phi}{\partial t} - \rho g y$$
$$= -\rho g y + \rho A g\cos(kx-\omega t)\frac{\cosh k(y+h)}{\cosh kh}$$

Velocity field of Airy wave

$$u = \frac{\partial \phi}{\partial x} = A\omega \frac{\cosh k(y+h)}{\sinh kh} \cos(kx - \omega t)$$

$$v = \frac{\partial \phi}{\partial y} = A\omega \frac{\sinh k(y+h)}{\sinh kh} \sin(kx - \omega t)$$

Review of Chapter 7

Dispersion relation of Airy wave

$$\omega^2 = gk\frac{\sinh kh}{\cosh kh} = gk\tanh kh$$

$$c^2 = \frac{\omega^2}{k^2} = \frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi h}{\lambda}\right)$$

Water particle orbit of Airy wave

$$\frac{\left(x_P - \overline{x}\right)^2}{a^2} + \frac{\left(y_P - \overline{y}\right)^2}{b^2} = 1$$

$$a = A\frac{\cosh k\left(\overline{y} + h\right)}{\sinh kh},$$

$$b = A\frac{\sinh k\left(\overline{y} + h\right)}{\sinh kh}$$

Characteristics of Airy waves

	deep water $kh > 3$	shallow water $kh \ll 1$
dispersion relation	$\omega^2 = gk$ $c_d^2 = \frac{g}{k} = \frac{g\lambda}{2\pi}$	$\omega^2 = ghk^2$ $c_s^2 = gh$
velocity	$\frac{u}{U_0} \approx e^{ky}$ $\frac{v}{V_0} \approx e^{ky}$	$\frac{u}{U_0} \approx 1$ $\frac{v}{V_0} \approx 1 + \frac{y}{h}$

Review of Chapter 7

Characteristics of Airy waves

	deep water $kh > 3$	shallow water $kh << 1$
Water particle orbit	$a = b = Ae^{k\bar{y}}$	$a = \dfrac{A}{kh}$ $b = A\left(1 + \dfrac{\bar{y}}{h}\right)$
Pressure field	$p - p_a$ $= \rho g\left(\eta e^{ky} - y\right)$	$p - p_a$ $= \rho g\left(\eta - y\right)$

Review of Chapter 7

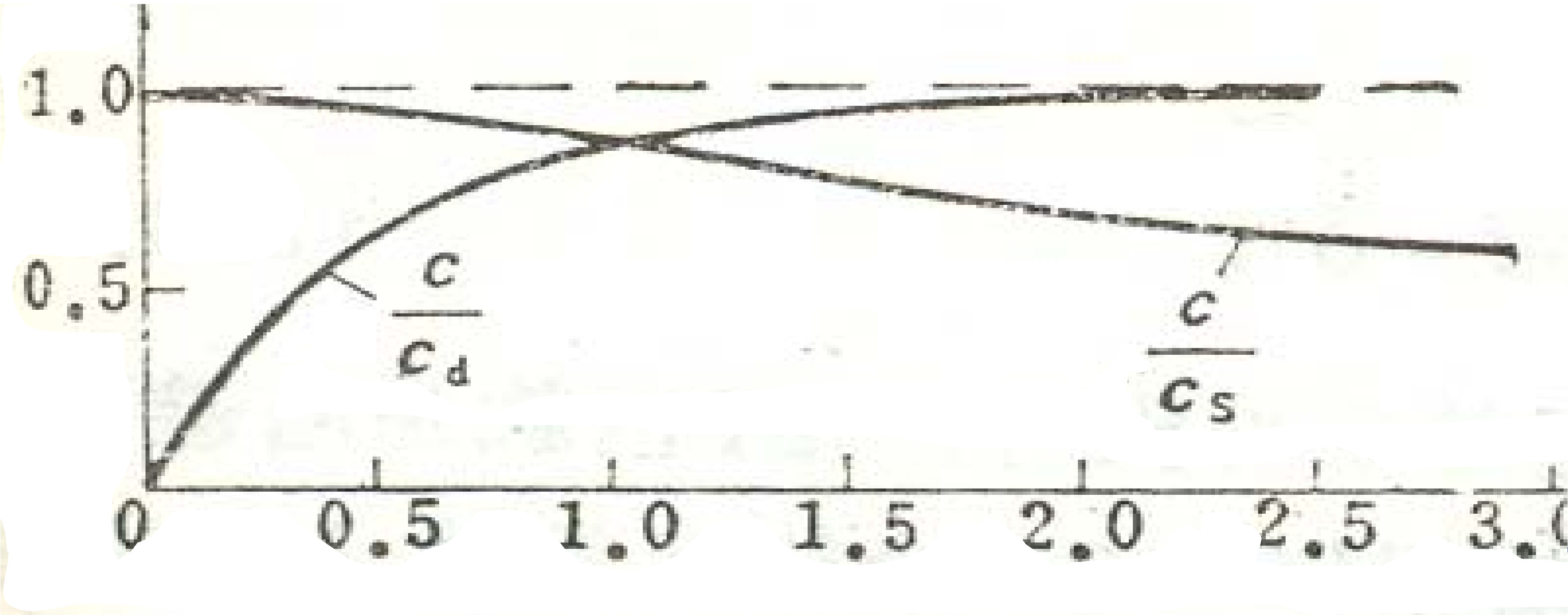
Characteristics of Airy waves

	deep water $kh > 3$	shallow water $kh \ll 1$
Water particle orbit	$a = b = Ae^{k\bar{y}}$	$a = \dfrac{A}{kh}$ $b = A\left(1 + \dfrac{\bar{y}}{h}\right)$
Pressure field	$p - p_a = \rho g\left(\eta e^{ky} - y\right)$	$p - p_a = \rho g\left(\eta - y\right)$

Review of Chapter 7

For a fixed water depth, since $c_s = \sqrt{gh}$ is a constant, **phase velocity** c will decrease with **wave length** λ . That is, *longer waves travel faster, and shorter waves travel slower.*

For waves with fixed wave length, since $c_d = \sqrt{\frac{g\lambda}{2\pi}}$ is a constant, phase velocity c will increase with the water depth h. That is, *deep water waves travel faster, and shallow water waves travel slower.*



$\frac{2\pi h}{\lambda}$

Review of Chapter 7

Linear plane progressive waves

$$\phi = \frac{gA}{\omega}\frac{\cosh k\left(y+h\right)}{\cosh kh}\sin\left(kx\cos\theta + kz\sin\theta - \omega t\right)$$

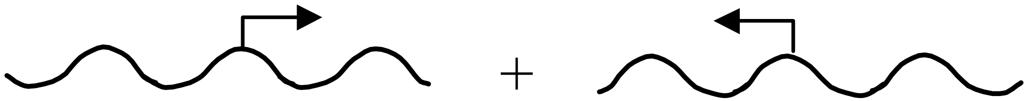
$$\eta = A\cos\left(kx\cos\theta + kz\sin\theta - \omega t\right)$$

$$\omega^2 = gk\tanh kh$$

$$k_x = k\cos\theta, \quad k_z = k\sin\theta, \quad k = \sqrt{k_x^2 + k_z^2}$$

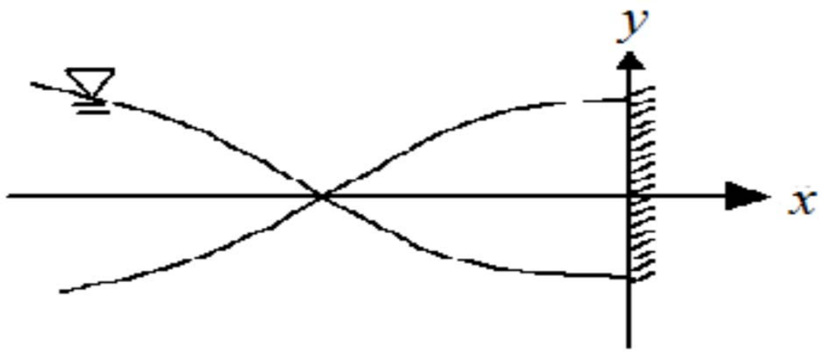
Linear superposition law

Standing Waves ---- a superposition of two Airy waves with same parameters, but propagating at opposite directions.



Same A, k, ω, no phase shift

$$\eta = 2A\cos kx \cos \omega t$$



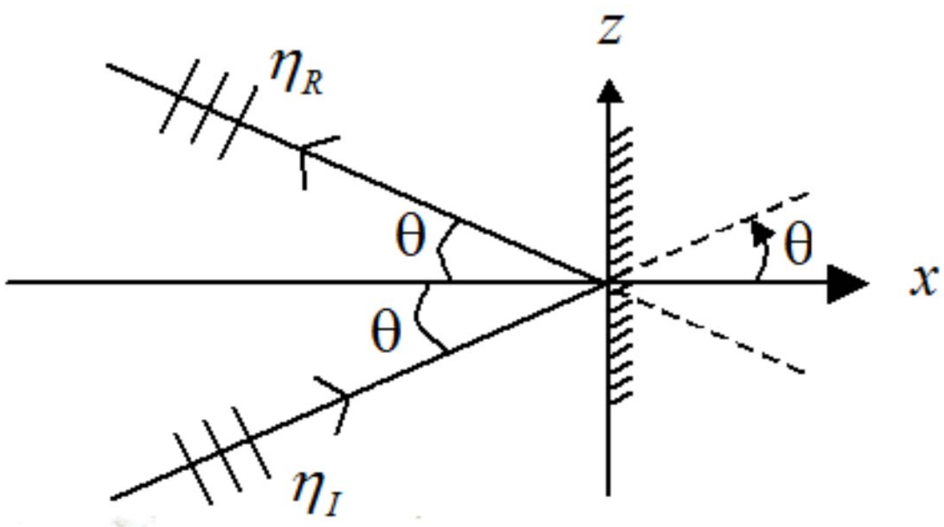
$$\phi = -\frac{2gA}{\omega}\frac{\cosh k\left(y+h\right)}{\cosh kh}\cos kx \sin \omega t$$

Review of Chapter 7

Oblique standing waves -- superposition of two plane progressive waves propagating not collinear.

$$\eta_T = 2A\cos\left(kx\cos\theta\right)\cos\left(kz\sin\theta - \omega t\right)$$

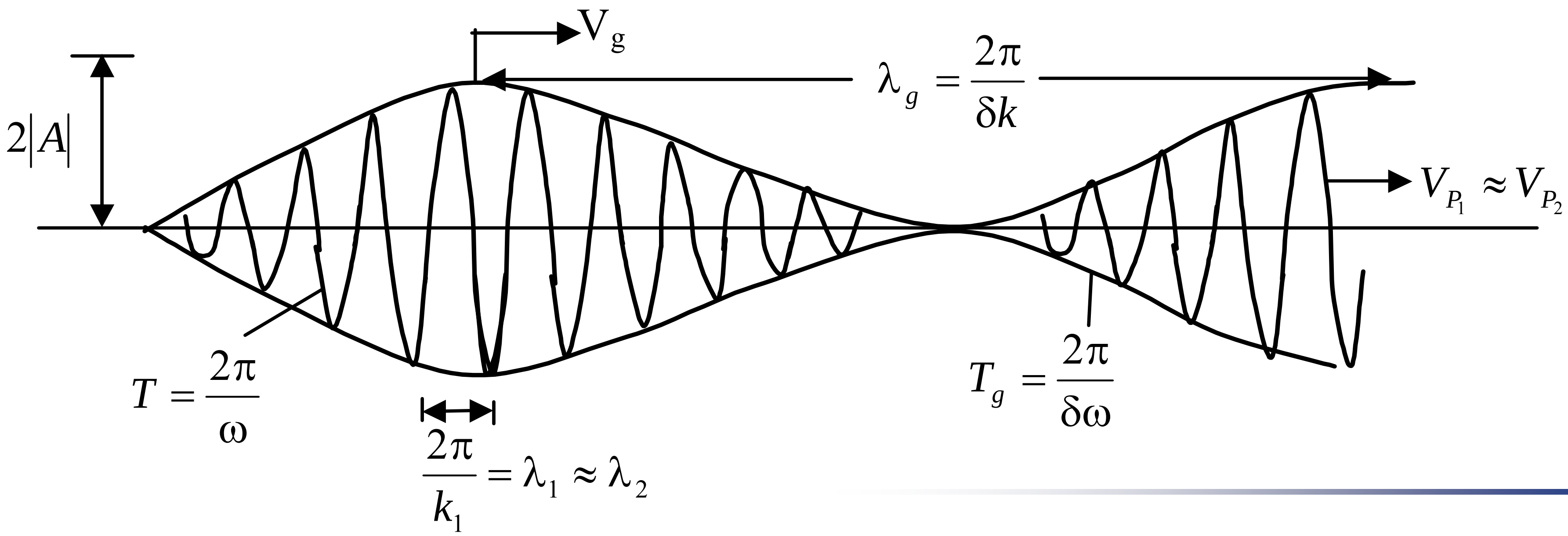
$$\phi = -\frac{2gA}{\omega}\frac{\cosh k\left(y+h\right)}{\cosh kh}\cos\left(kx\cos\theta\right)\sin\left(kz\sin\theta - \omega t\right)$$



Review of Chapter 7

Wave Group: a superposition of different progressive waves with similar frequencies propagating along the same direction.

$$\eta_T = \eta_1 + \eta_2 = \mathrm{Re}\left\{ A e^{i(k_1 x - \omega_1 t)} \left[1 + e^{i(\delta k\, x - \delta\omega t)} \right] \right\}$$



Review of Chapter 7

Group velocity

$$V_g = \frac{\delta\omega}{\delta k} = \frac{d\omega}{dk}$$

Group velocity of Airy waves $(1/2 < C < 1)$

$$V_g = \frac{d\omega}{dk} = \underbrace{\frac{\omega}{k}}_{V_p} \underbrace{\frac{1}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right)}_{C} = CV_p$$

For deep water, $C = \frac{V_g}{V_p} = \frac{1}{2}$; **For shallow water,** $C = \frac{V_g}{V_p} = 1$

Review of Chapter 7

Wave energy of Airy wave

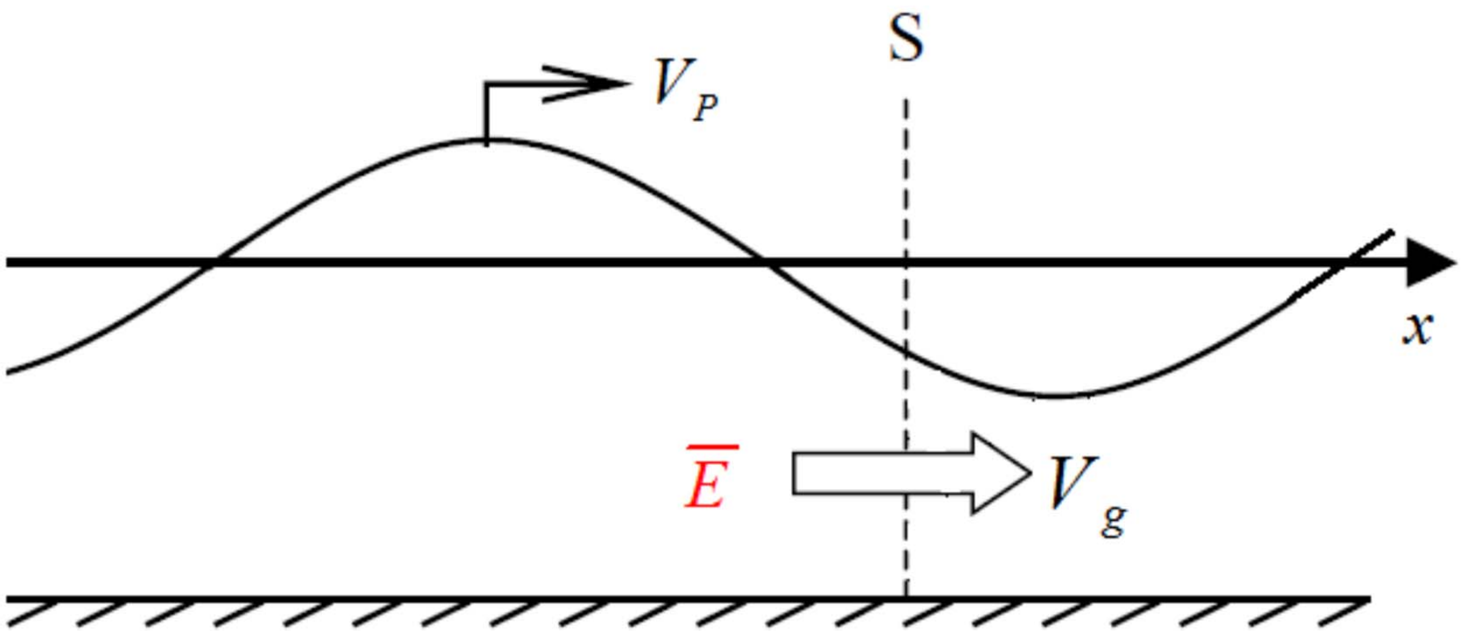
$$\overline{E} = \overline{PE}_{\text{wave}} + \overline{KE}_{\text{wave}} = \frac{\rho g A^2}{4} + \frac{\rho g A^2}{4} = \frac{\rho g A^2}{2}$$

Waver energy propagation equation

$$\frac{\partial \overline{E}}{\partial t} + \frac{\partial \left(V_g \overline{E} \right)}{\partial x} = 0$$

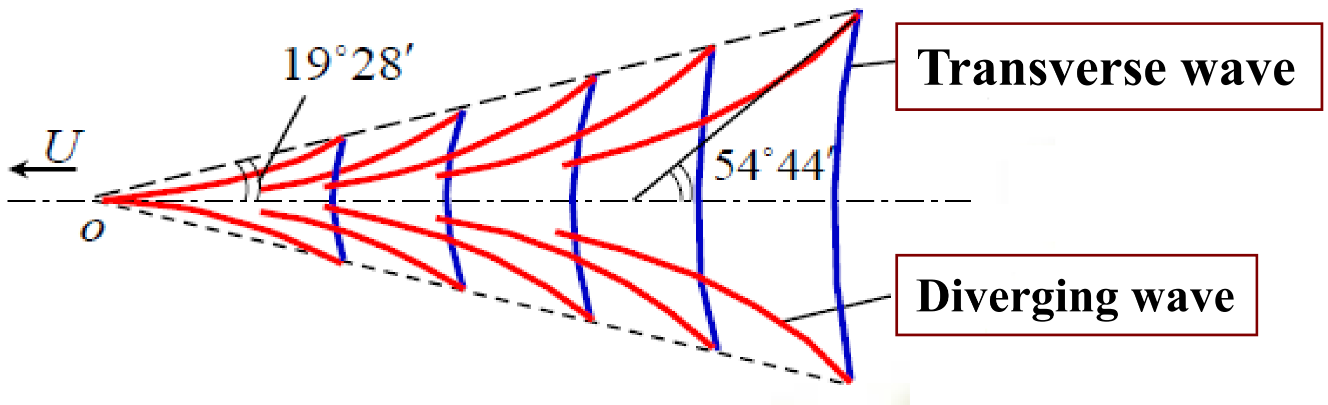
Review of Chapter 7

Wave energy propagates at group velocity, V_g, that is, wave energy propagates through vertical plane, perpendicular to the wave propagating direction, at group velocity.



Review of Chapter 7

Kelvin waves



Two kind of waves in a moving point pressure generated wave systems

1. *Transverse waves*, nearly perpendicular to forward speed, of wave length

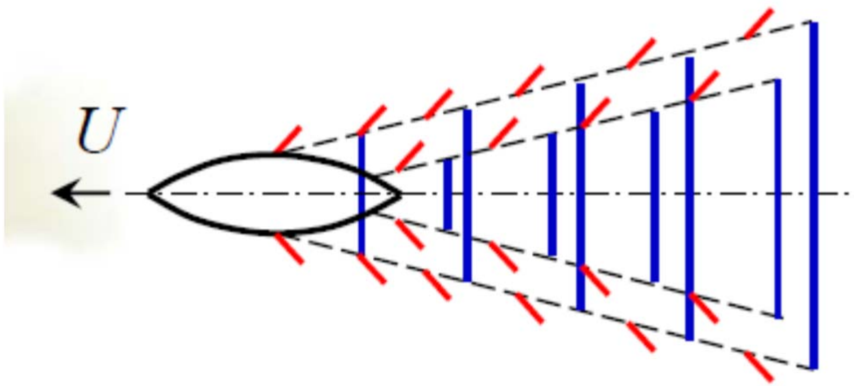
$$\lambda = 2\pi U^2/g$$

2. *Diverging waves*

Waves are confined to a sector of semi-angle 19° 28′, known as ***Kelvin angle***.

Review of Chapter 7

A ship, advancing in calm water, can be approximately represented by a superposition of the ***bow waves*** and the ***stern waves***.



Wave making resistance of a ship has relationship with the ***interference of bow wave and stern wave***. ***Lower ship speed corresponds remarkable transverse wave, and higher speed corresponds remarkable diverging wave.*** Diverging waves disappear more rapidly than transverse waves.

Review of Chapter 7

Wave making resistance of a moving ship is equal to half of the average wave energy.

$$R_W = \frac{1}{2}\overline{E} = \frac{1}{4}\rho g A^2$$

Wave making resistance is proportional to the square of the wave amplitude, which is determined by the ship speed, hull form and the interference between bow waves and stern waves.

Review of Chapter 7

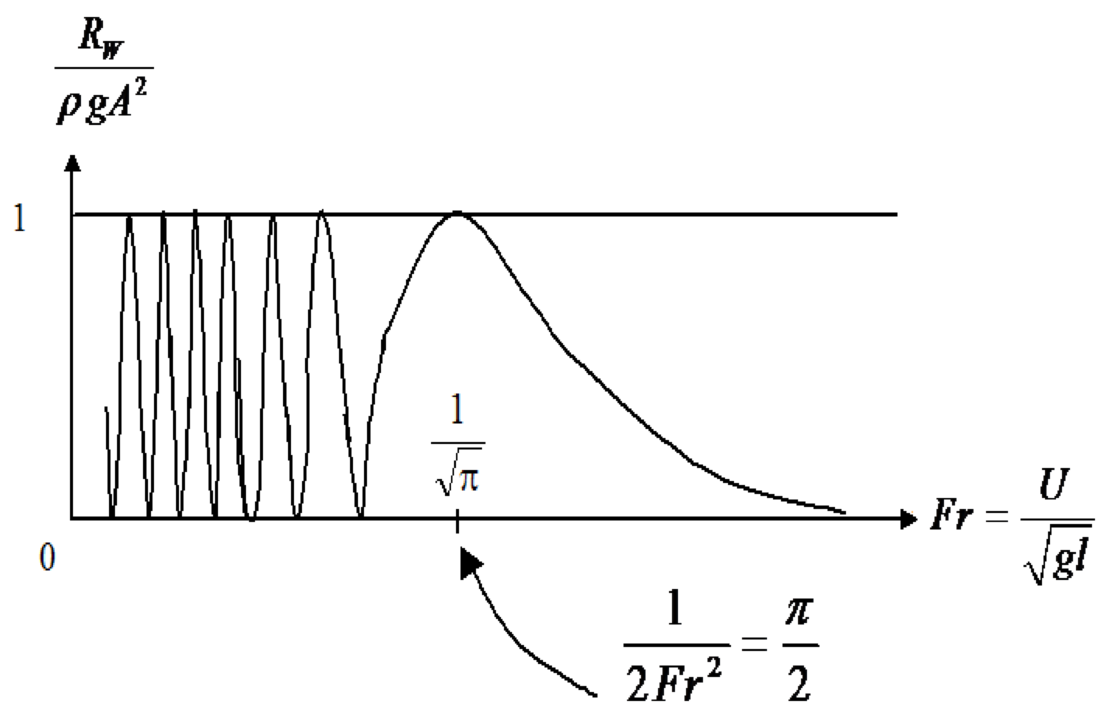
Waves behind a ship can be approximately looked as a superposition of a *bow wave* and a *stern wave*. Then, the wave resistance is expressed as

$$R_W = \rho g A^2 \sin^2\left(\frac{1}{2Fr^2}\right)$$

For relatively lower speed , if the ship is well designed, the bow wave and the stern wave may cancel out with each other and wave resistance is small. But for higher speed ship, wave resistance is unavoidable, would not vanish.

Review of Chapter 7

If *Froude number* $F_r > 0.56$, *wave resistance* is unavoidable.



$$\frac{1}{2Fr^2} = \frac{\pi}{2}$$

$$\Rightarrow \quad Fr = \frac{1}{\sqrt{\pi}}$$

$$\approx 0.56$$

$$\Rightarrow \quad U \approx 0.56\sqrt{gl}$$

$$\cong 0.56\sqrt{gL}$$

Chapter 8

Fundamental Theory of Viscous Incompressible Fluid Flow

8.1 Viscous Flow Phenomena

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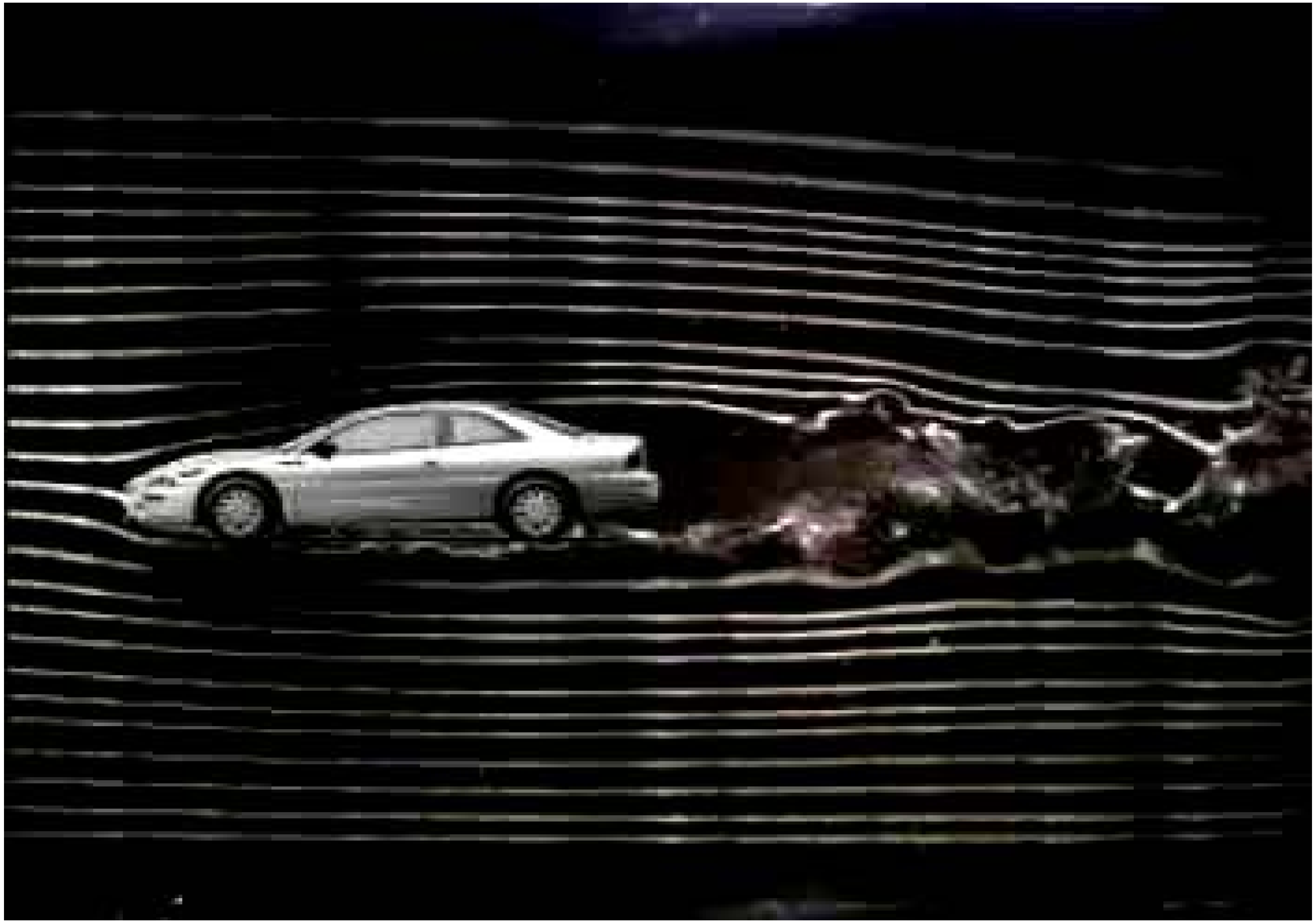

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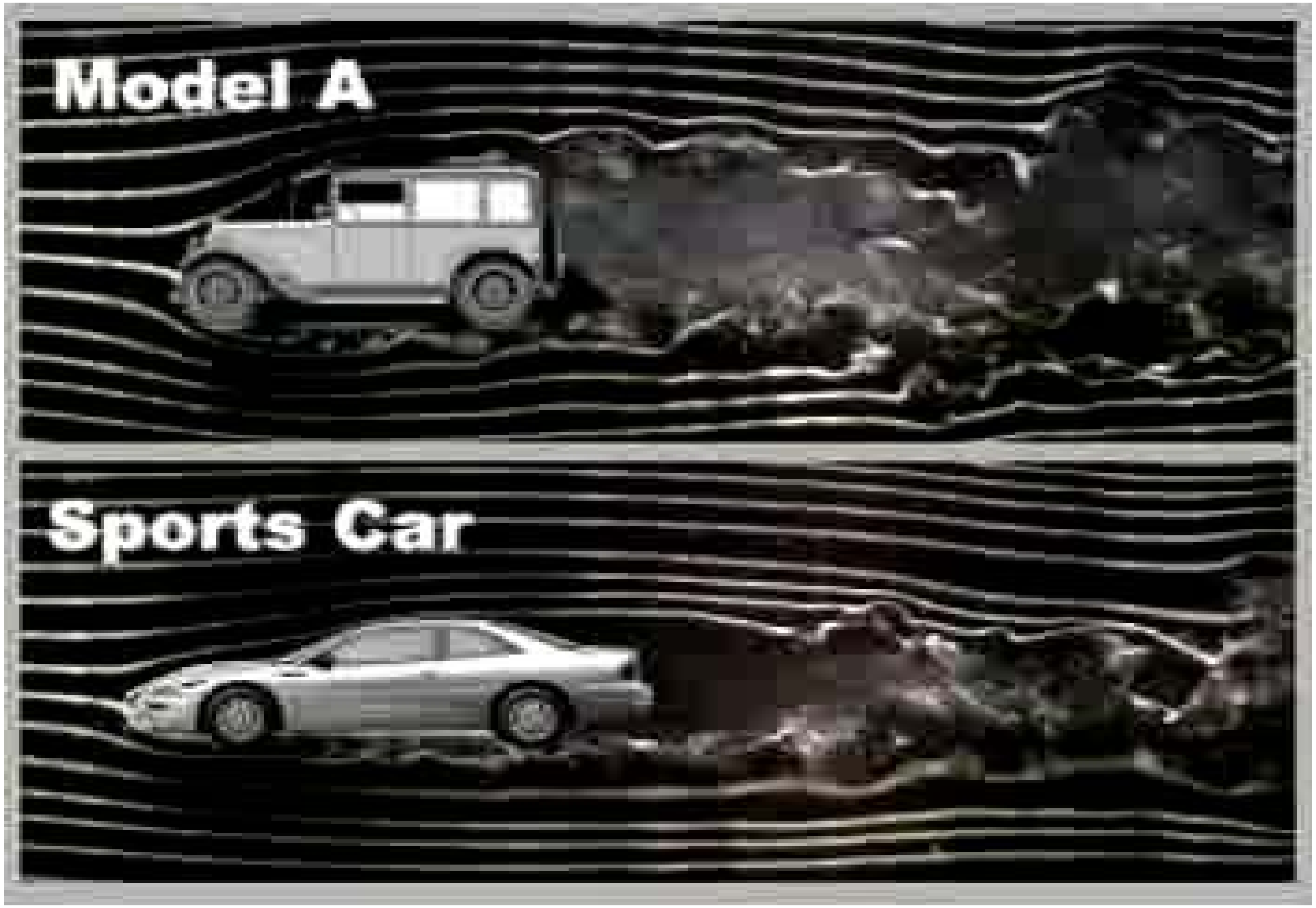

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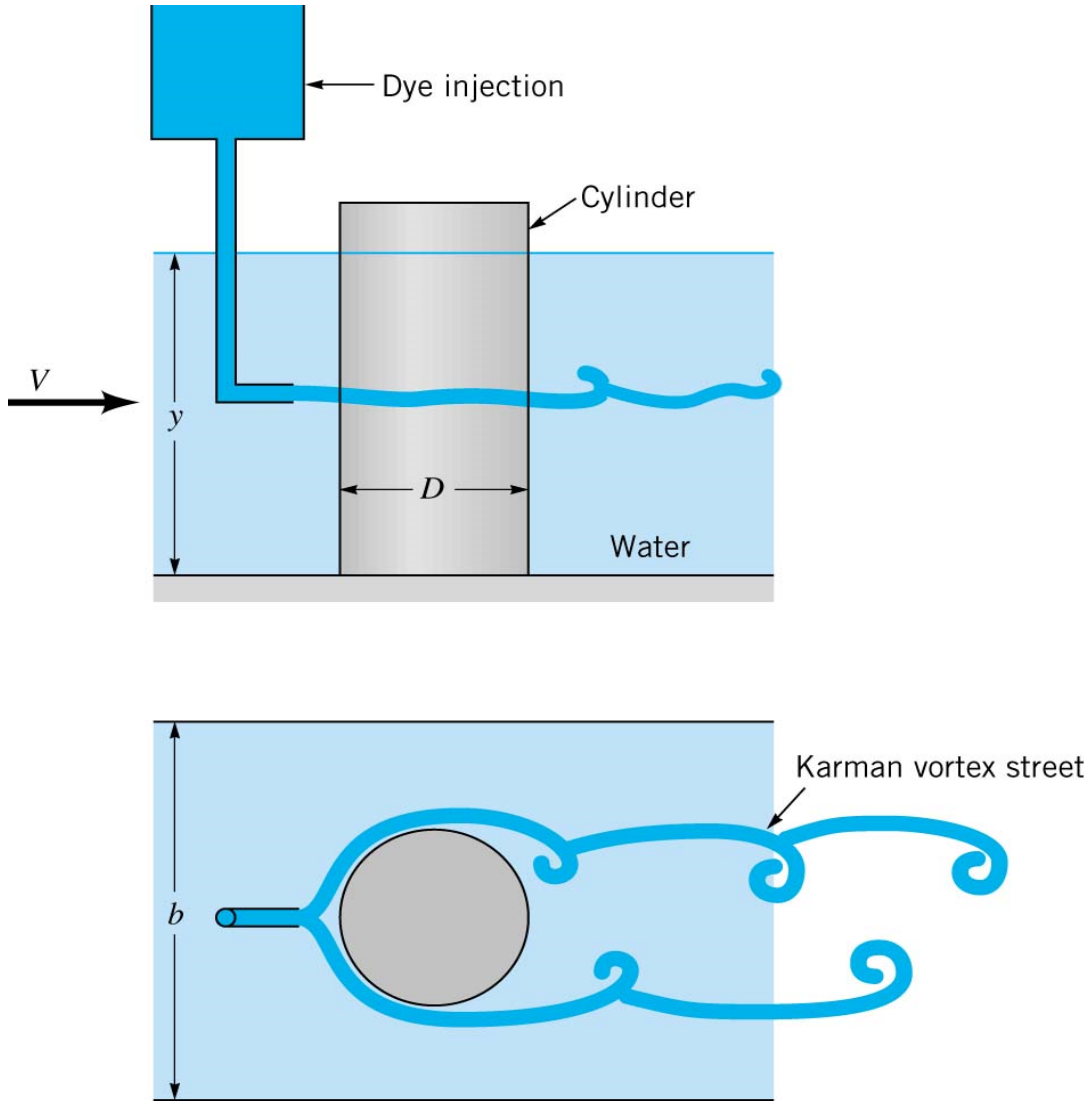

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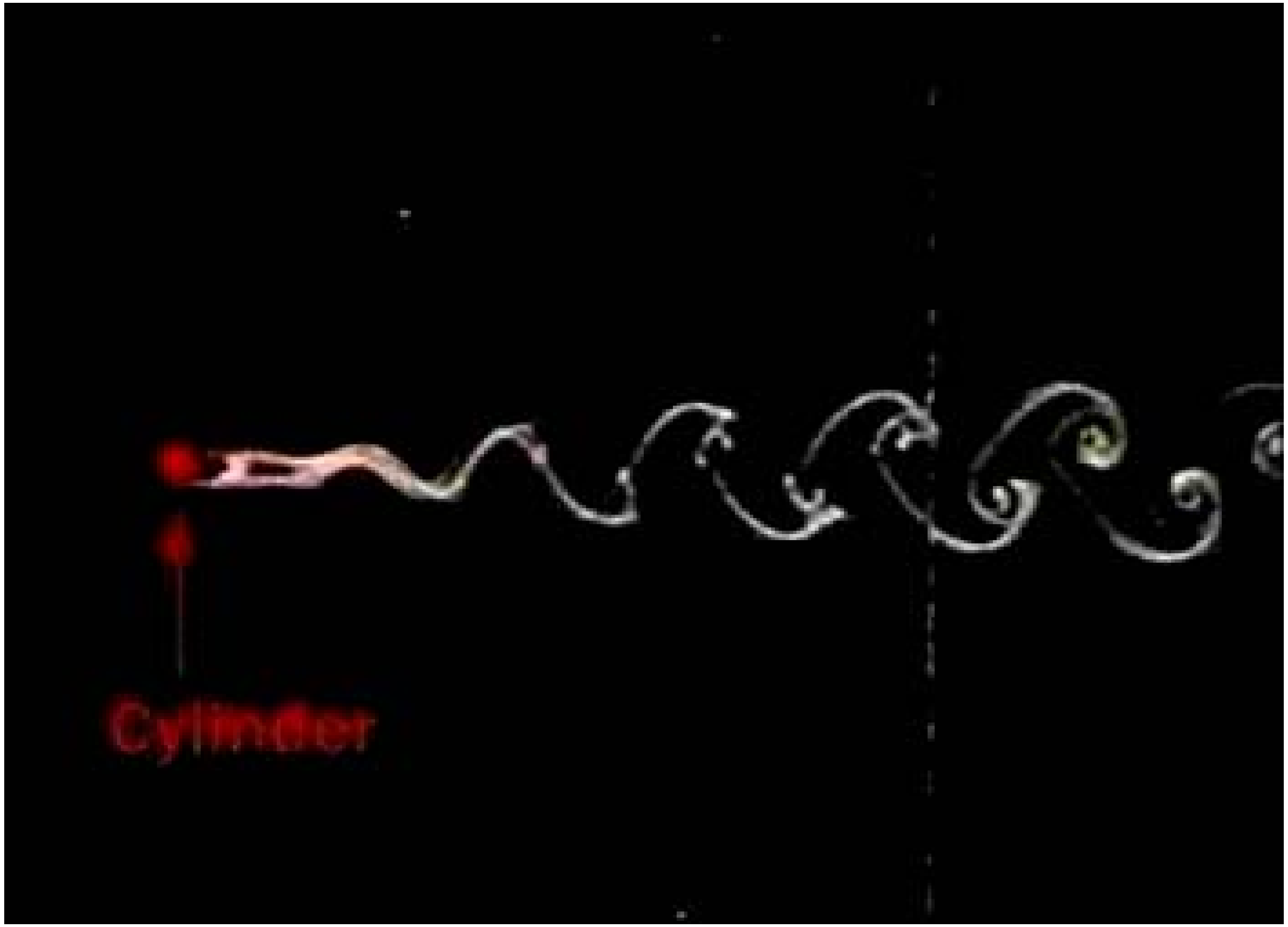

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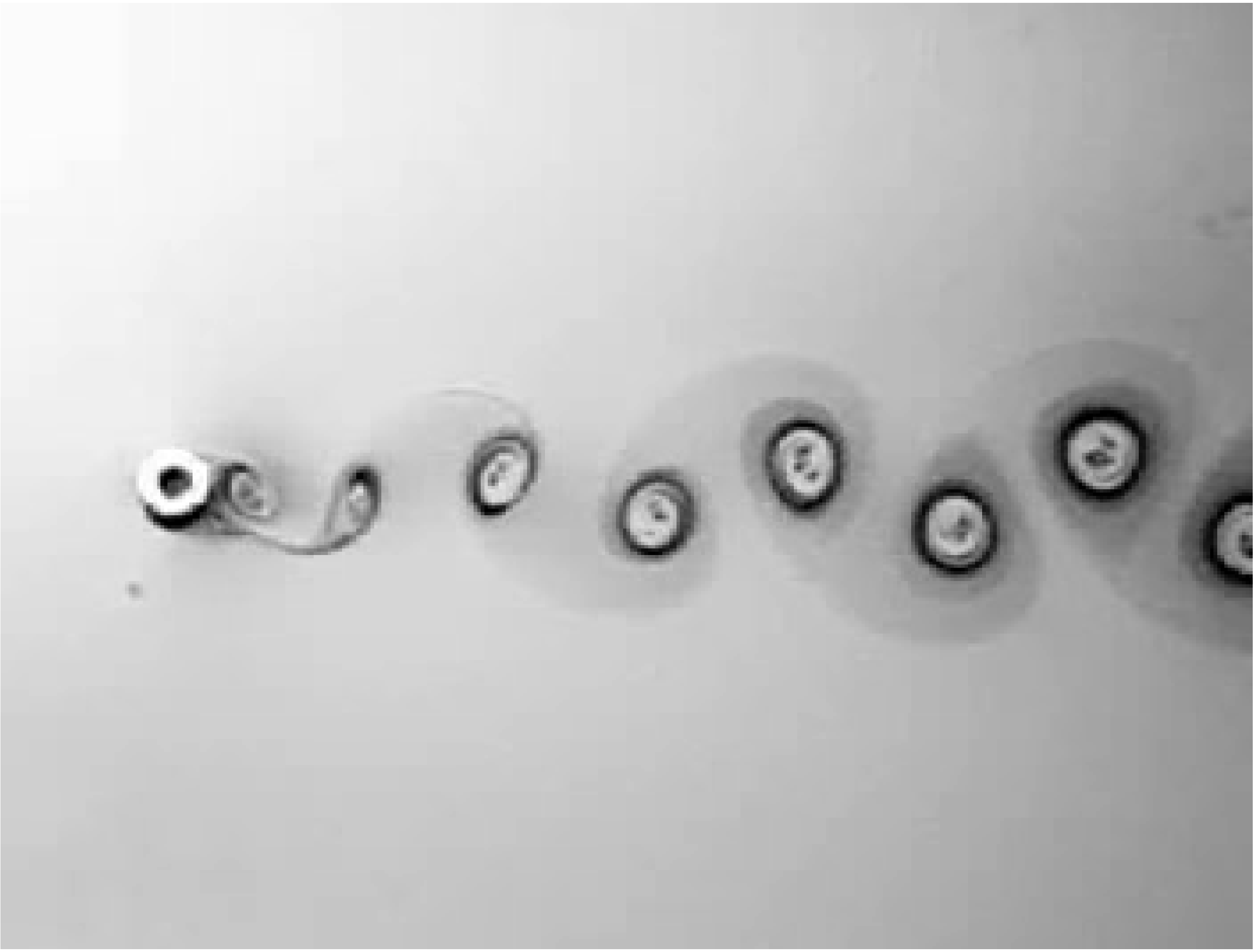

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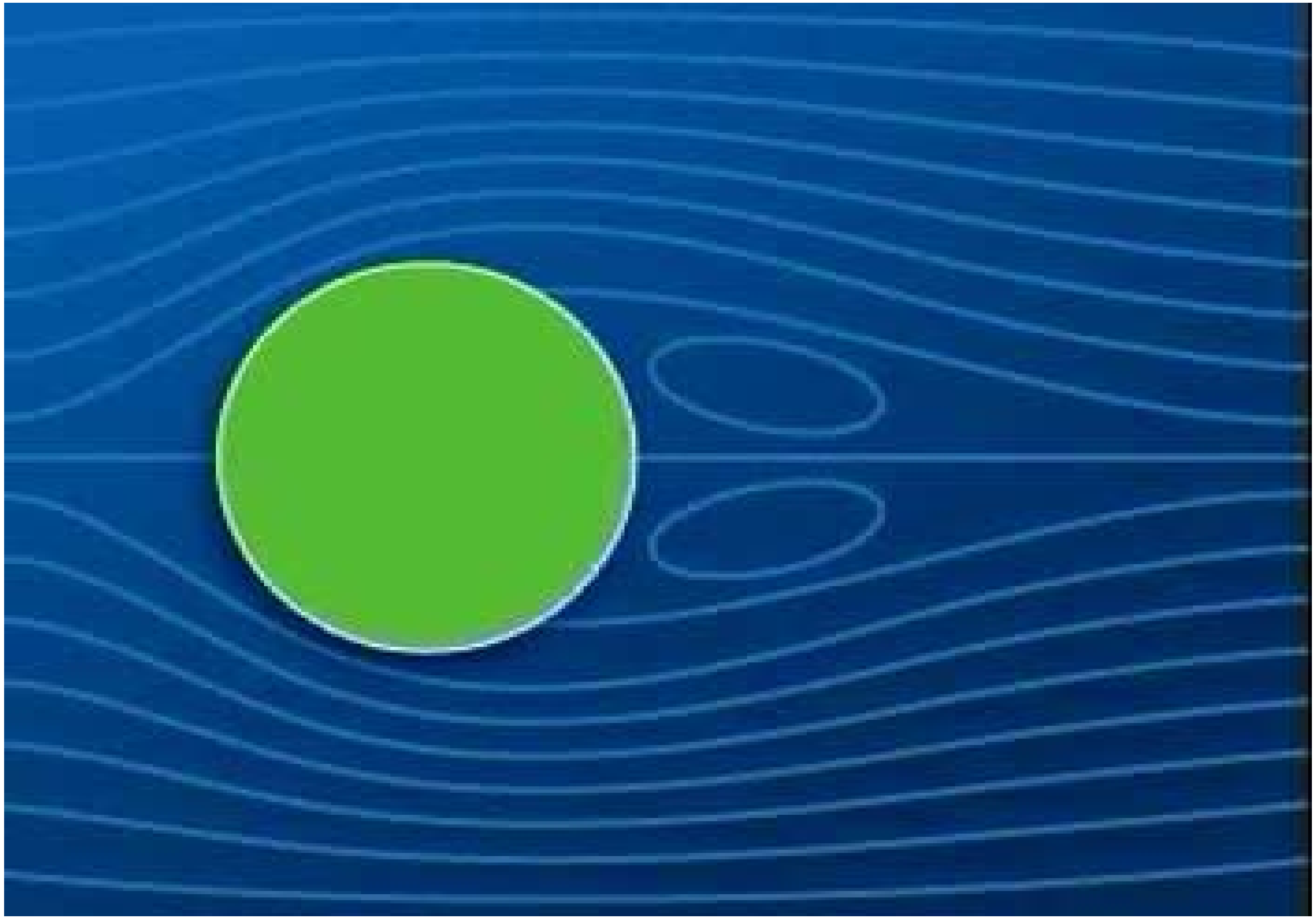

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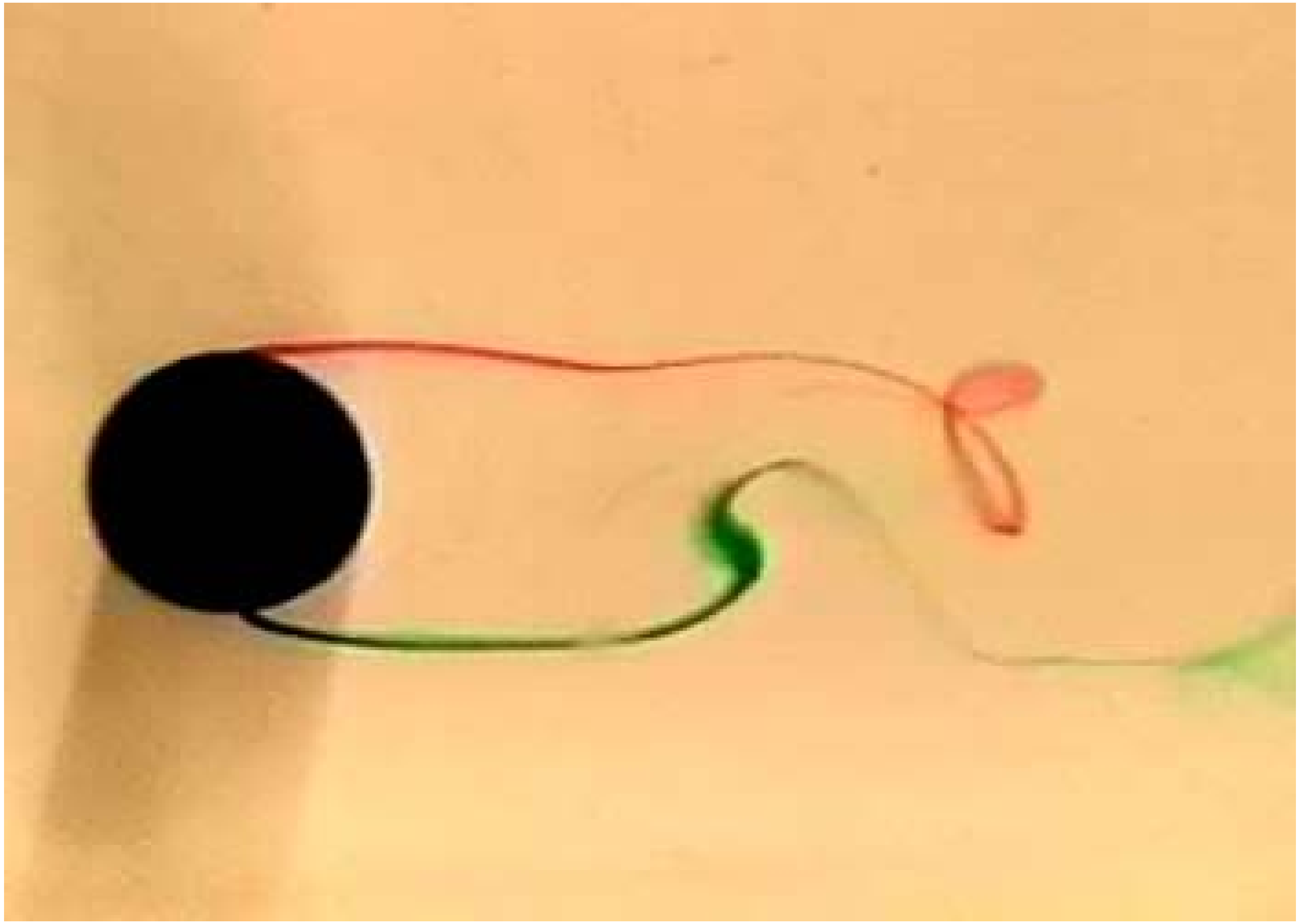

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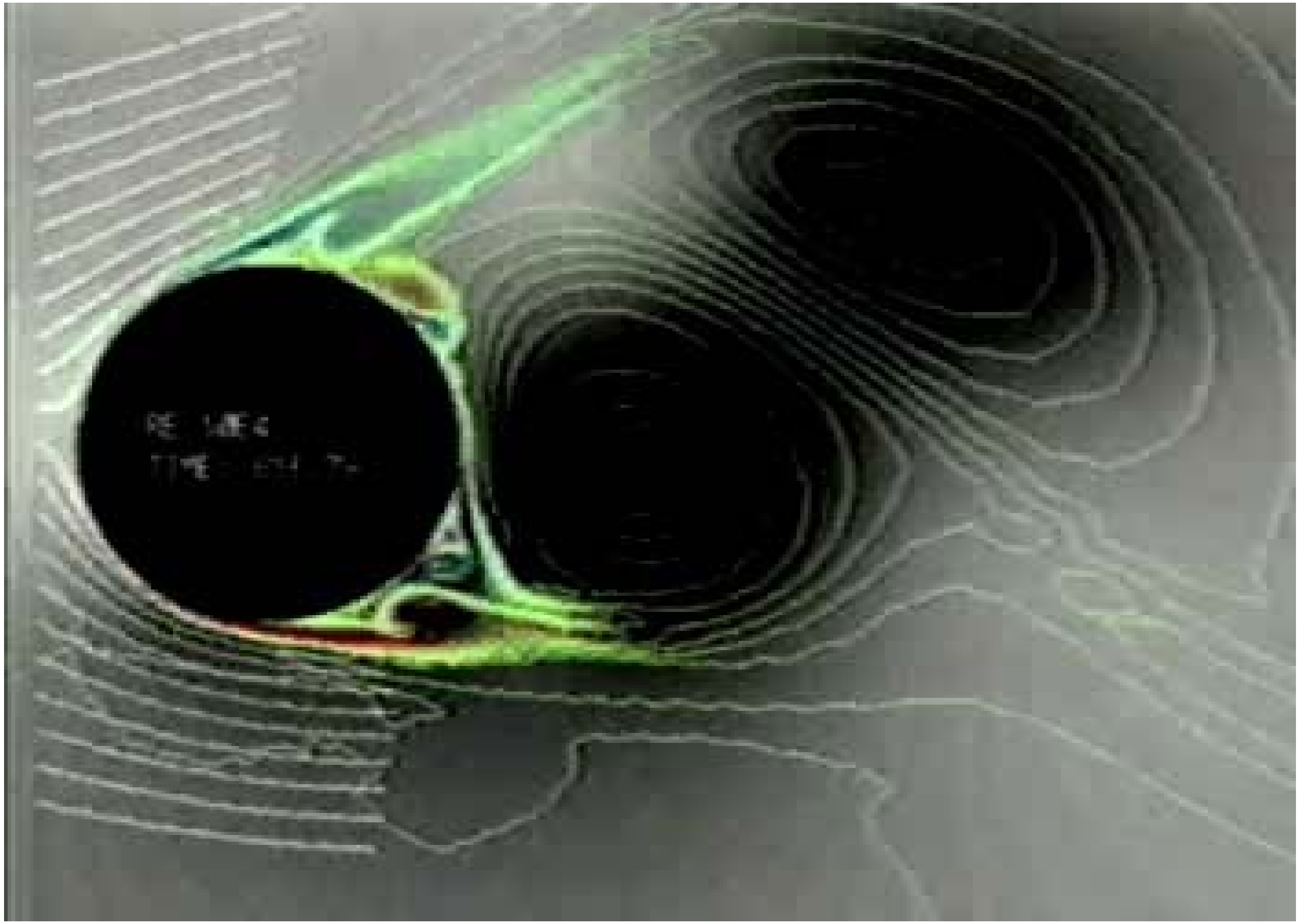

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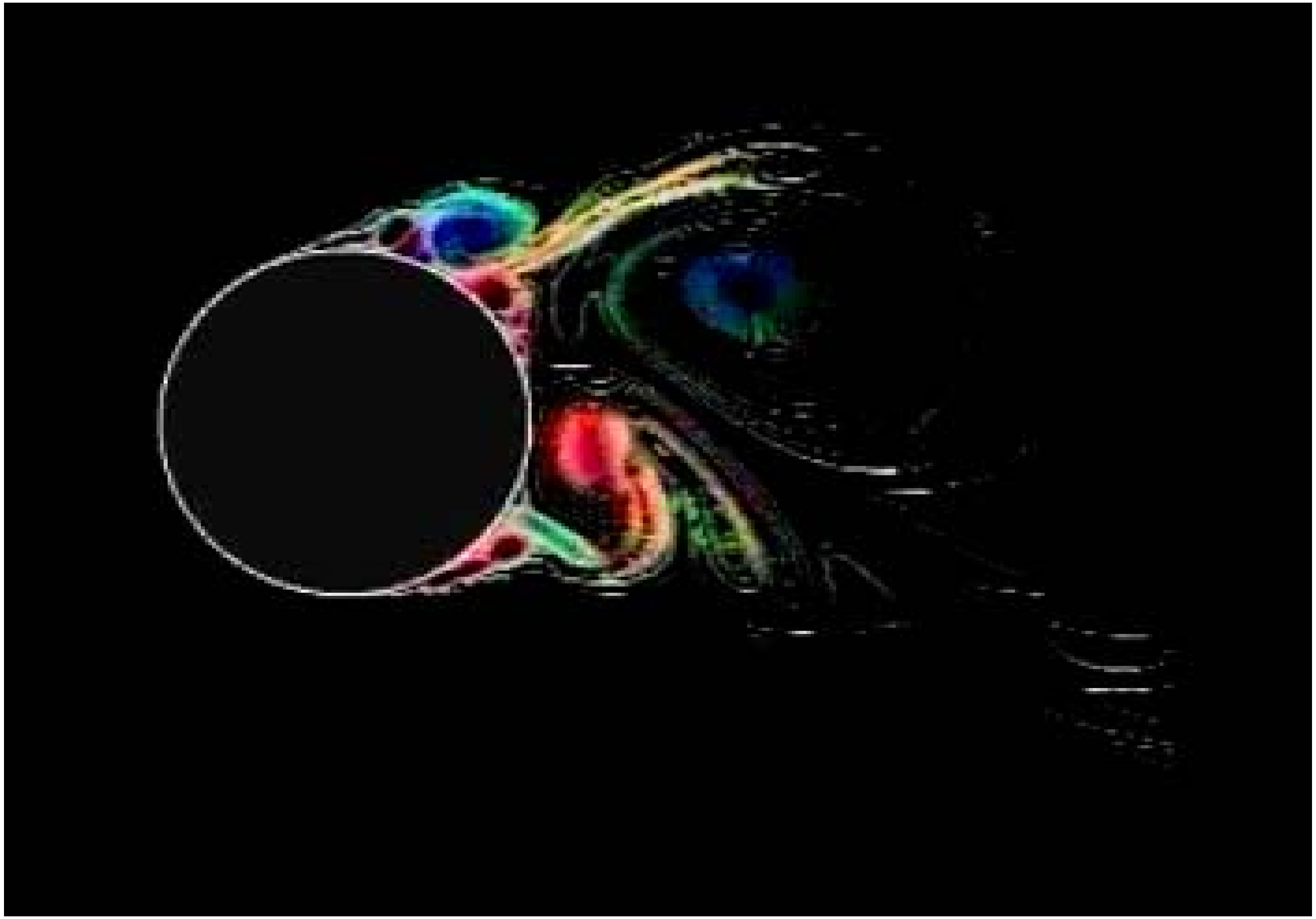

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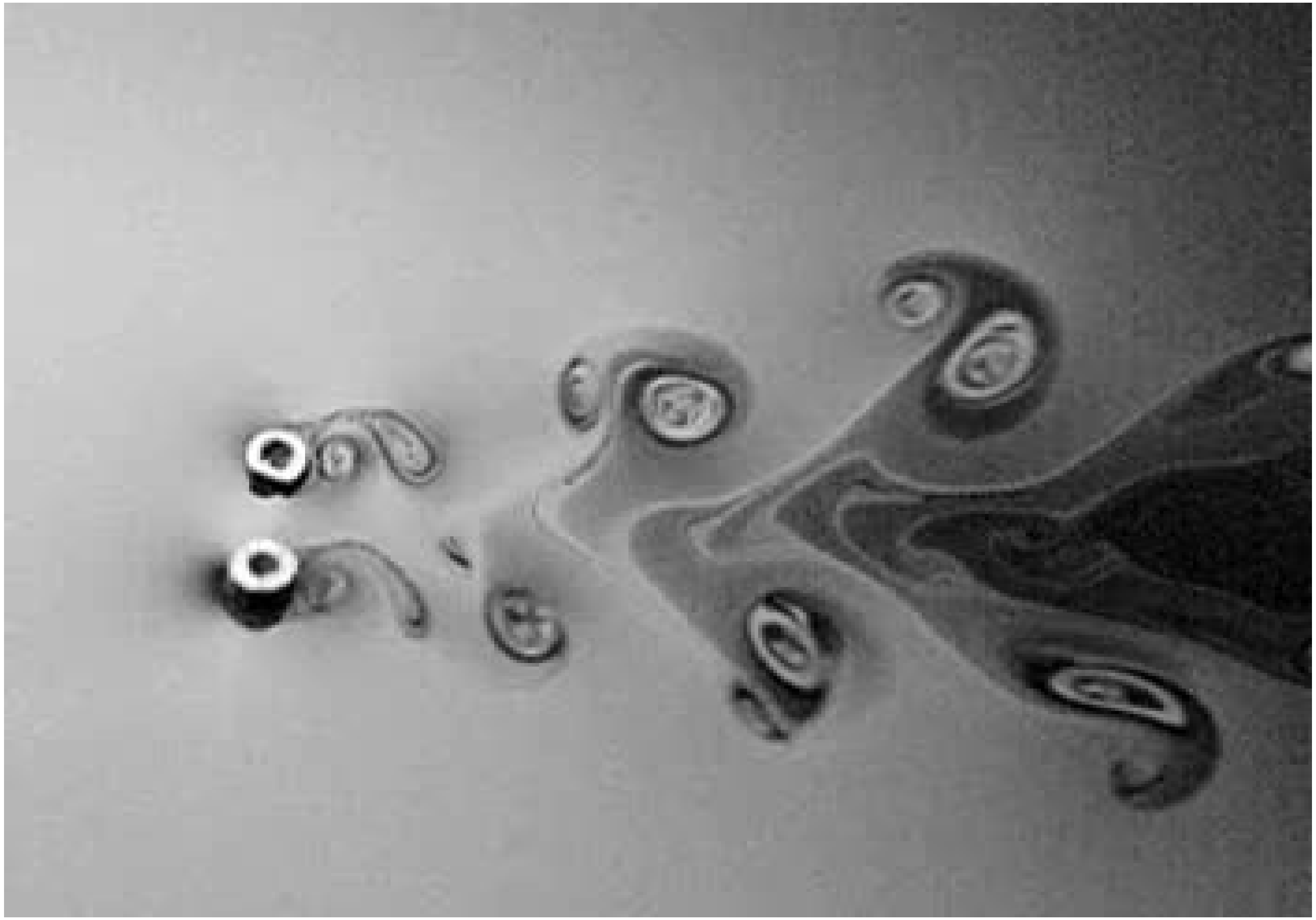

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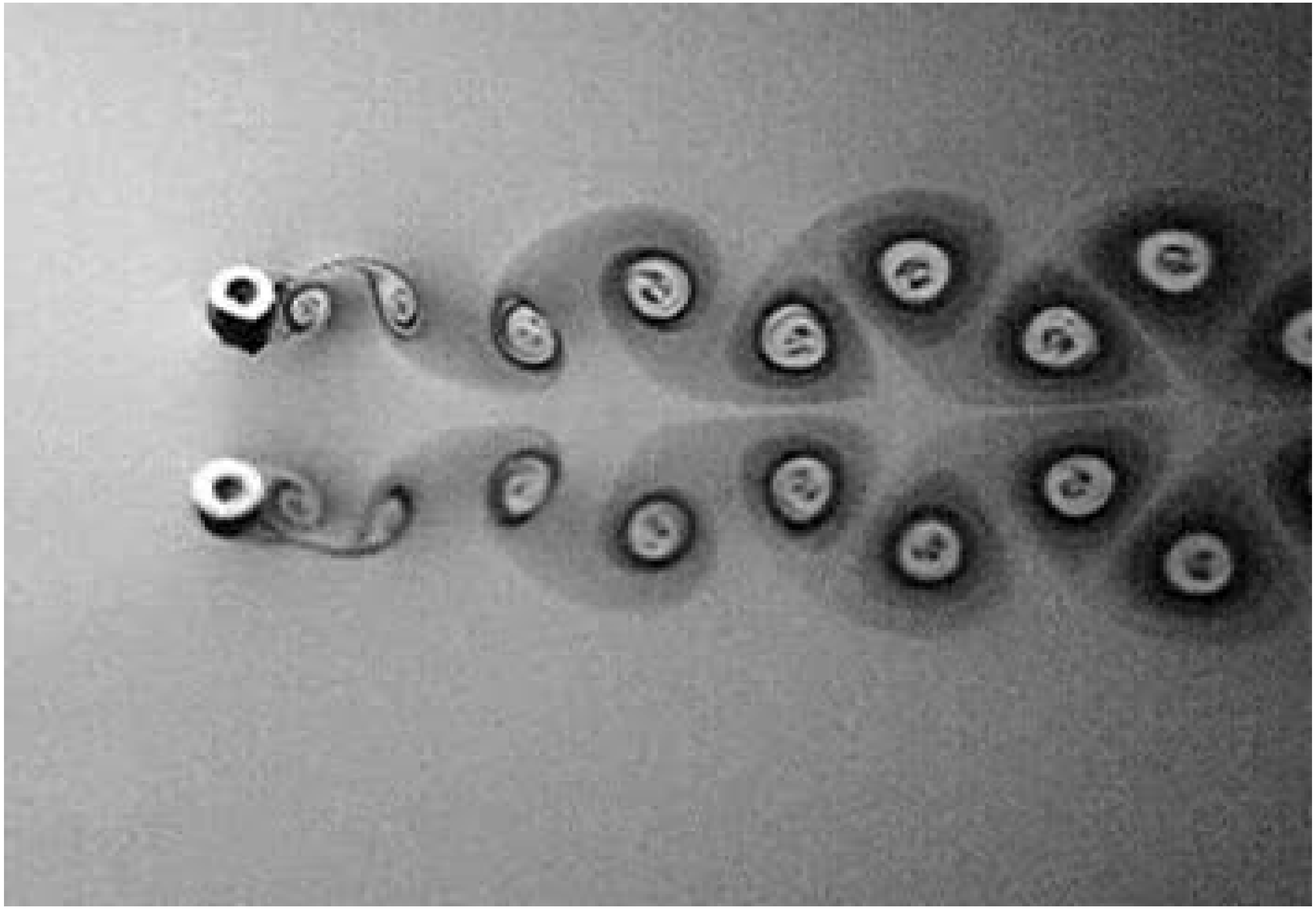

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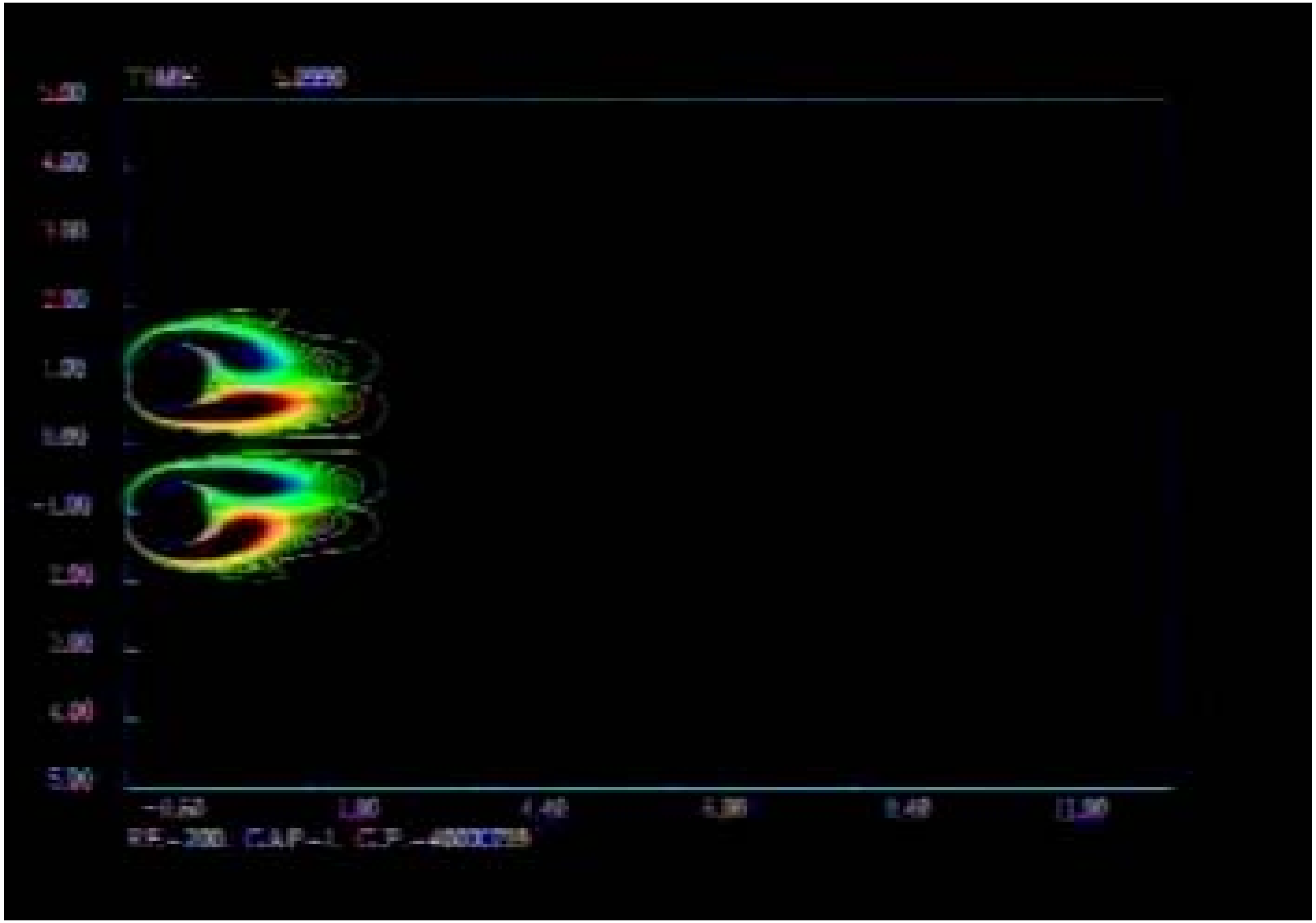

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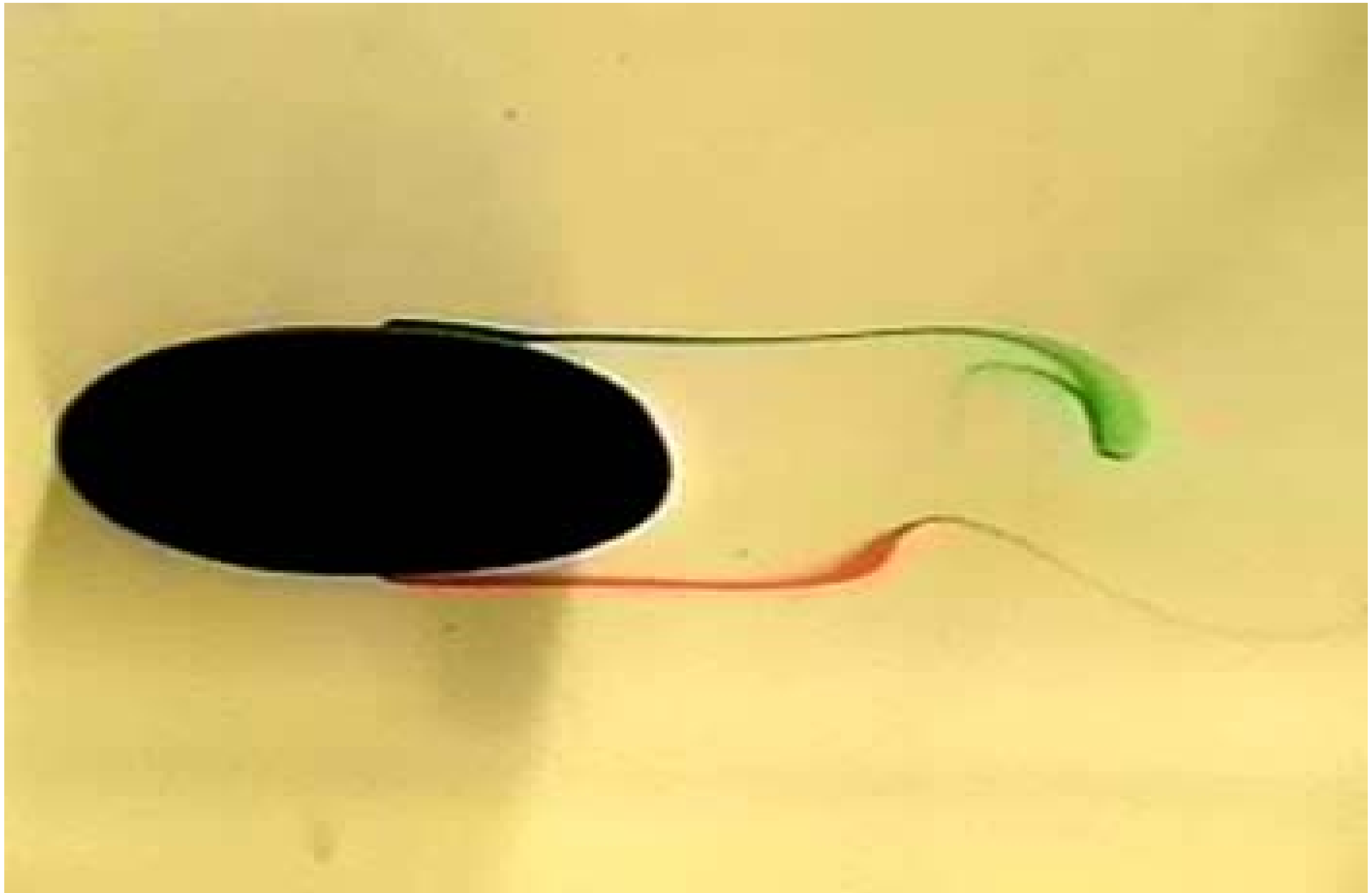

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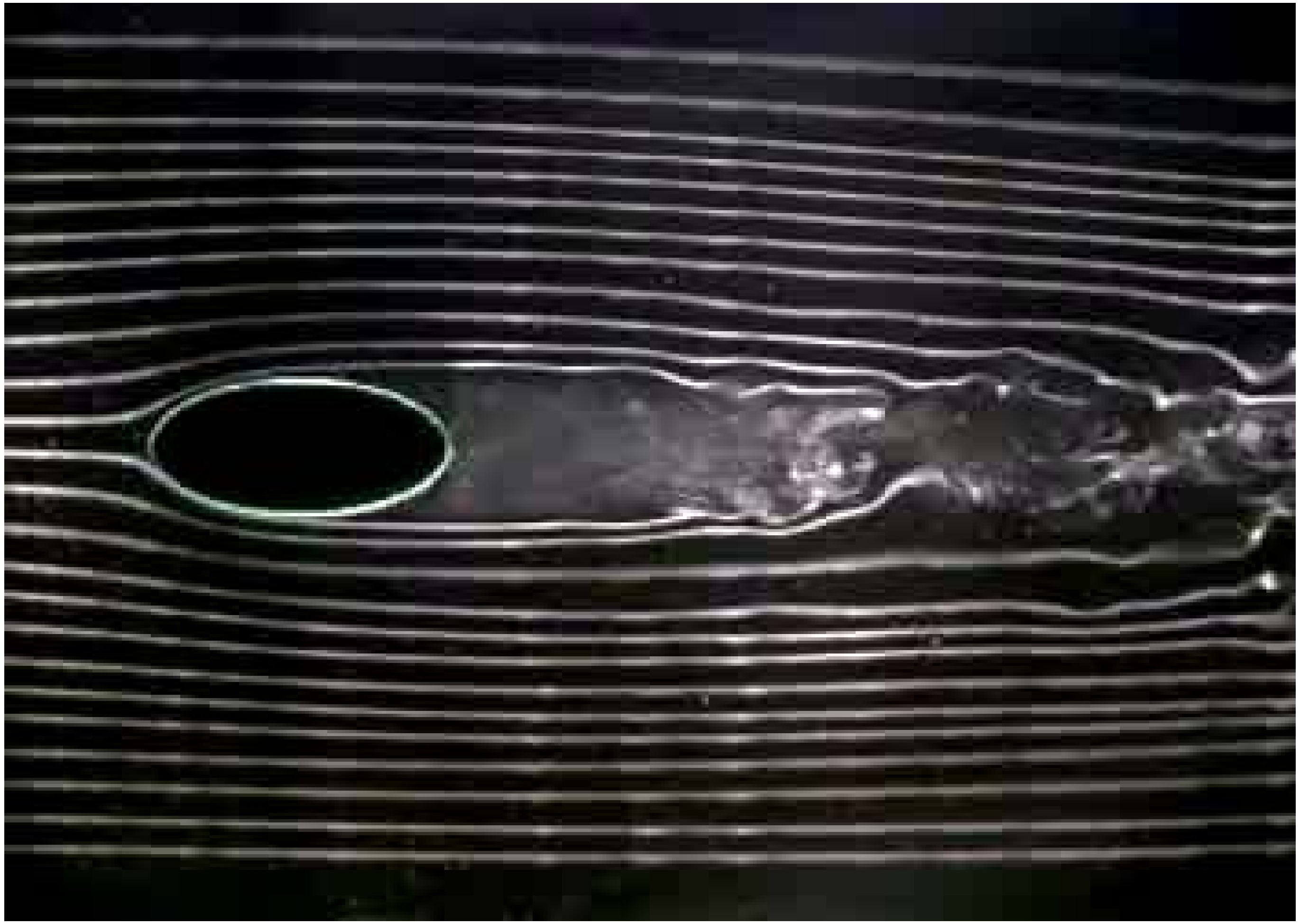

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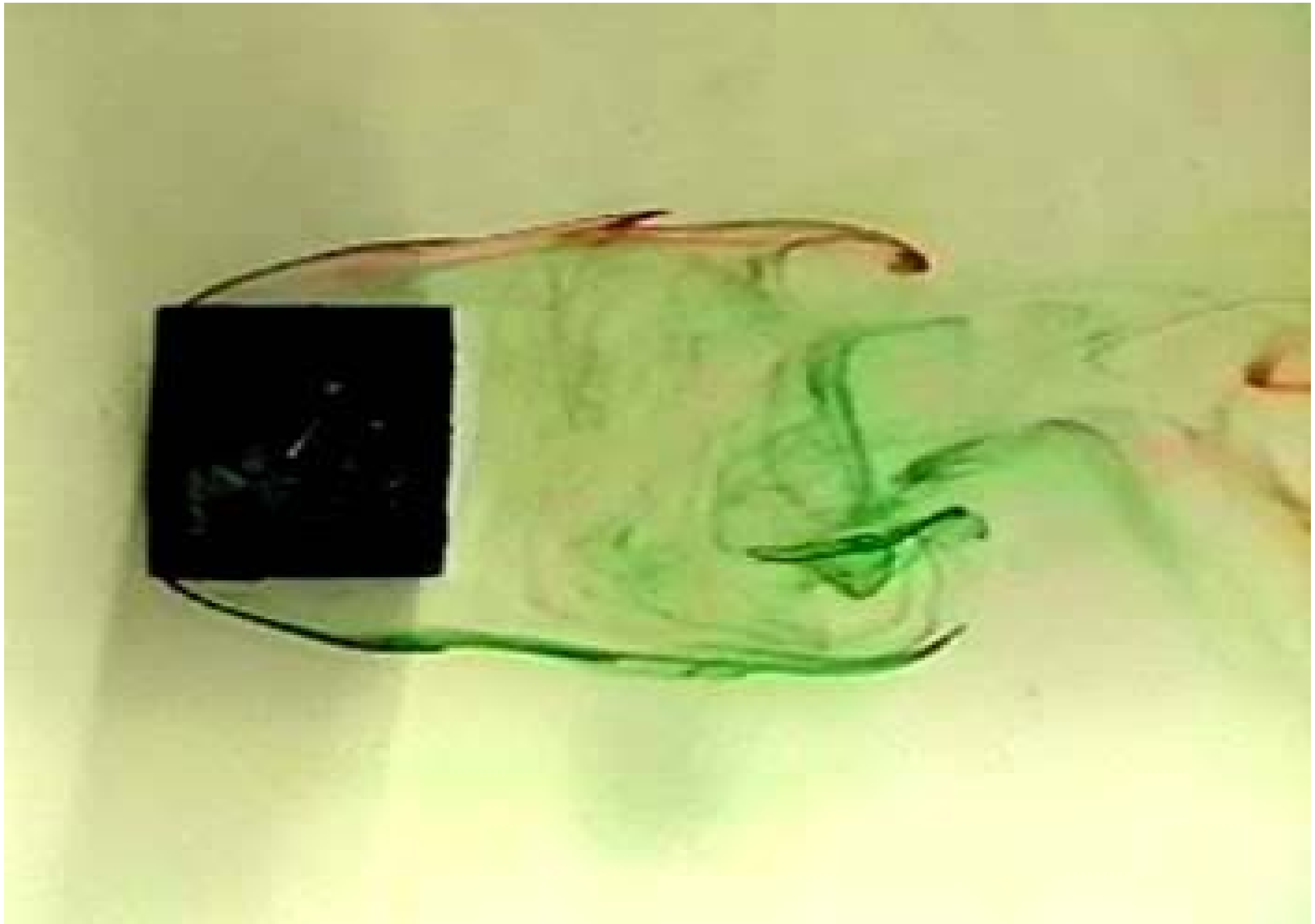

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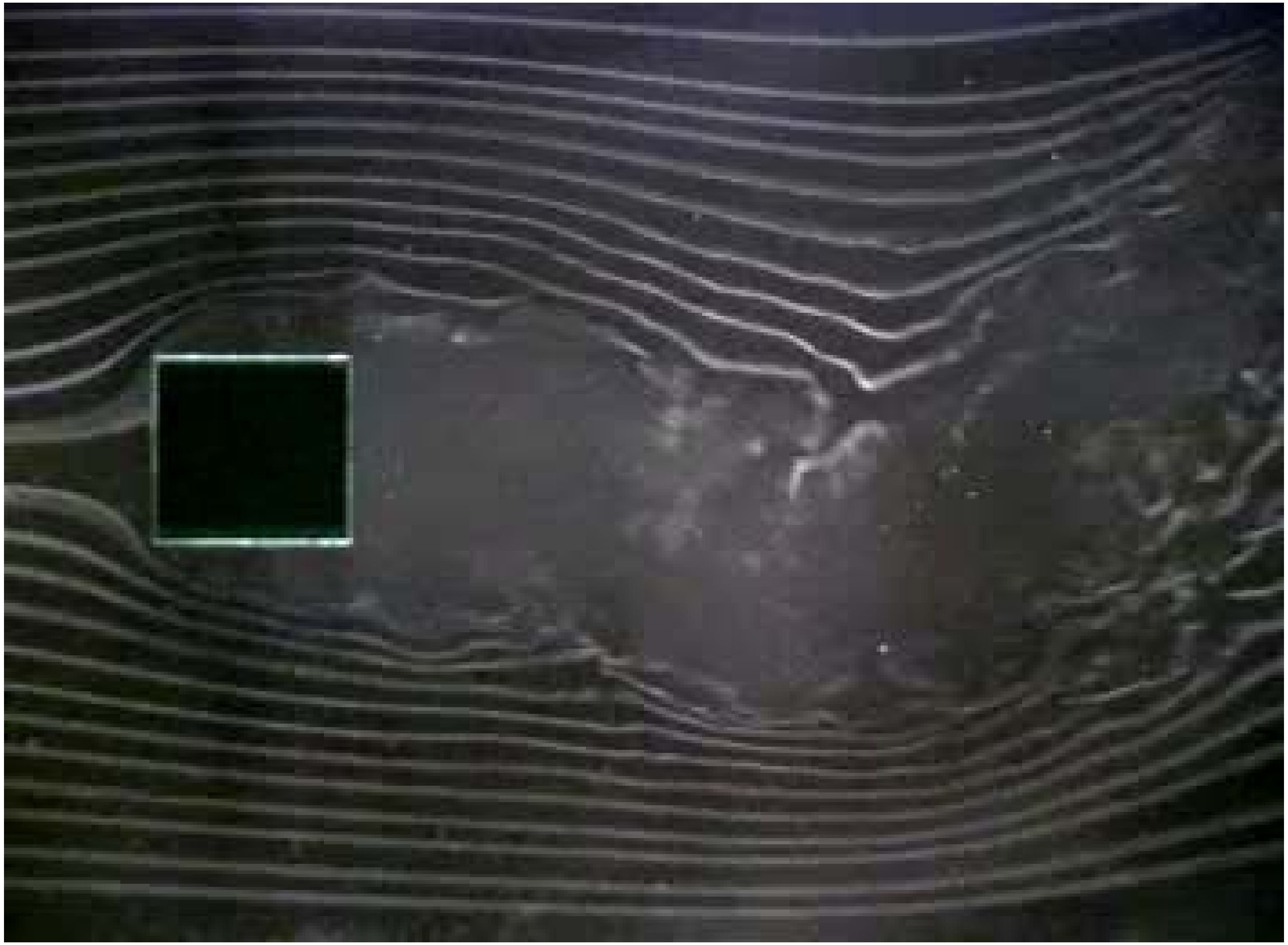

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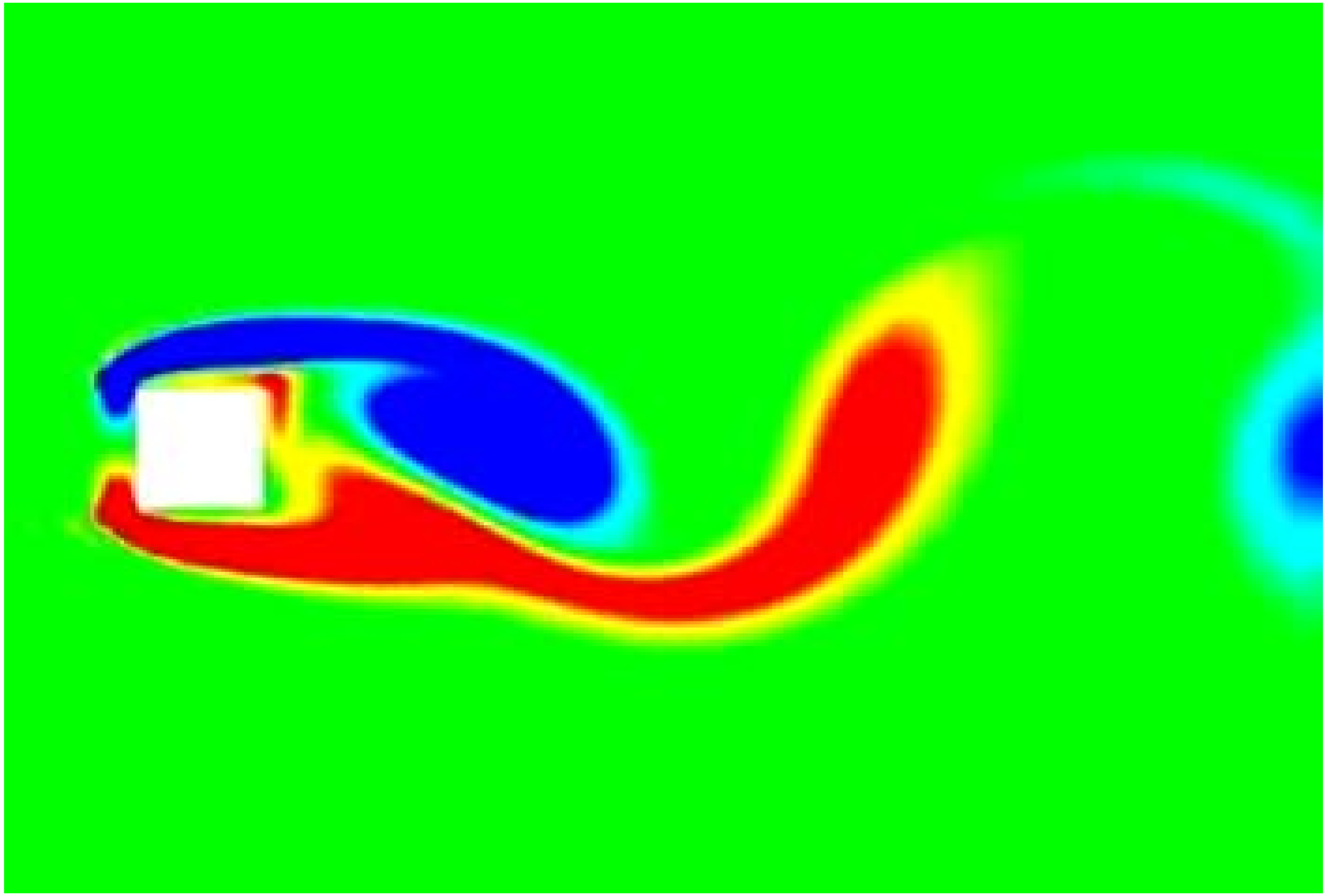

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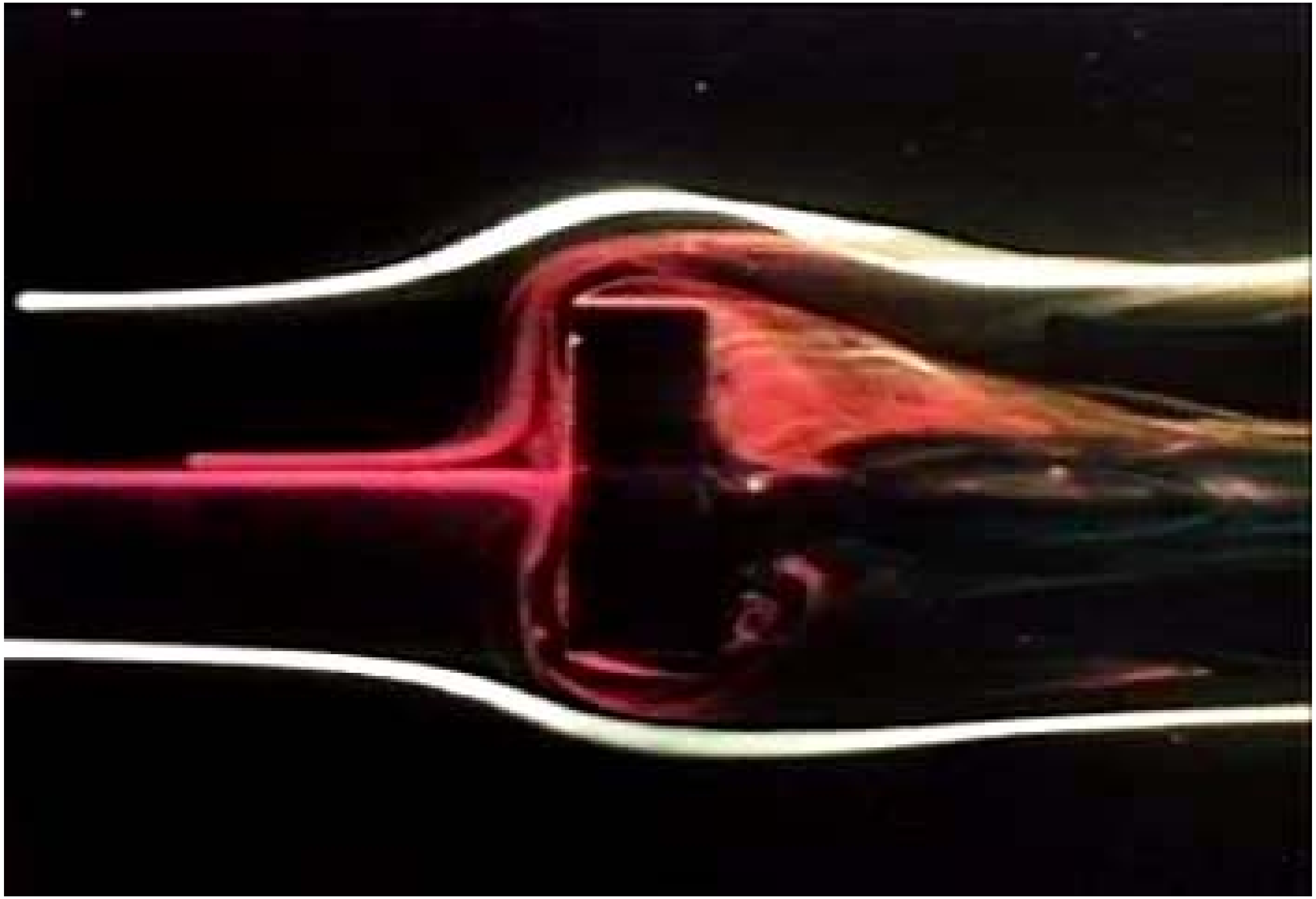

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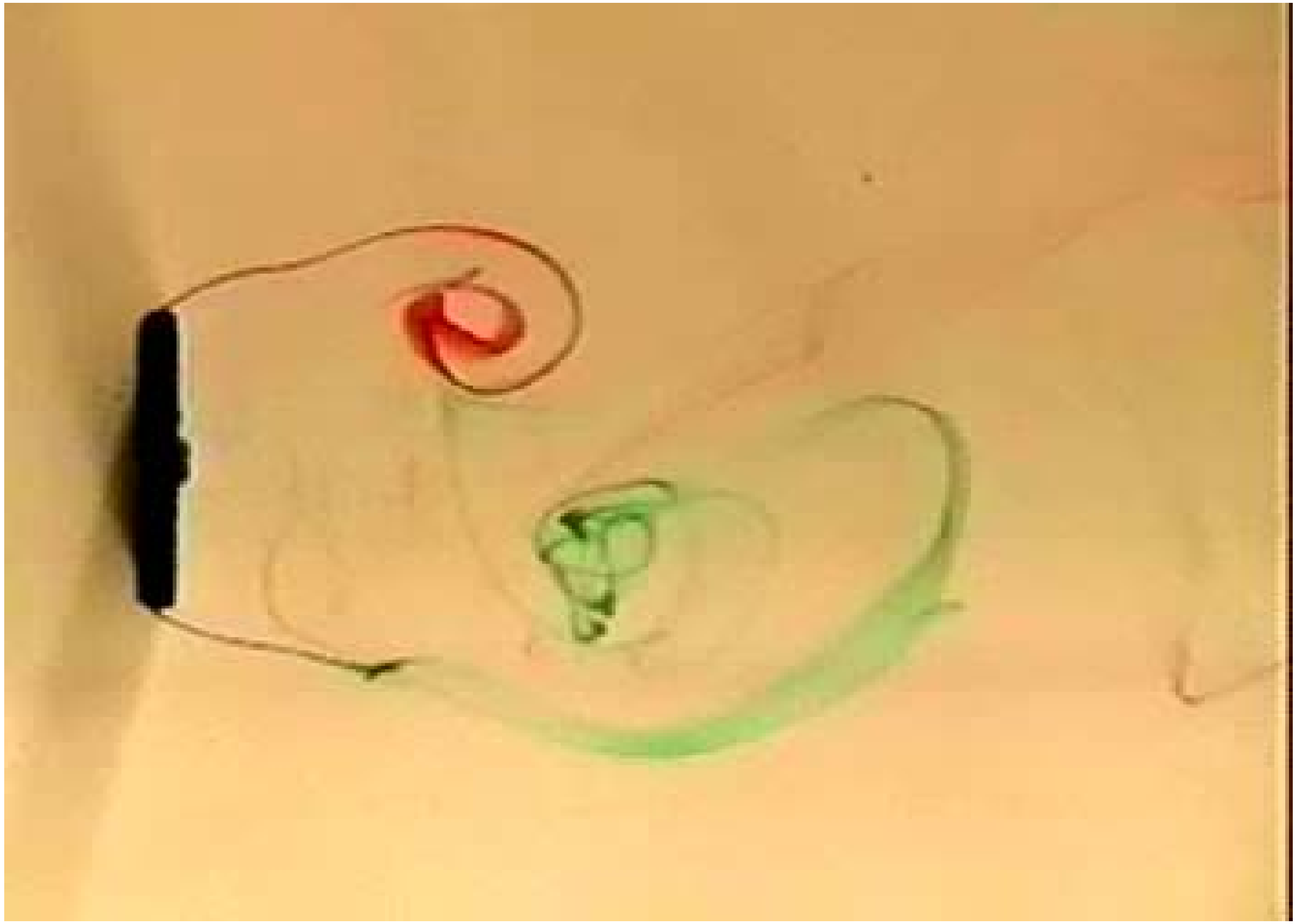

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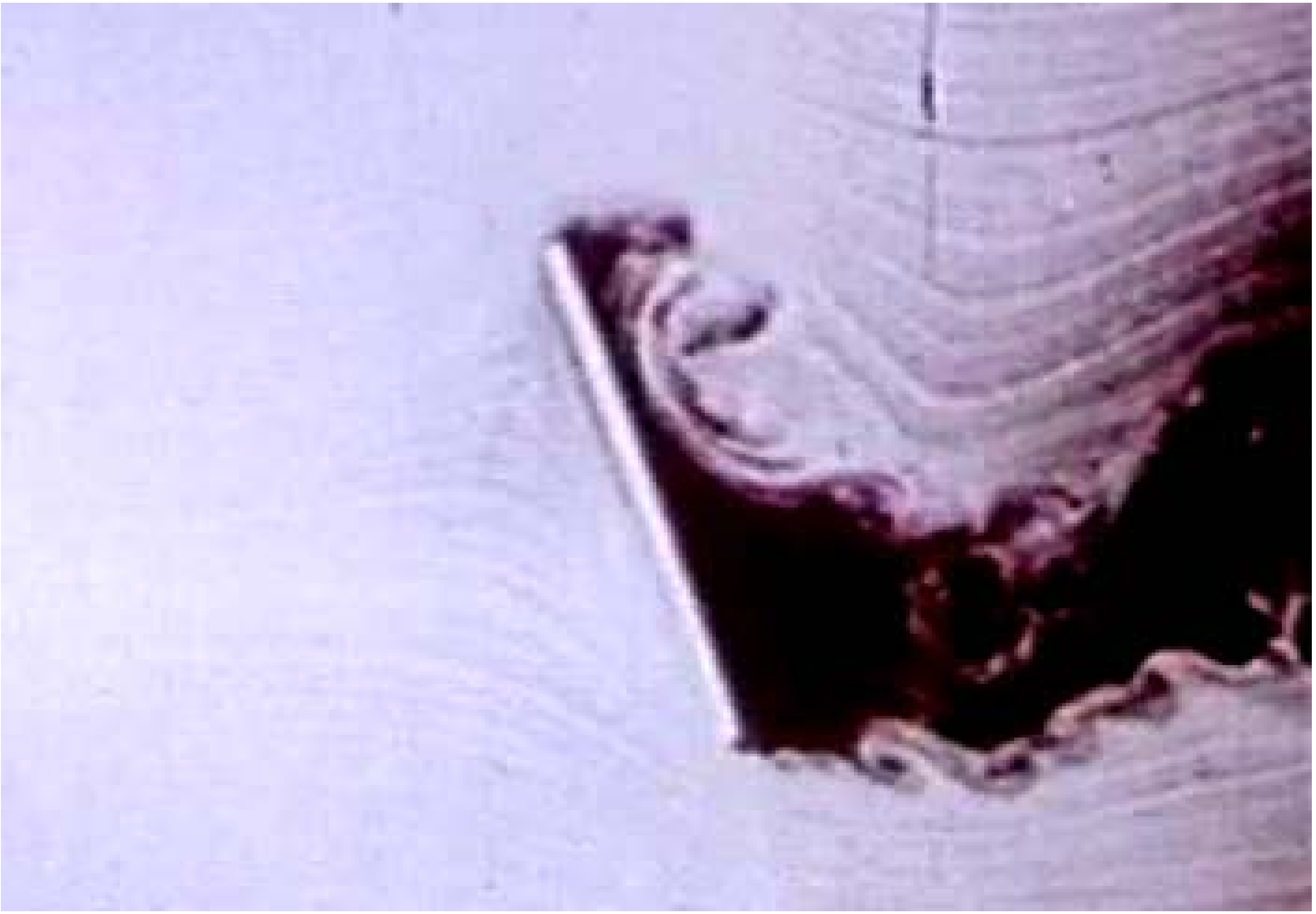

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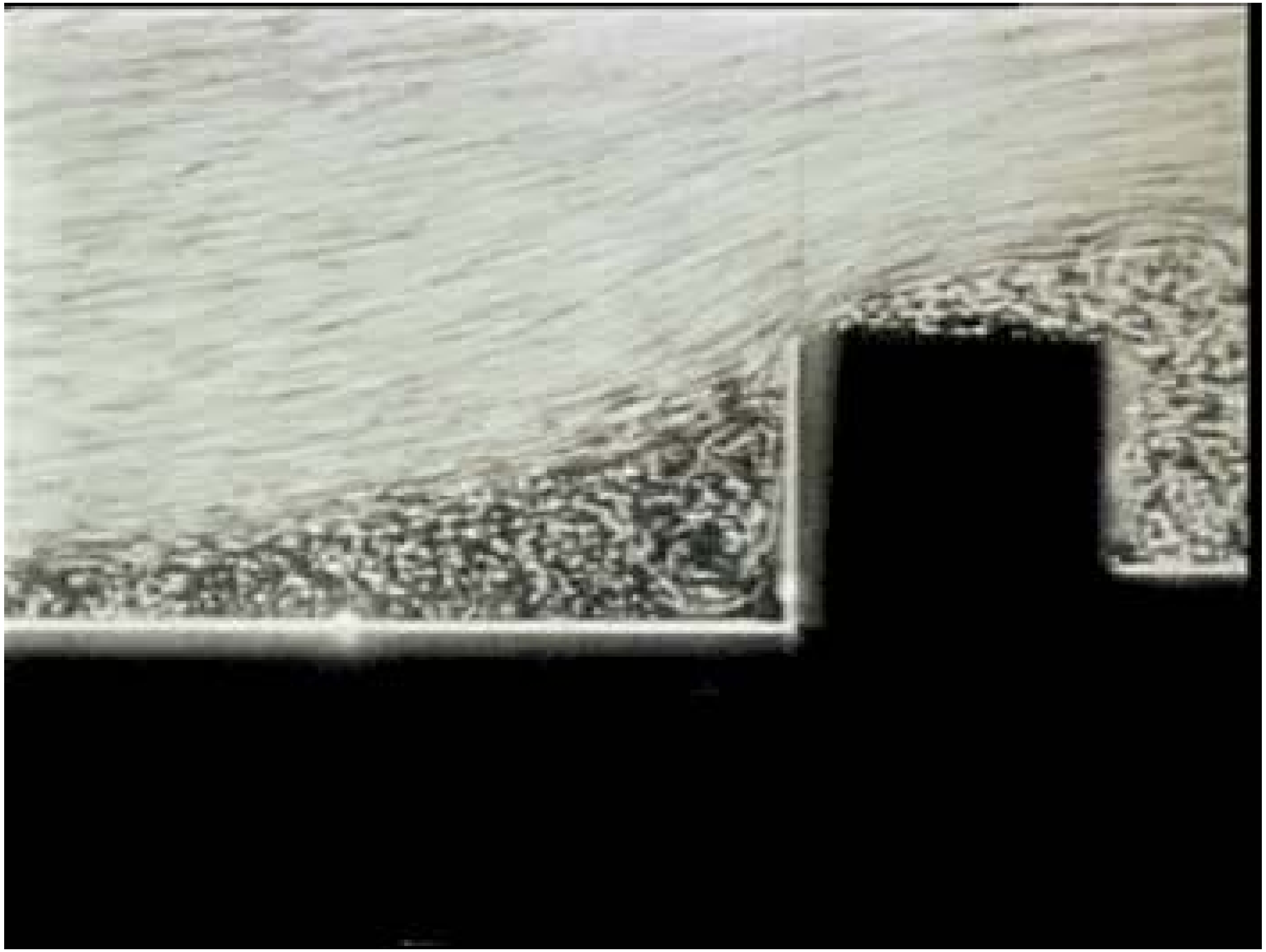

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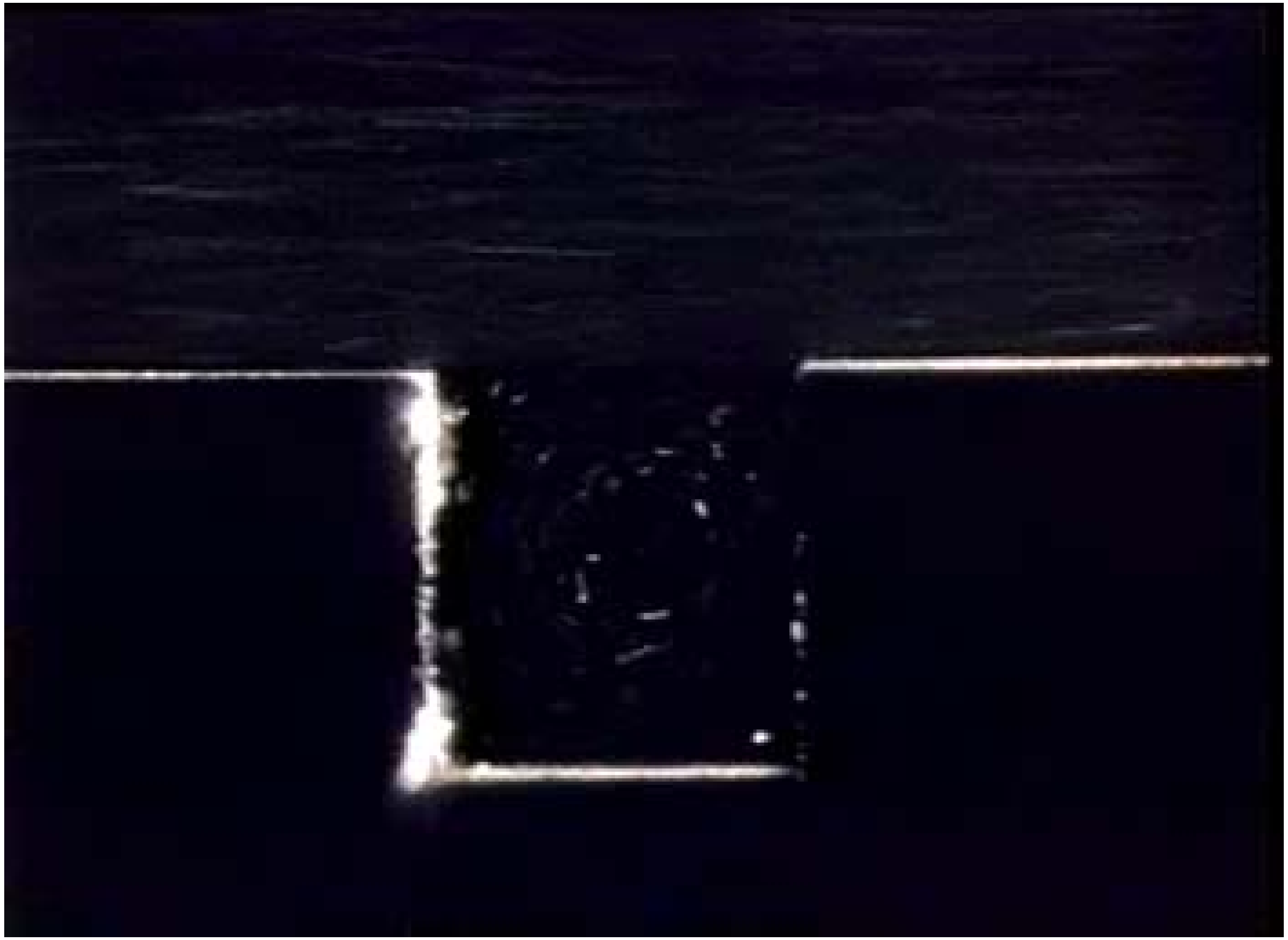

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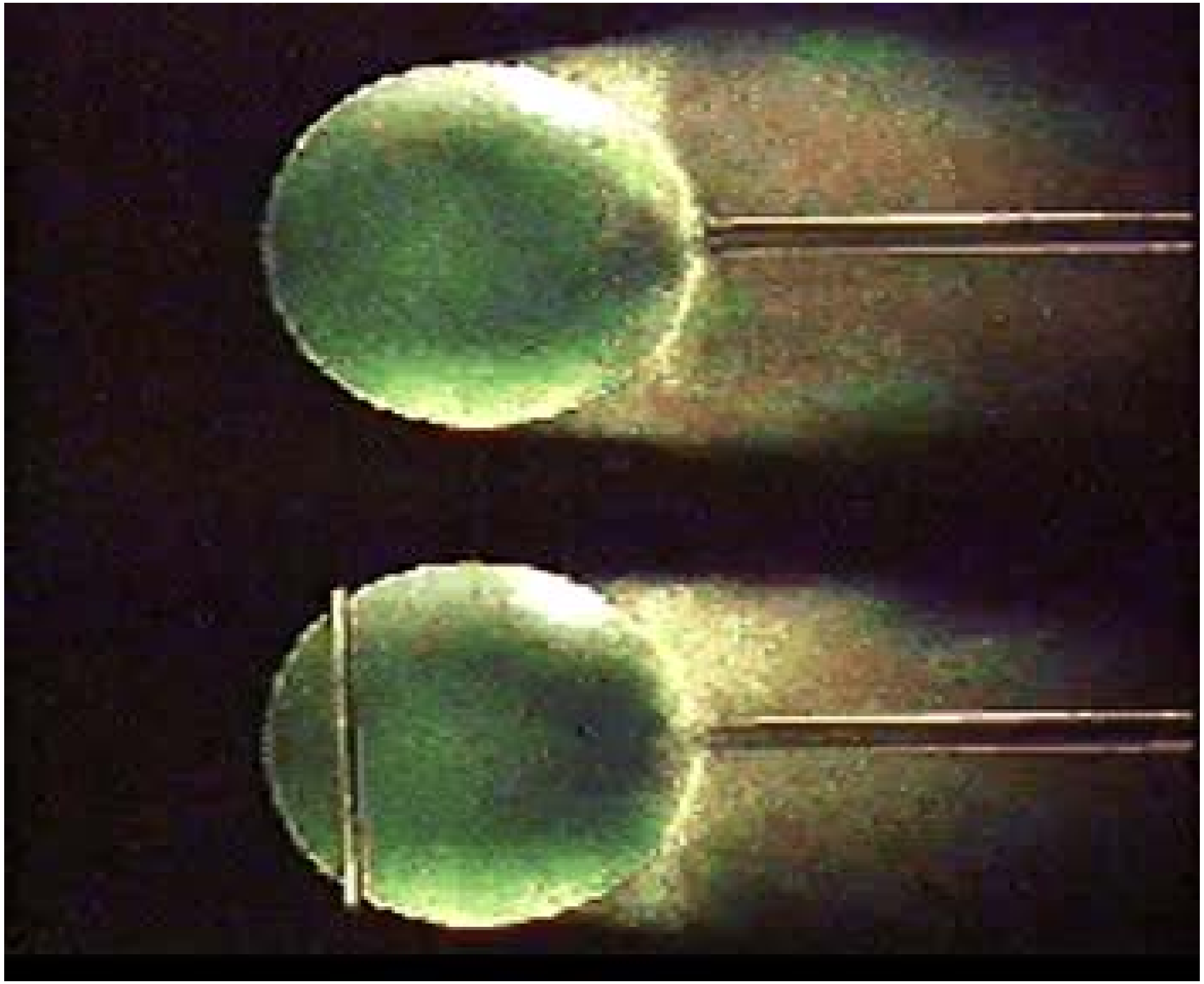

8.1 Viscous Flow Phenomena

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It can be concluded that viscous flows have characters as follows.

1) **Rotational:** vorticity may be non-zero

2) **Dissipation:** mechanical energy may be changed to other energy

3) **Diffusion:** physical quantity may diffuse due to its gradient

4) **Unsteady:** physical quantity may vary with time

5) **Unstable:** physical quantity may change essentially due to some small disturbance

6) **Random:** physical quantity may change in a random fashion and become indeterminable