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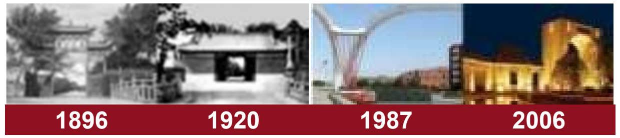


Introduction to Marine Hydrodynamics

(NA235)

Department of Naval Architecture and Ocean Engineering

School of Naval Architecture, Ocean & Civil Engineering

Shanghai Jiao Tong University

Chapter 5
Vorticity Dynamics

上海交通大学 Shanghai Jiao Tong University

5.11 Induced Velocity Field

For a fluid at rest, to make it in motion, there should be either source/sink in the flow field (divergence of velocity is not 0, $\nabla \cdot \mathbf{V} \neq 0$), or vortex in the flow field (vorticity is not 0, $\nabla \times \mathbf{V} \neq 0$). Therefore, source/sink and vortex are two factors to induce the motion of fluid.

In a mathematical sense: if there is divergence field or vorticity field in the flow field, then the velocity field **V** of a fluid can be uniquely determined, i.e.,

$$\left.\begin{aligned} &\nabla \cdot \mathbf{V} = H(x, y, z, t), && \text{divergence field} \\ &\nabla \times \mathbf{V} = \mathbf{\Omega}(x, y, z, t), && \text{vorticity field} \\ &\mathbf{V} \cdot \mathbf{n}\big|_{\Sigma} = \mathbf{U}_{\Sigma}(x, y, z, t), && \text{boundary condition on solid surfaces} \end{aligned}\right\} \Rightarrow$$

Determine the velocity field **V** uniquely

5.11 Induced Velocity Field

Verification of the uniqueness: Suppose there are two velocity fields V_1 and V_2, both satisfying equations of divergence field, vorticity field and boundary conditions. Verify that $V_1=V_2$.

Verification: Let $V = V_1 - V_2$, then V satisfies:

$$\nabla \cdot \mathbf{V} = 0, \quad \nabla \times \mathbf{V} = 0, \quad (\mathbf{V} \cdot \mathbf{n})\big|_{\Sigma} = 0$$

Because the curl of V is 0, i.e., for irrotational flow, there must be a velocity potential ϕ, i.e., $\mathbf{V} = \nabla \phi$

And since the divergence of V is 0, then the velocity potential ϕ satisfies Laplace equation: $\nabla^2 \phi = 0, \quad \frac{\partial \phi}{\partial \mathbf{n}} = (\mathbf{V} \cdot \mathbf{n})\big|_{\Sigma} = 0$

Based on the theory of Laplace equation, solution for the equation above is: ϕ= Const, thus: $\mathbf{V} = \nabla \phi \equiv 0$

Which verifies that $V_1=V_2$, the solution is unique.

5.11 Induced Velocity Field

Problem: In an infinite region, if there is no solid boundary, and it satisfies the divergence equation and curl equation. Determine the velocity field, i.e., :

$$\left.\begin{aligned}\nabla\cdot\mathbf{V}&=H(x,y,z,t)\\ \nabla\times\mathbf{V}&=\mathbf{\Omega}(x,y,z,t)\end{aligned}\right\}\Rightarrow$$

Determine velocity field V

Solution: According to the uniqueness of the solution, the velocity field exists and is unique. Because both equations of divergence field and vorticity field are linear, their solution, the velocity field can be decomposed into two parts: $V = V_e + V_v$, where:

1) V_e satisfies: $\nabla\cdot\mathbf{V}_e=H(x,y,z,t),\quad \nabla\times\mathbf{V}_e=0$

2) V_v satisfies: $\nabla\cdot\mathbf{V}_v=0,\quad \nabla\times\mathbf{V}_v=\mathbf{\Omega}(x,y,z,t)$

5.11 Induced Velocity Field

In this way, the problem is changed to a linear superposition of two problems:

$$\left.\begin{aligned}\nabla\cdot\mathbf{V}&=H(x,y,z,t)\\ \nabla\times\mathbf{V}&=\mathbf{\Omega}(x,y,z,t)\end{aligned}\right\} \Leftrightarrow \left.\begin{aligned}\nabla\cdot\mathbf{V}_e&=H(x,y,z,t)\\ \nabla\times\mathbf{V}_e&=0\end{aligned}\right\} + \left.\begin{aligned}\nabla\cdot\mathbf{V}_v&=0\\ \nabla\times\mathbf{V}_v&=\mathbf{\Omega}(x,y,z,t)\end{aligned}\right\}$$

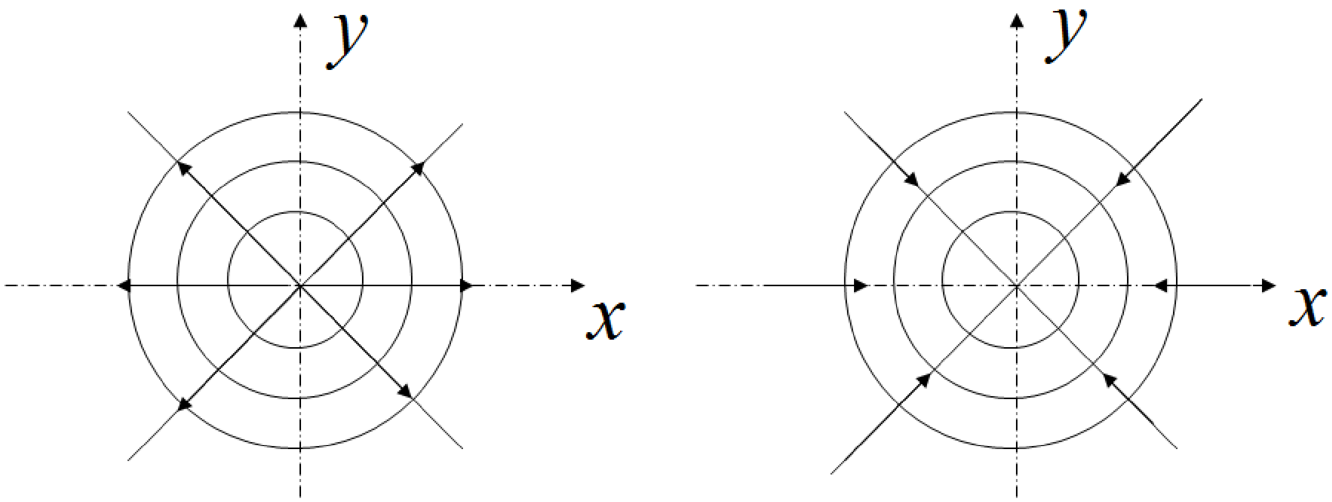
1) First to solve $\mathbf{V}_e$

$$\mathbf{V}_e=\frac{1}{4\pi}\iiint H(\xi,\eta,\zeta,t)\frac{\mathbf{r}}{r^3}d\xi\, d\eta\, d\zeta$$

5.11 Induced Velocity Field

Source and sink

The fluid flows radially outward through a line from the origin, is called a **source** flow, the origin is a source point; if the fluid flows radially inward through a line toward the origin, is called a **sink** flow, the origin is a sink point. For both source and sink, there is only radial velocity, since the flow is a purely radial flow.



(a) (b)

5.11 Induced Velocity Field

From the continuity principle of the fluid: in polar (spherical) coordinates, the **volume flow rate *m*** (also called as the strength of the source/sink flow) per unit height of a fluid through any cylindrical surface (spherical surface) is constant.

For 2D: $V_r \cdot 2\pi r = \pm m \quad \Rightarrow \quad V_r = \pm \dfrac{m}{2\pi r}, \quad V_\theta = 0$

For 3D: $V_r \cdot 4\pi r^2 = \pm m \quad \Rightarrow \quad V_r = \pm \dfrac{m}{4\pi r^2}, \quad V_\theta = 0, \quad V_z = 0$

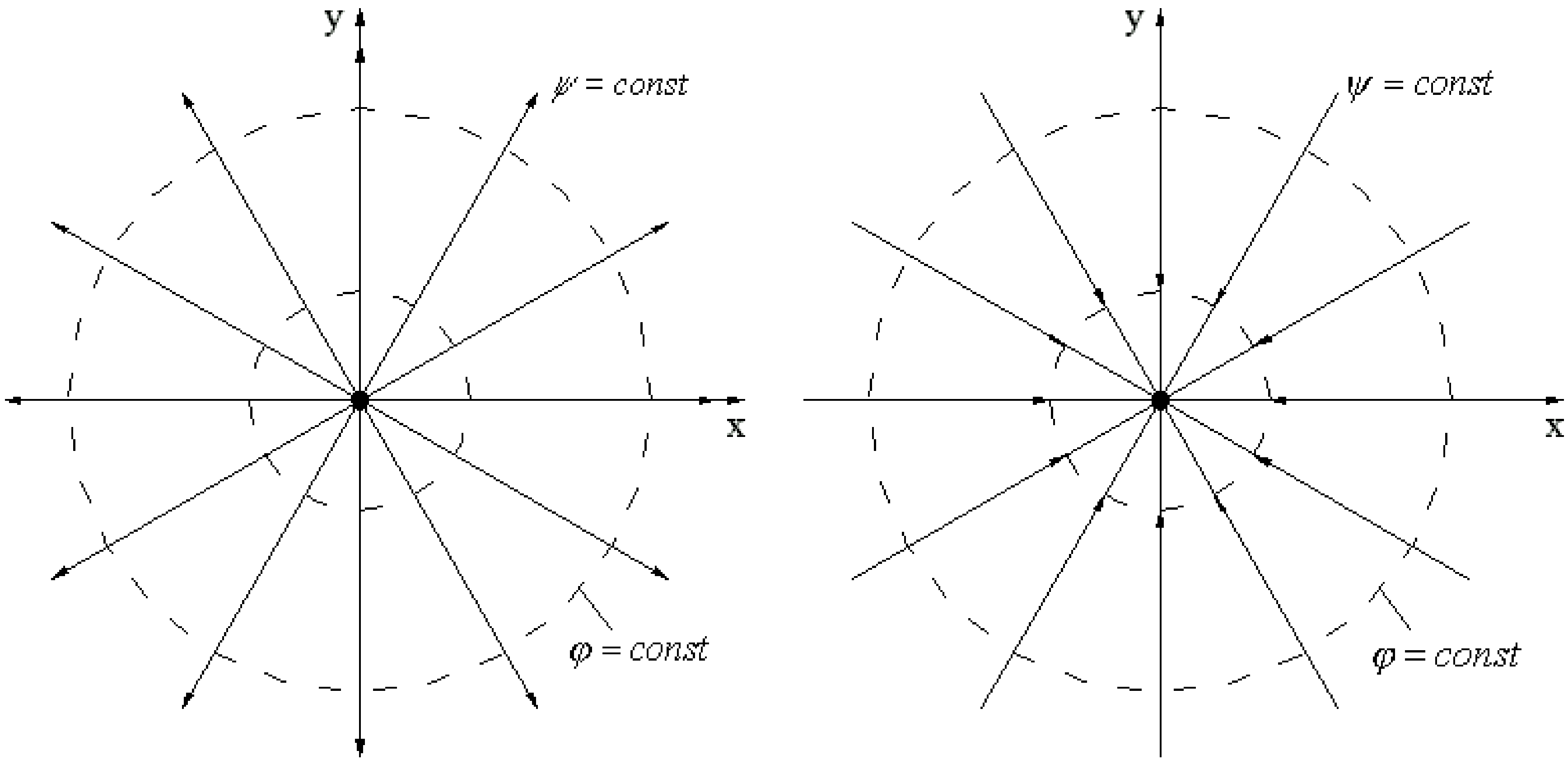
It can be verified that except for the source and sink, the above-mentioned flow field is irrotational and the divergence is 0, then:

For 2D: $$\phi = \frac{m}{2\pi} \ln r, \quad \psi = \frac{m}{2\pi} \theta$$

For 3D: $$\phi = \frac{m}{4\pi r}$$

5.11 Induced Velocity Field

According to the derived stream function and velocity potential function, equipotential lines are a series of concentric circles with different radii, streamlines are a series of radial lines with different polar angles.



5.11 Induced Velocity Field

2) Secondly, to solve V_v, which satisfies:

$$\nabla \cdot \mathbf{V}_v = 0, \quad \nabla \times \mathbf{V}_v = \mathbf{\Omega}(x, y, z, t)$$

The divergence of curl is 0, and $\nabla \cdot \mathbf{V}_v = 0$, so there must be a vector potential (function) **B**, satisfying:

$$\mathbf{V}_v = \nabla \times \mathbf{B}$$

Substituting it into the vorticity equation, and applying the following tensor formula:

$$\nabla^2 \mathbf{V} = \nabla(\nabla \cdot \mathbf{V}) - \nabla \times (\nabla \times \mathbf{V})$$

We get: $\nabla \times (\nabla \times \mathbf{B}) = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \mathbf{\Omega}$

$$\begin{cases} \nabla^2 \mathbf{B} = -\mathbf{\Omega} \\ \nabla \cdot \mathbf{B} = 0 \end{cases} \Longrightarrow \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \mathbf{\Omega}$$

5.11 Induced Velocity Field

The first equation is a Poisson equation, its solution is:

$$\mathbf{B} = \frac{1}{4\pi}\iiint \frac{\mathbf{\Omega}(\xi,\eta,\zeta,t)}{\sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}}\,d\xi\, d\eta\, d\zeta$$

Next to check if the solution satisfies the second equation:

$$\nabla\cdot\mathbf{B} = \frac{1}{4\pi}\iiint \nabla\cdot\left(\frac{\mathbf{\Omega}}{r}\right) d\xi\, d\eta\, d\zeta = \frac{1}{4\pi}\iint \frac{\mathbf{\Omega}\cdot\mathbf{n}}{r}\, dS$$

To make $\nabla\cdot\mathbf{B}=0$ then there must be: $\mathbf{\Omega}\cdot\mathbf{n}\big|_S = 0$, which requires the boundary at infinity or in local region should be a vortex surface.

In this case, **B** is the original solution to a vorticity equation.

5.11 Induced Velocity Field

Under this condition, the induced velocity distribution of a vorticity field can be obtained:

$$\mathbf{V}_v = \nabla \times \mathbf{B} = \nabla \times \left(\frac{1}{4\pi} \iiint \frac{\boldsymbol{\Omega}(\xi,\eta,\zeta,t)}{\sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\varsigma)^2}} d\xi d\eta d\zeta \right)$$

$$= -\frac{1}{4\pi} \iiint \boldsymbol{\Omega}(\xi,\eta,\zeta,t) \times \nabla \left(\frac{1}{r} \right) d\xi d\eta d\zeta = \frac{1}{4\pi} \iiint \frac{\boldsymbol{\Omega}(\xi,\eta,\zeta,t) \times \mathbf{r}}{r^3} d\xi d\eta d\zeta$$

i.e.,

$$\mathbf{V}_v = \frac{1}{4\pi} \iiint \frac{\boldsymbol{\Omega}(\xi,\eta,\zeta,t) \times \mathbf{r}}{r^3} d\xi d\eta d\zeta$$

Therefore, if the vorticity field $\boldsymbol{\Omega}$ is known, the induced velocity distribution of the motion of surrounding spatial points by a vortex can be determined.

5.11 Induced Velocity Field

Thus, the total induced velocity by divergence field (source/sink) and vorticity field (vortex) is:

$$\mathbf{V} = \mathbf{V}_e + \mathbf{V}_v$$

$$= \frac{1}{4\pi}\iiint H(\xi,\eta,\zeta,t)\frac{\mathbf{r}}{r^3}d\xi d\eta d\zeta + \frac{1}{4\pi}\iiint \frac{\mathbf{\Omega}(\xi,\eta,\zeta,t)\times\mathbf{r}}{r^3}d\xi d\eta d\zeta$$

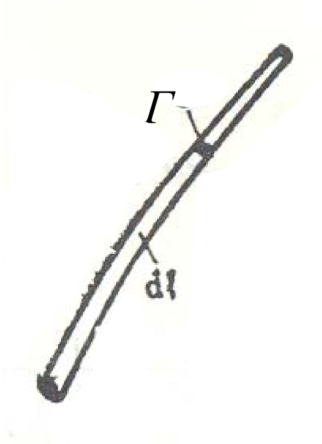
$$= \frac{1}{4\pi}\iiint \frac{1}{r^3}\left[H(\xi,\eta,\zeta,t)\mathbf{r} + \mathbf{\Omega}(\xi,\eta,\zeta,t)\times\mathbf{r}\right]d\xi d\eta d\zeta$$

where (x, y, z) is a spatial point, (ξ, η, ς) is a integral variable

$$\mathbf{r} = (x-\xi, y-\eta, z-\zeta)^{\mathrm{T}}, \quad r = |\mathbf{r}| = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2}$$

5.11 Induced Velocity Field

Application 2: Determine the induced velocity distribution of a vortex filament (length is *L*, vortex strength is Γ)



Solution: A vortex filament is the vorticity concentrates on a vortex tube whose cross-section is of extremely small dimension, so the vortex filament can be approximated as a line. Take a small segment *dl* from this thin tube, the cross-sectional area is *S*, the volume is *Sdl*. Let the vorticity be Ω, then:

$$\lim_{\substack{\Delta S\to 0\\ |\boldsymbol{\Omega}|\to\infty}} S\left|\boldsymbol{\Omega}\right| = \Gamma, \qquad \boldsymbol{\Omega}\, d\xi d\eta d\varsigma = \boldsymbol{\Omega}\, d\tau = \left|\boldsymbol{\Omega}\right| S d\mathbf{l} = \Gamma d\mathbf{l}$$

Substitute into the vortex-induced velocity equation, thus:

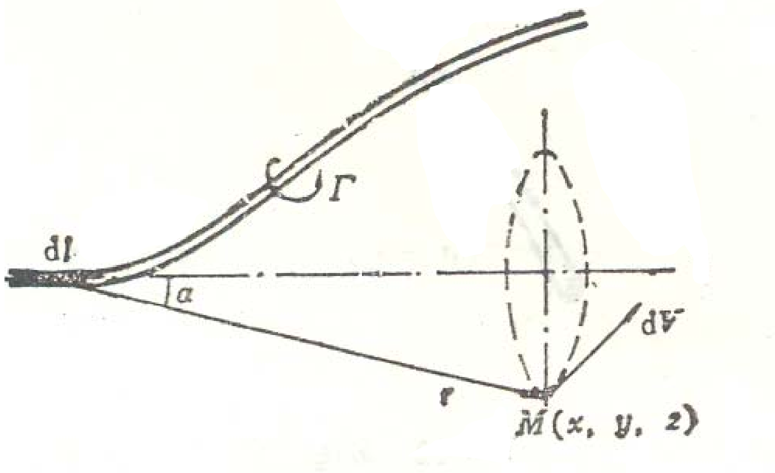
$$\mathbf{V}_v = \frac{1}{4\pi}\iiint \frac{\boldsymbol{\Omega}(\xi,\eta,\varsigma,t)\times\mathbf{r}}{r^3} d\xi d\eta d\varsigma = \frac{\Gamma}{4\pi}\int_L \frac{d\mathbf{l}\times\mathbf{r}}{r^3}$$

5.11 Induced Velocity Field

This equation is identical to the Biot-Savart equation in electromagnetics, which can be used for liquid-electricity analogy. In electromagnetics, the Biot-Savart equation is used to determine the induced magnetic strength field around a live wire; while in fluid mechanics, this equation is used to determine the induced velocity field around a vortex. Let the angle between r and dl be α, then Biot-Savart equation turns to be:

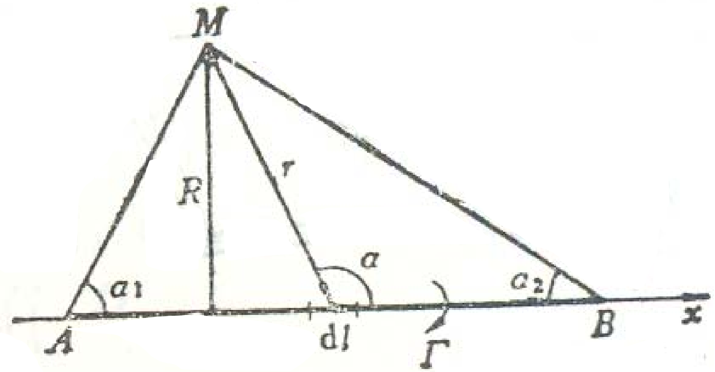
$$|\mathbf{V}_v| = \frac{\Gamma}{4\pi}\int_L \frac{\sin\alpha}{r^2}\,dl$$

Direction of induced velocity is the direction of the cross product of *dl* and r.



5.11 Induced Velocity Field

Application 3: A section of straight vortex filament AB is shown in the figure. Its vortex strength is Γ, the direction is the same as the positive direction of *x* axis. The distance between a spatial point M and the vortex filament is *R*. Determine the induced velocity at point M by the vortex filament.



Solution: Take a 1-D vortex filament dl from AB, from the geometrical relationship:

$$x = R\,\mathrm{tg}\left(\alpha - \frac{\pi}{2}\right) = -R\cot\alpha,\ dl = dx = R\,\mathrm{cse}^2\alpha\, d\alpha,\ r = R\sec\left(\alpha - \frac{\pi}{2}\right) = -R\,\mathrm{cse}\,\alpha$$

Substitute it into the Biot-Savart equation of vortex-induced velocity:

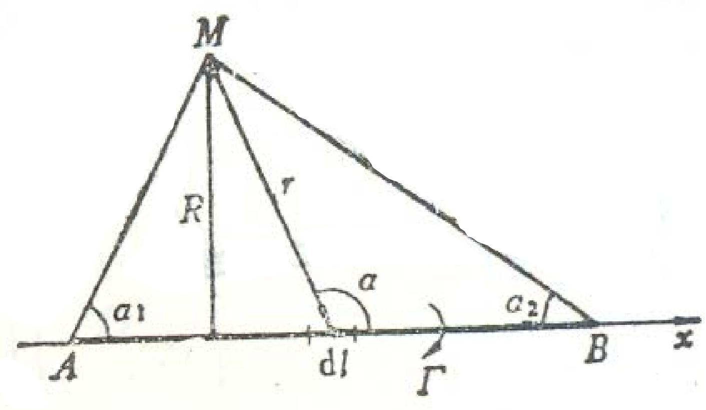
$$V = \frac{\Gamma}{4\pi}\int_L \frac{\sin\alpha}{r^2}dl = \frac{\Gamma}{4\pi R}\int_{\alpha_1}^{\pi-\alpha_2}\sin\alpha\, d\alpha = \frac{\Gamma}{4\pi R}(\cos\alpha_1 + \cos\alpha_2)$$

5.11 Induced Velocity Field

For semi-infinite vortex filament: $\alpha_1 = \frac{\pi}{2}, \ \alpha_2 = 0$

Substitute into the equation:

$$V = \frac{\Gamma}{4\pi R}$$



For infinite vortex filament: $\alpha_1 = 0, \ \alpha_2 = 0$

Substitute into the equation:

$$V = \frac{\Gamma}{2\pi R}$$

For an infinite vortex filament, in any plane perpendicular to the vortex filament, the induced velocity are the same, therefore, it can be treated as the 2D vortex induces a 2D flow, the velocity distribution is:

$$V_r = 0, \ V_\theta = \frac{\Gamma}{2\pi r}$$

where *r* is the distance between a spatial point and the vortex.

5.11 Induced Velocity Field

Vortex:

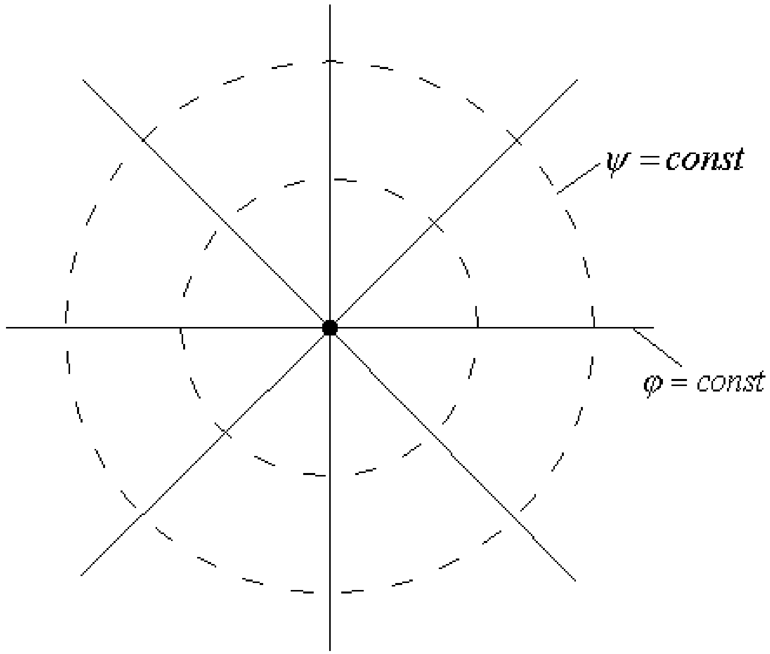
If the radius of a straight vortex filament $r_b \rightarrow 0$, then the flow in the plane perpendicular to this vortex filament is called vortex or free vortex flow, the center of the vortex flow is called the vortex point. Except for the vortex point, the flow field is irrotational and its divergence is 0, then:

$$V_r = \frac{\partial \phi}{\partial r} = 0, \quad V_\theta = \frac{\partial \phi}{r \partial \theta} = \frac{\Gamma}{2\pi r} \Rightarrow \phi = \frac{\Gamma}{2\pi}\theta$$

$$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 0, \quad V_\theta = -\frac{\partial \psi}{\partial r} \Rightarrow \psi = -\frac{\Gamma}{2\pi}\ln r$$

5.11 Induced Velocity Field

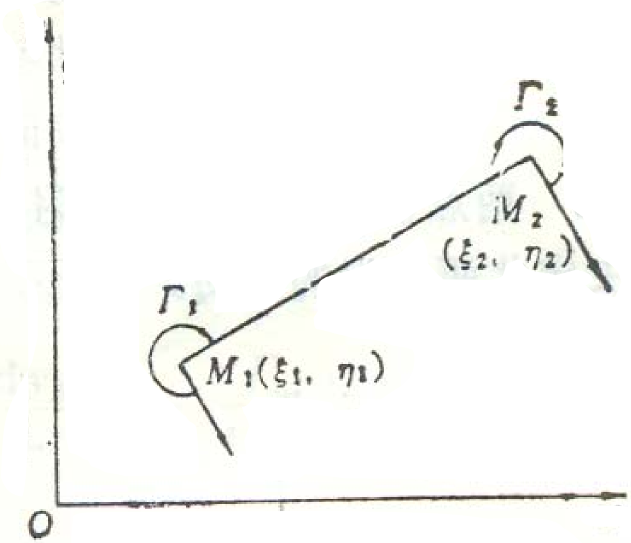
The equipotential lines of a vortex flow field are radial lines with different polar angles, i.e., ϕ=constant; The streamlines are concentric circles with different radii, i.e., ψ=constant. **Opposite** to the source/sink. The strength of a vortex Γ is the circulation around the axis of the vortex.



When Γ>0, the circulation is counterclockwise; when Γ<0, the circulation is clockwise. From Stokes' theorem, the strength Γ of a vortex depends on the vortex strength (vortex flux).

5.11 Induced Velocity Field

Application 4: The original position of two vortices is shown in the figure. Their vortex strengths are Γ_1 and Γ_2. Analyze the motion of these two vortices.



Solution: If regard the two vortices as a whole.

When $\Gamma_1 + \Gamma_2 = 0$, their vortex strengths are the same but with different directions, the two vortices only have the translational motion as a whole, there is no rotation or relative motion. Conversely, there will be rotation and relative motion.

The motion of point M_1, is induced by the vortex at point M_2, from the vortex-induced Biot-Savart equation, then:

$$u_{M_1} = \frac{d\xi_1}{dt} = -\frac{\Gamma_2}{2\pi}\frac{\eta_1 - \eta_2}{r^2}, \quad v_{M_1} = \frac{d\eta_1}{dt} = \frac{\Gamma_2}{2\pi}\frac{\xi_1 - \xi_2}{r^2}$$

5.11 Induced Velocity Field

The motion of point M_2, is induced by the vortex at point M_1, from the vortex-induced Biot-Savart equation, then:

$$u_{M_2} = \frac{d\xi_2}{dt} = -\frac{\Gamma_1}{2\pi}\frac{\eta_2 - \eta_1}{r^2}, \quad v_{M_2} = \frac{d\eta_2}{dt} = \frac{\Gamma_1}{2\pi}\frac{\xi_2 - \xi_1}{r^2}$$

Where $r = \sqrt{(\xi_1 - \xi_2)^2 + (\eta_1 - \eta_2)^2}$ is the distance between the two vortices.

If $\Gamma_1 + \Gamma_2 = 0$, obviously, $u_{M_1} = u_{M_2}, \ v_{M_1} = v_{M_2}$ the two vortices have the uniform translational motion, there is no relative motion.

If $\Gamma_1 + \Gamma_2 \neq 0$, there are relative motion and rotational motion of the two vortices.

The first equation times Γ_1, the second equation times Γ_2, then sum up the two equations and by integrating:

$$\Gamma_1\xi_1 + \Gamma_2\xi_2 = \text{const}, \quad \Gamma_1\eta_1 + \Gamma_2\eta_2 = \text{const}$$

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5.11 Induced Velocity Field

Because $\Gamma_1+\Gamma_2 \neq 0$, and $\Gamma_1+\Gamma_2$= constant, then:

$$\xi_c = \frac{\Gamma_1\xi_1 + \Gamma_2\xi_2}{\Gamma_1 + \Gamma_2} = \text{const}, \qquad \eta_c = \frac{\Gamma_1\eta_1 + \Gamma_2\eta_2}{\Gamma_1 + \Gamma_2} = \text{const}$$

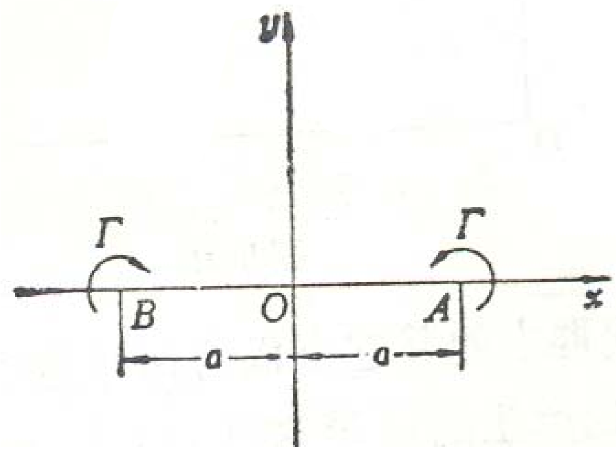
If there are several vortices, and $\sum \Gamma_i \neq 0$, $\sum \Gamma_i = \text{const}$, similarly, it can be derived that:

$$\xi_c = \frac{\sum \Gamma_i \xi_i}{\sum \Gamma_i} = \text{const}, \quad \eta_c = \frac{\sum \Gamma_i \eta_i}{\sum \Gamma_i} = \text{const}$$

Where point (ξ_c, η_c) is called the center of gravity of the vortex group, which is similar to the equation of computing the centroid of several material particle. It should be noted that, although there are relative motions between vortices, the center of gravity of the vortex group does not change during the motion. The rotational motion and relative motion of vortices are around the center of gravity of the vortex group as the origin.

5.11 Induced Velocity Field

Application 5: The original position of two vortices is shown in the figure. Their vortex strengths have the same magnitude but opposite directions. Determine the motion of these two vortices.



Solution: The motion of a vortex must be induced by other vortex.

Because $\Gamma_A+\Gamma_B=0$, the two vortices have a translational motion as a whole. The motion of point A, is induced by the vortex at point B, from the vortex-induced Biot-Savart equation, then:

$$u_A=\frac{dx_A}{dt}=0,\quad v_A=\frac{dy_A}{dt}=-\frac{\Gamma}{2\pi\cdot 2a}=-\frac{\Gamma}{4\pi a}$$

Integrating: $x_A=C_1,\quad y_A=-\dfrac{\Gamma}{4\pi a}t+C_2$

At $t=0$, $x_A=a$, $y_A=0$, then: $C_1=a,\ C_2=0$

5.11 Induced Velocity Field

The motion of point B, is induced by the vortex at point A, from the vortex-induced Biot-Savart equation, then:

$$u_B = \frac{dx_B}{dt} = 0, \quad v_B = \frac{dy_B}{dt} = -\frac{\Gamma}{2\pi \cdot 2a} = -\frac{\Gamma}{4\pi a}$$

Integrating: $x_B = C_3, \quad y_B = -\frac{\Gamma}{4\pi a} t + C_4$

At t=0, x_B= -a, y_B=0, then: C_3 = -a, C_4 = 0

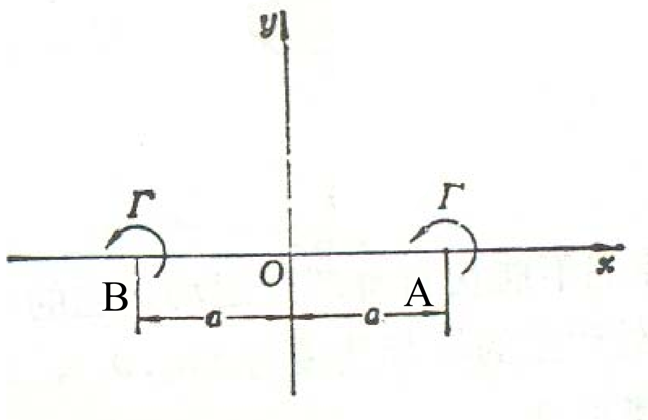
Therefore, the motion equation s of points A and B are:

$$x_A = a, \quad y_A = -\frac{\Gamma}{4\pi a} t$$

$$x_B = -a, \quad y_B = -\frac{\Gamma}{4\pi a} t$$

5.11 Induced Velocity Field

Application 6: The original position of two vortices is shown in the figure. Their vortex strengths have the same magnitude and direction. Determine the motion of these two vortices.



Solution: Because $\Gamma_A+\Gamma_B\neq 0$, the two vortices have relative motion and rotational motion. The center of gravity of the vortex group (fixed point) is:

$$\xi_c = \frac{\Gamma a - \Gamma a}{2\Gamma} = 0, \quad \eta_c = \frac{0-0}{2\Gamma} = 0$$

So the two vortices rotates around the origin 0.

5.11 Induced Velocity Field

The motion of point A, is induced by the vortex at point B, from the vortex-induced Biot-Savart equation, then:

$$u_A = \frac{dx_A}{dt} = 0, \quad v_A = \frac{dy_A}{dt} = \frac{\Gamma}{2\pi \cdot 2a} = \frac{\Gamma}{4\pi a}$$

Similarly, the motion of point B, is induced by the vortex at point A, from the vortex-induced Biot-Savart equation, then:

$$u_B = \frac{dx_B}{dt} = 0, \quad v_B = \frac{dy_B}{dt} = -\frac{\Gamma}{2\pi \cdot 2a} = -\frac{\Gamma}{4\pi a}$$

Both point A and B rotate around the origin 0 with the same angular velocity ω:

$$\omega = \frac{\left|v_{A(B)}\right|}{a} = \frac{\Gamma}{4\pi a^2}$$

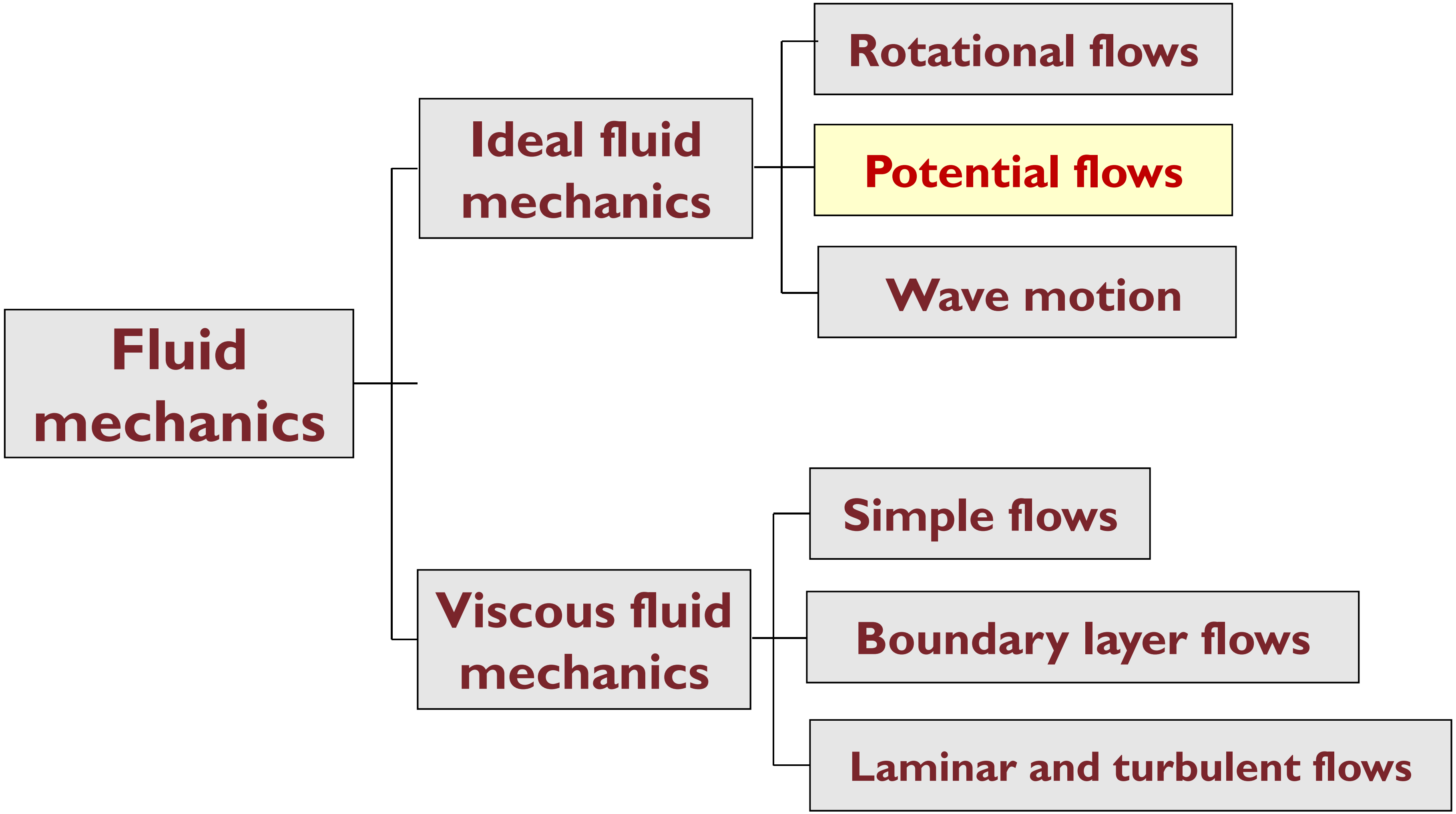
5.11 Induced Velocity Field

Expressed in polar coordinates, the equation of motion for point A and B are:

$$r_A = a, \quad \theta_A = \frac{\Gamma}{4\pi a^2} t$$

$$r_B = a, \quad \theta_B = \pi + \frac{\Gamma}{4\pi a^2} t$$

Chapter 6
Potential Flow Theory


Fluid mechanics
Ideal fluid mechanics
Rotational flows
Potential flows
Wave motion
Viscous fluid mechanics
Simple flows
Boundary layer flows
Laminar and turbulent flows

6.1 Governing Equations

We consider an ***incompressible inviscid potential flow***. The flow is governed by ***Euler's equation*** (*inviscid*) and ***continuum equation*** (*incompressible*).

$$\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{V}=-\frac{1}{\rho}\nabla p+\mathbf{f}, \qquad \nabla\cdot\mathbf{V}=0$$

$$\frac{\partial \mathbf{V}}{\partial t}+\nabla\left(\frac{|\mathbf{V}|^2}{2}\right)-\cancel{\mathbf{V}\times\boldsymbol{\Omega}}=-\frac{1}{\rho}\nabla p+\mathbf{f}, \qquad \nabla\cdot\mathbf{V}=0$$

For *irrotational* flow, there exists a velocity potential, ϕ ,

$$\mathbf{V}=\nabla\phi$$

Substituting it in continuum equation, ***Lapalce's equation*** results

$$\nabla\cdot\nabla\phi=0 \quad\Rightarrow\quad \nabla^2\phi=0$$

6.1 Governing Equations

In addition, if the body force is a potential force, then ***Euler's equation*** is rewritten

$$\nabla\left(\frac{\partial\phi}{\partial t}\right)+\nabla\left(\frac{|\nabla\phi|^2}{2}\right)=-\nabla\left(\frac{p}{\rho}\right)-\nabla\Pi$$

$$\nabla\left(\frac{\partial\phi}{\partial t}+\frac{|\nabla\phi|^2}{2}+\frac{p}{\rho}+\Pi\right)=0$$

$$\frac{\partial\phi}{\partial t}+\frac{|\nabla\phi|^2}{2}+\frac{p}{\rho}+\Pi=C(t)$$

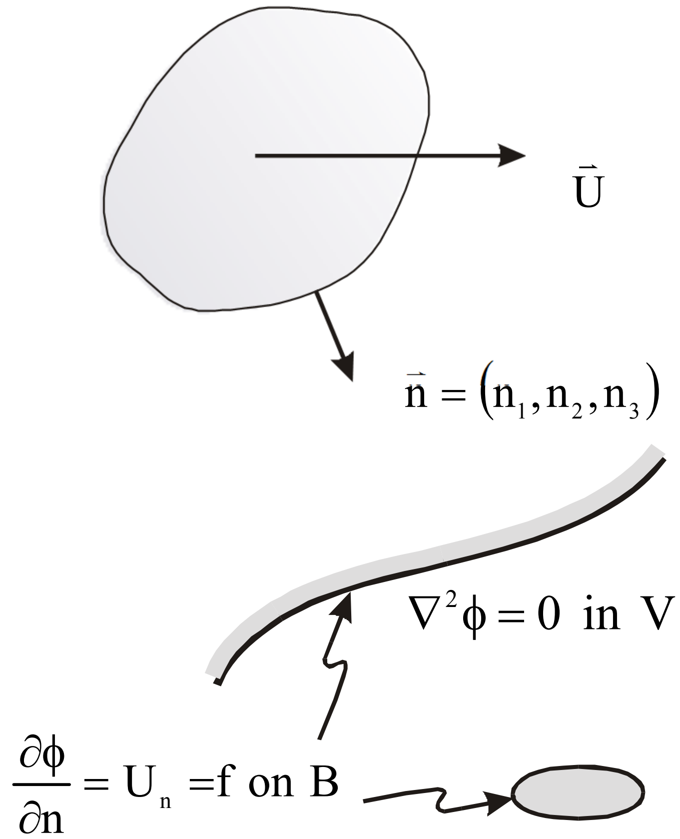
That is the general ***Bernoulli equation***, usually used as a ***dynamic boundary condition*** in potential flow solution, from which pressure distribution can be determined when velocity potential is obtained.

6.1 Governing Equations

Furthermore, generally fluid is assumed not able to enter into or flow out from the wall of a body, but has to move with it, i.e. body surface is considered as ***impermeable***. This is a boundary condition governed on body surface,

$$\mathbf{V} \cdot \mathbf{n} = \mathbf{U} \cdot \mathbf{n} \quad \Rightarrow \quad \nabla \phi \cdot \mathbf{n} = \mathbf{U}_n$$

$$\frac{\partial \phi}{\partial \mathbf{n}} = \mathbf{U}_n$$



Named ***kinematic boundary condition***.

6.1 Governing Equations

Besides, at very far field, both velocity and pressure have to be given. Also the whole flow field has to be given at an instant, generally when flow starts and named as an initial instant.

The ***far field condition*** at infinity

$$\nabla \phi = \mathbf{U}_{\infty}, \qquad p = p_{\infty}$$

The general ***initial condition***

$$\nabla \phi \big|_{t=0} = \mathbf{U}_0(\mathbf{x}), \qquad p \big|_{t=0} = p_0(\mathbf{x})$$