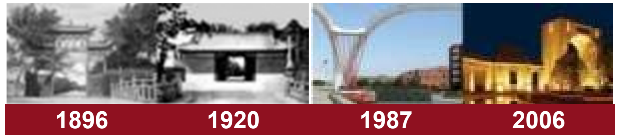
上海交通大學
SHANGHAI JIAO TONG UNIVERSITY



Introduction to Marine Hydrodynamics

(NA235)

Department of Naval Architecture and Ocean Engineering

School of Naval Architecture, Ocean & Civil Engineering

Shanghai Jiao Tong University

Review

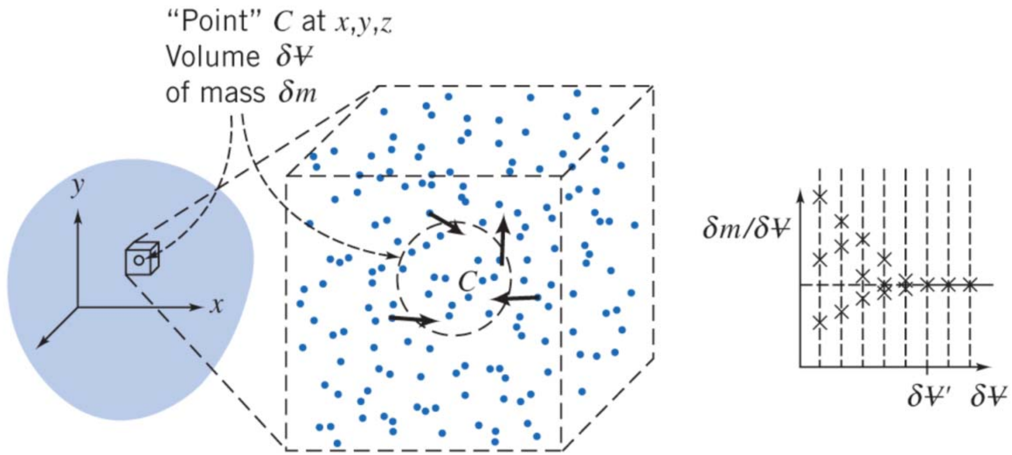
- Definition of fluids
- Properties of fluids
  - Fluidity
  - Deformability
  - Viscosity
    - Newton's law of viscosity
    - Real fluids and Ideal fluids
    - Newtonian fluids and non-Newtonian fluids
  - Compressibility
    - Compressible versus incompressible flows

Review

◆ Continuum hypothesis

Fluid as a continuum: the variation in fluid properties is so smooth that differential calculus can be used to analyze the flow.

◆ Fluid particle



1.4 Forces on Fluids

- Body force
- Surface force
- Surface tension

1.4.1 Body Force

A **body force** is a force that acts throughout the entire volume of a fluid and is proportional to the mass of the volume. It can be expressed in terms of either the force per unit volume or the force per unit mass, therefore body force is also called volume force or mass force.

For instance, gravity is the most common body force (G=mg)

Other body forces are, such as: magnetic forces in electromagnetic fields; electric forces in electric field; inertial forces in an accelerated motion.

Expression: unit body force f(x, y, z)

$$\mathbf{f}(x,y,z)=\lim_{\Delta m\to 0}\frac{\Delta \mathbf{F}}{\Delta m}=\frac{1}{\rho}\lim_{\Delta V\to 0}\frac{\Delta \mathbf{F}}{\Delta V}=\frac{d\mathbf{F}}{\rho\, dV}$$

Unit: m/s^2

1.4.1 Body Force

Unit body force: $\mathbf{f} = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$

Body force (volume integral): $\mathbf{F} = \iiint_V \rho\,\mathbf{f}(x,y,z,t)\,dV$

$$f_x = \lim_{\Delta m \to 0}\frac{\Delta F_x}{\Delta m},\quad f_y = \lim_{\Delta m \to 0}\frac{\Delta F_y}{\Delta m},\quad f_z = \lim_{\Delta m \to 0}\frac{\Delta F_z}{\Delta m}$$

In a gravitational field:

$$f_x = \frac{G_x}{m} = 0$$

$$f_y = \frac{G_y}{m} = 0$$

$$f_z = \frac{G_z}{m} = -g$$

$$\mathbf{f} = \mathbf{g} = -g\mathbf{k}$$

1.4.2 Surface Force

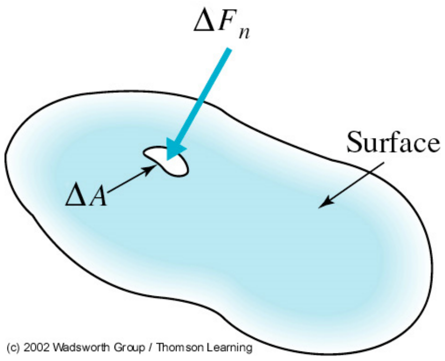
Surface force: acts on the surface of the fluid element and is proportional to the surface area.

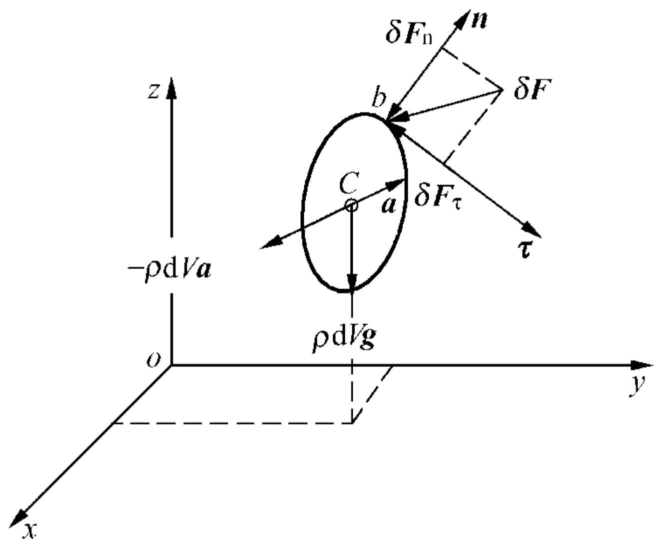
Normal force: perpendicular to the surface of a fluid element

$$\overline{p_{nn}} = \lim_{\delta A \to 0} \frac{\delta \overline{F_n}}{\delta A} = \frac{d\overline{F_n}}{dA}$$

Tangential force: tangent to the surface of a fluid element

$$\overline{p_{n\tau}} = \lim_{\delta A \to 0} \frac{\delta \overline{F_\tau}}{\delta A} = \frac{d\overline{F_\tau}}{dA} = \tau$$



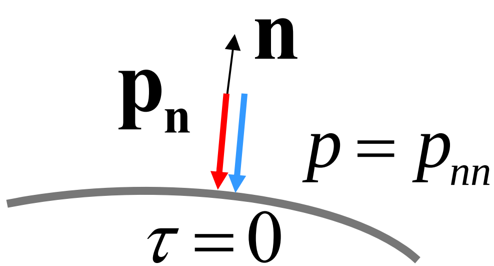


1.4.2 Surface Force

Pressure at a point in a ideal (static) fluid

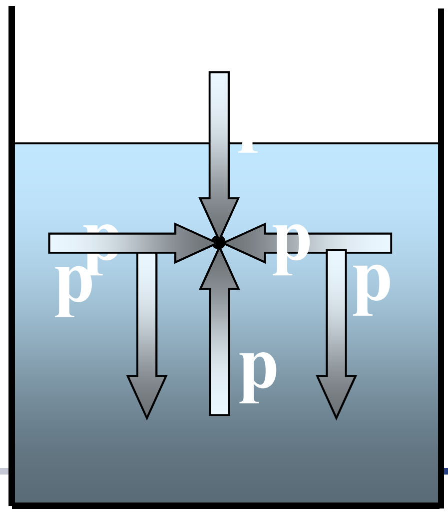
For a static fluid, the only stress is the **normal stress** since by definition a fluid subjected to a shear stress must deform and undergo a motion.

$p = -\mathbf{p}_\mathrm{n} \cdot \mathbf{n}$



Static pressure is the pressure at a nominated point in a fluid.

- Static pressure exerted by a static fluid on an object is always perpendicular to the surface of the object
- Static pressure at any point in a fluid is the same in all directions
- Static pressure is transmitted to solid boundaries or across arbitrary sections of fluid ***normal*** *to* these boundaries or sections at every point.



1.4.2 Surface Force

Units of pressure:

- Pa＝N/m^2;
- (standard atmosphere) atm;
- height of equivalent column of water;
- bar;
- kgf / cm^2;
- etc.

Relationships:

$$1\ \text{atm} = 101325\ \text{Pa}$$
$$= 1.033\ \text{Kgf/cm}^2$$
$$= 10.33\ \text{mH}_2\text{O}$$
$$= 760\ \text{mmHg}$$
$$1\ \text{bar} = 10^5\ \text{Pa}$$

1.4.2 Surface Force

Three ways of expressing the pressure:

Absolute pressure P_{abs}: zero-referenced against a perfect vacuum

(= gage pressure + atmospheric pressure)

Gage pressure P_{gage}: zero-referenced against local atmospheric pressure P_{atm}

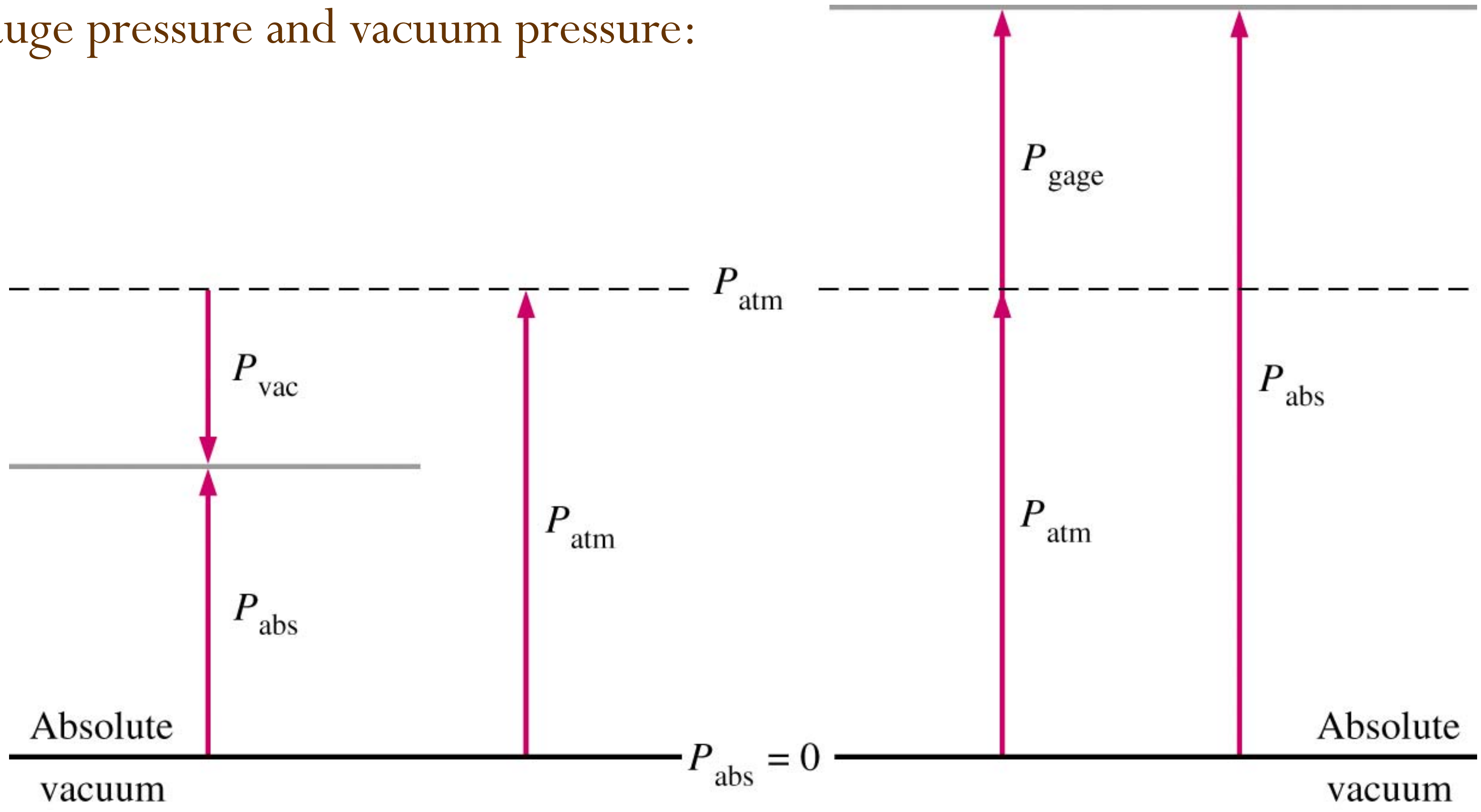
(= absolute pressure - atmospheric pressure)

Vacuum: pressures below atmospheric pressure; absolute pressure $\leq$ atmospheric pressure

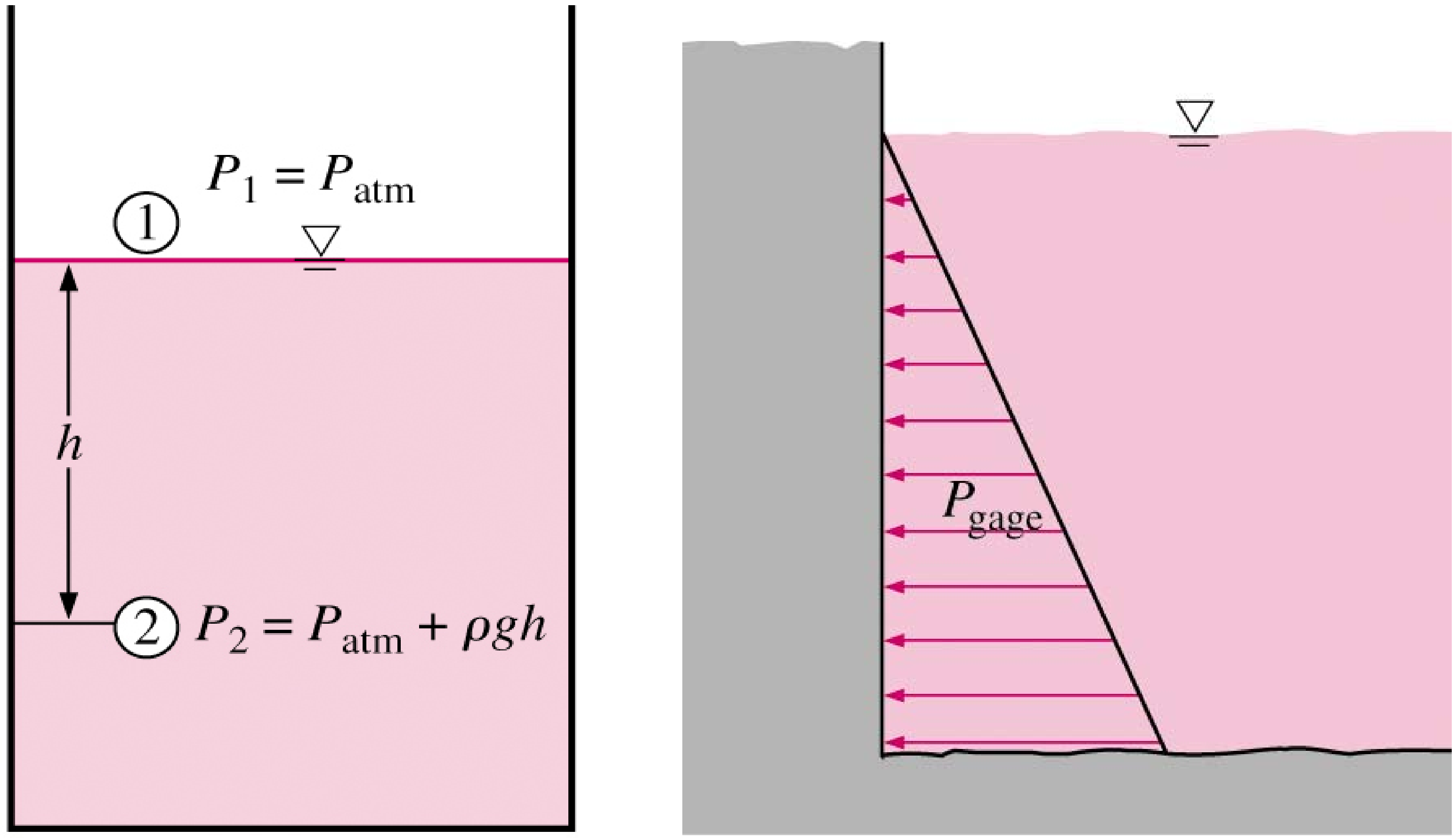
(= atmospheric pressure - absolute pressure)

1.4.2 Surface Force

Relationships among absolute pressure, gauge pressure and vacuum pressure:



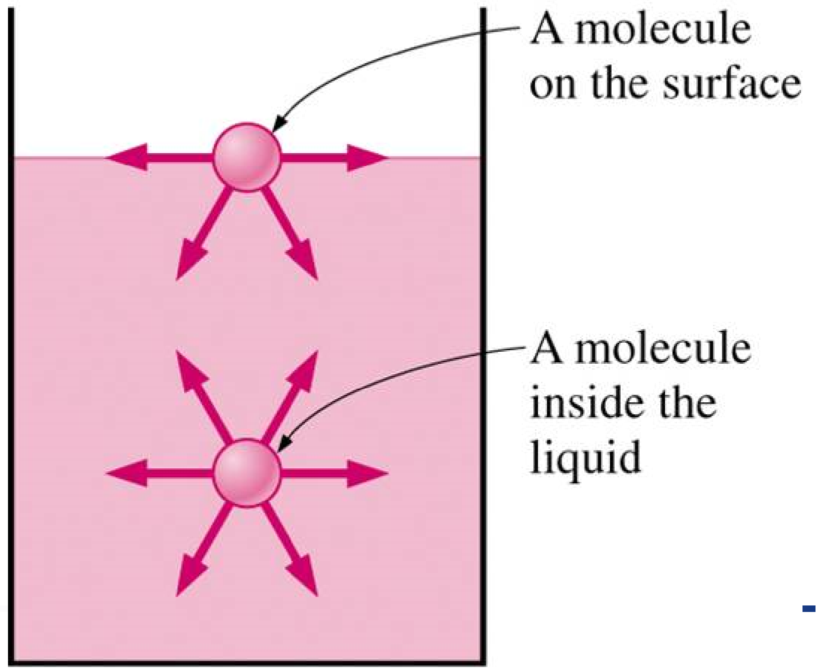
1.4.2 Surface Force



1.4.3 Surface Tension

A liquid exhibits the free surface. Near the interface between liquid and air, all the liquid molecules are trying to pull the molecules on the interface **inward**, the magnitude of the pulling force per unit length of a line on the interface is called **surface tension** (N/m).

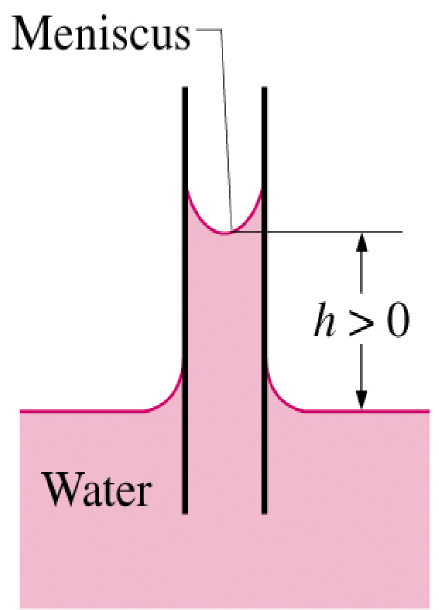
The **surface tension** is defined as the force along a line of unit length, therefore is also known as line force.

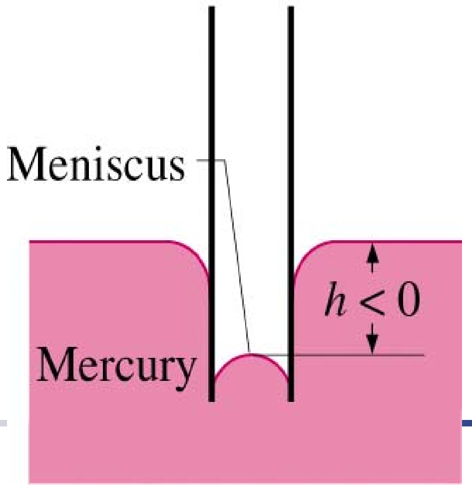


1.4.3 Surface Tension

The rise or fall of a liquid in a small-diameter tube inserted into the liquid:

The liquid molecules at a solid–liquid interface are subjected to both intermolecular cohesion by other liquid molecules and adhesive force by molecules of the solid. If intermolecular cohesion > adhesive force (mercury), the surface tension causes the liquid surface tends to be depressed; If intermolecular cohesion < adhesive force (water), the liquid surface tends to rise along the tube surface because of the surface tension. →

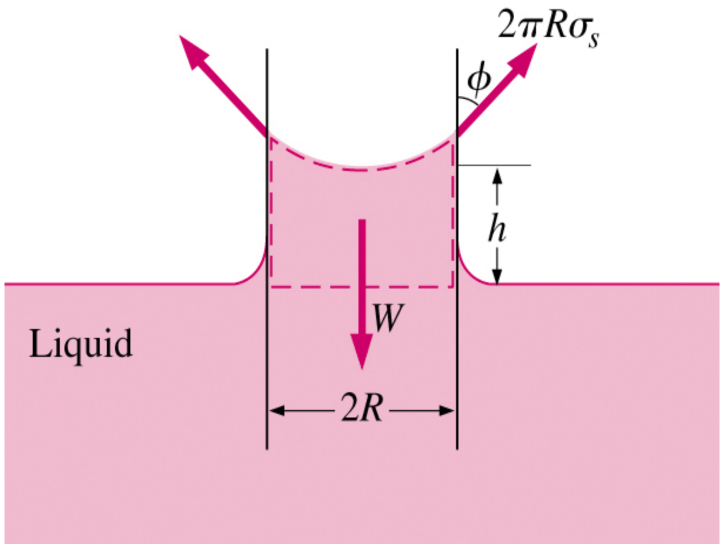




capillarity

1.4.3 Surface Tension

The magnitude of the capillary rise *h*:



weight of fluid column = surface tension pulling force

$$\Rightarrow \rho g\left(\pi R^2 h\right) = 2\pi R\sigma_s \cos\phi$$

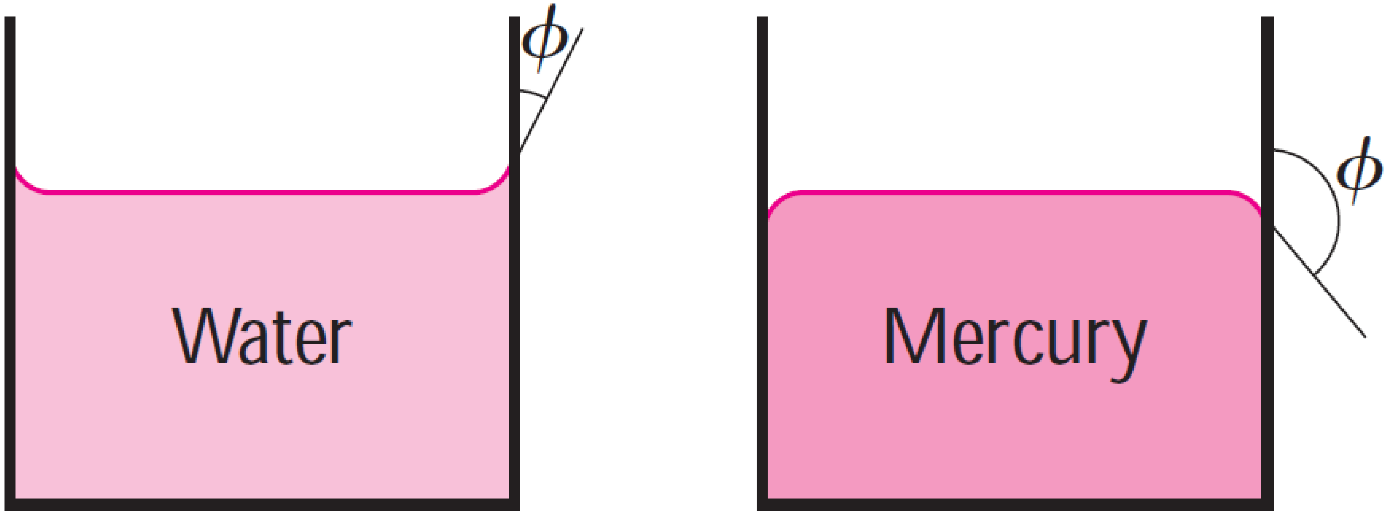
$$\Rightarrow h = \frac{2\sigma_s \cos\phi}{\rho g R}$$

σ_s is surface tension of the fluid in air

ϕ is wetting angle (tangent to the liquid surface with the solid surface)

1.4.3 Surface Tension

Strength of the capillary effect

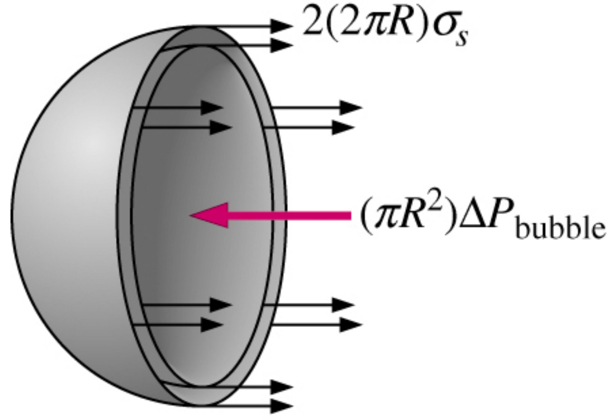


(a) Wetting fluid

(b) Nonwetting fluid

1.4.3 Surface Tension

Soap bubbles: lower the surface tension of water and enable it to penetrate through the small openings between fibers for more effective washing.

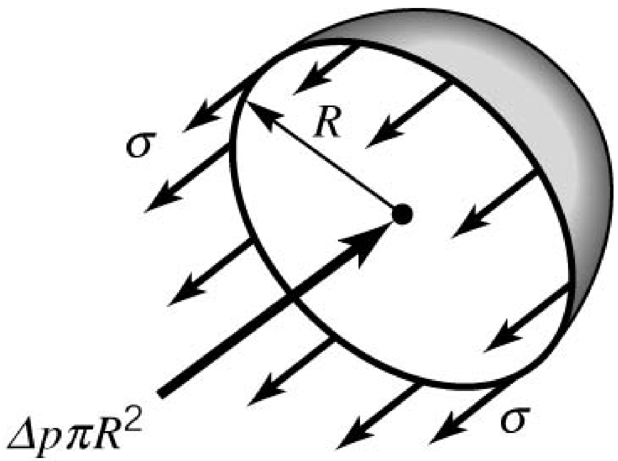


(*b*) Half a bubble

Surface tension acts along the circumference. The pressure acts on the area, horizontal force balances for the bubble give:

$$2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{bubble}}$$

$$\Rightarrow \quad \Delta P_{\text{bubble}} = 4\sigma_s / R$$



σ_s is surface tension of the fluid in air

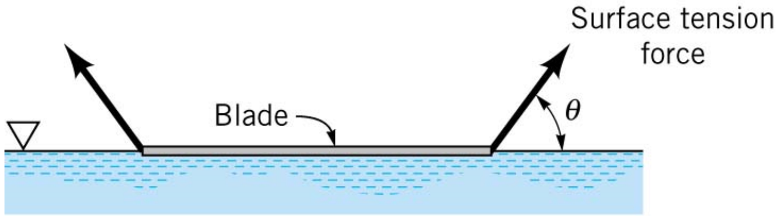
ΔP is the difference of the pressure inside and outside the bubble

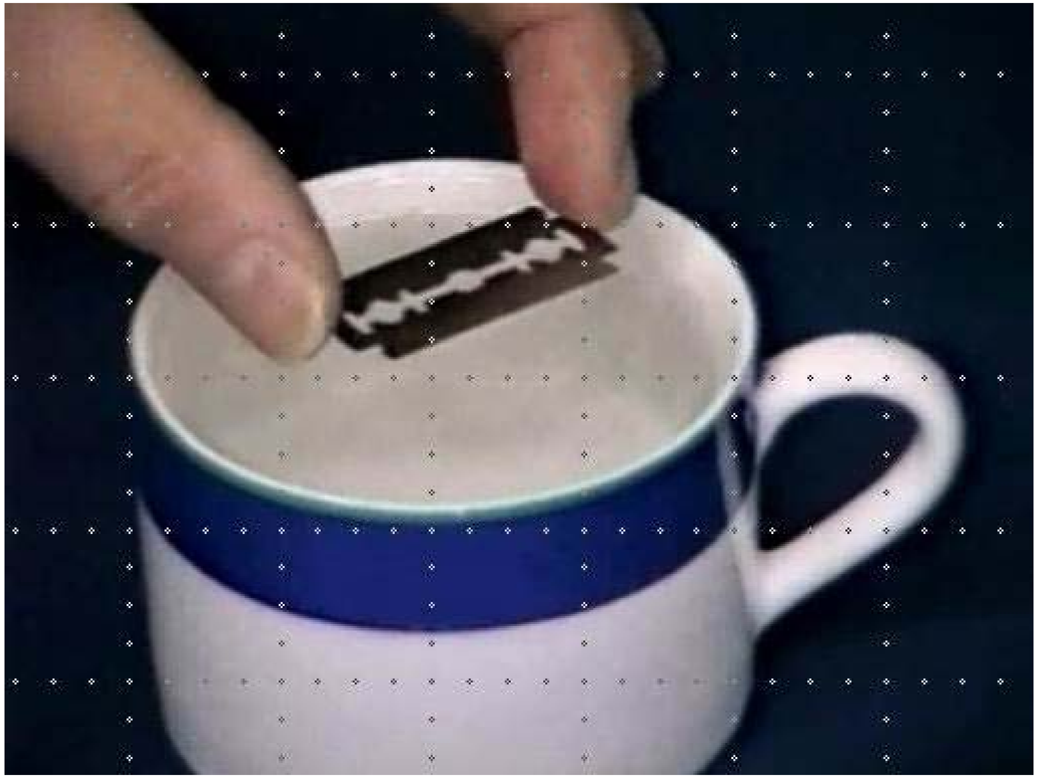
1.4.3 Surface Tension

Surface tension is negligible in most engineering situations, but the effects are important in:

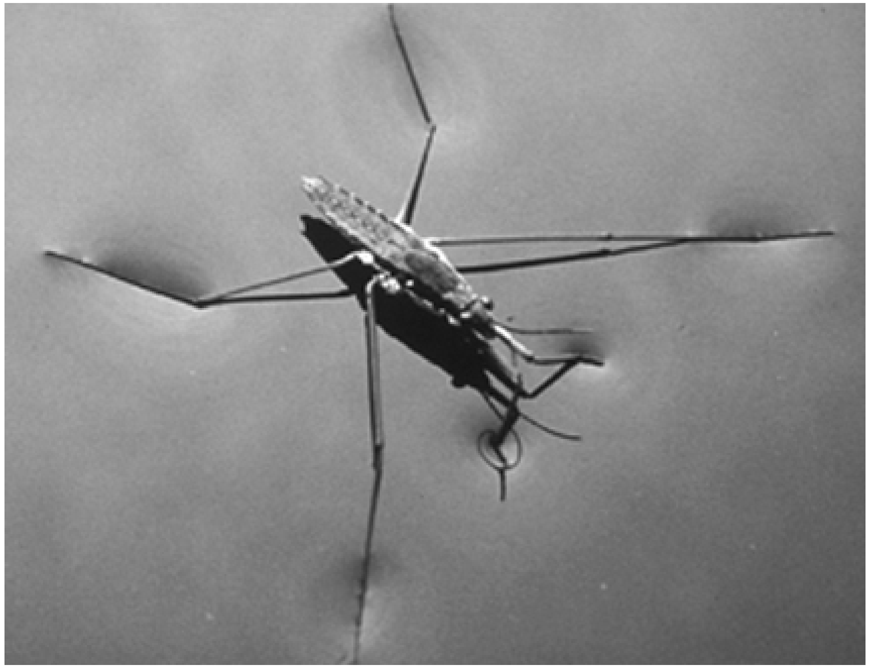
- capillary of liquid in narrow space
- bubble formation, liquid drop
- breakup of waves
- small scale model

1.4.3 Surface Tension





1.4.3 Surface Tension



Review: Key Points in Chapter 1

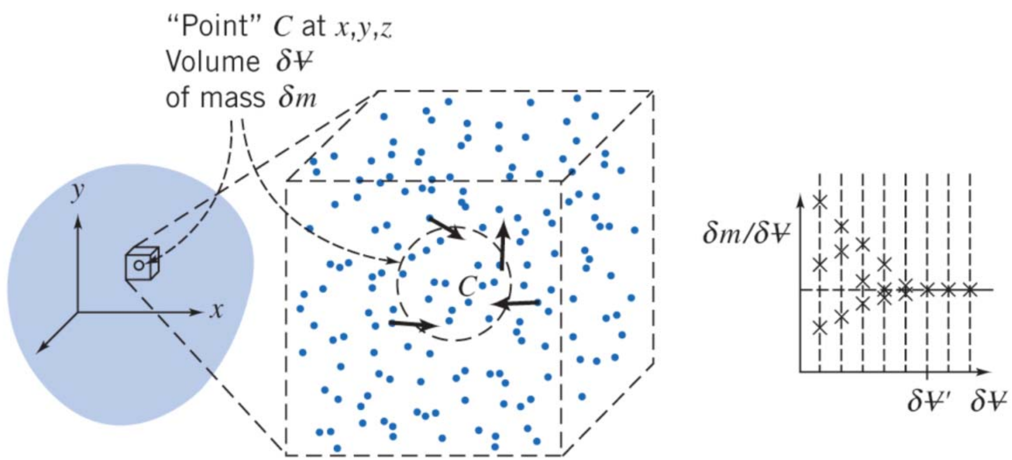
- Definition of fluids
- Properties of fluids
  - Fluidity
  - Deformability
  - Viscosity
    - Newton's law of viscosity
    - Real fluids and Ideal fluids
    - Newtonian fluids and non-Newtonian fluids
  - Compressibility
    - Compressible versus incompressible flows

Review: Key Points in Chapter 1

- **Continuum hypothesis**

  Fluid as a continuum: the variation in fluid properties is so smooth that differential calculus can be used to analyze the flow.

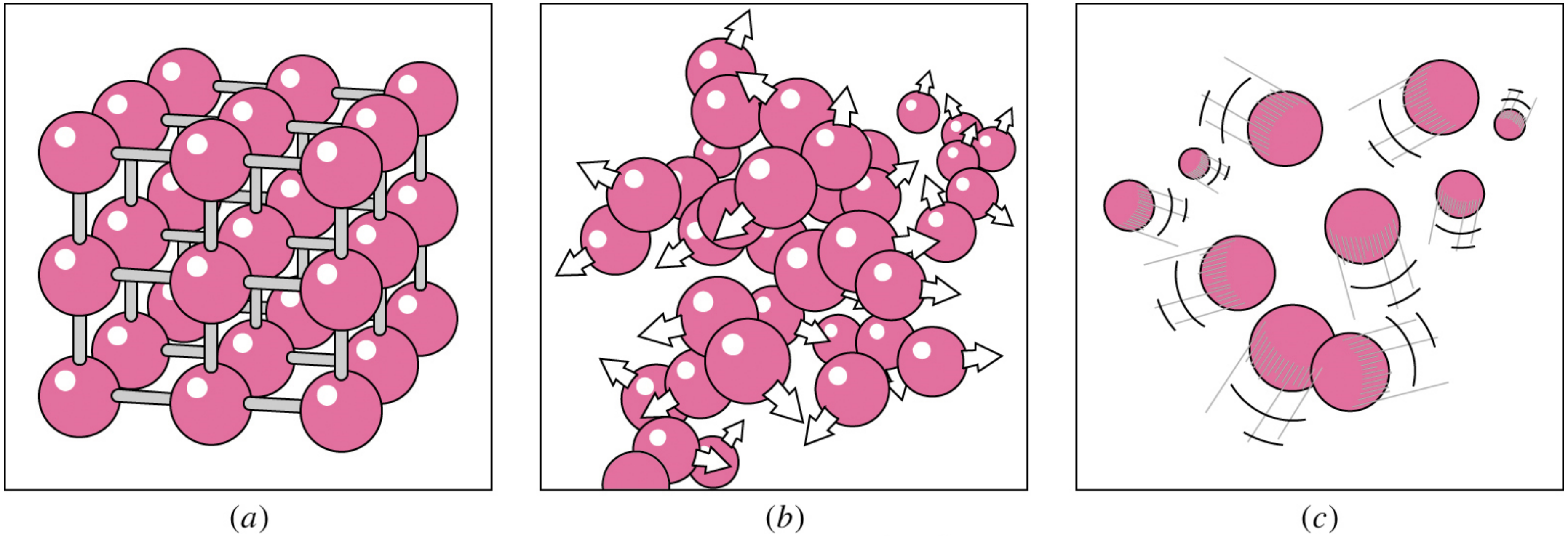
- **Fluid particle**

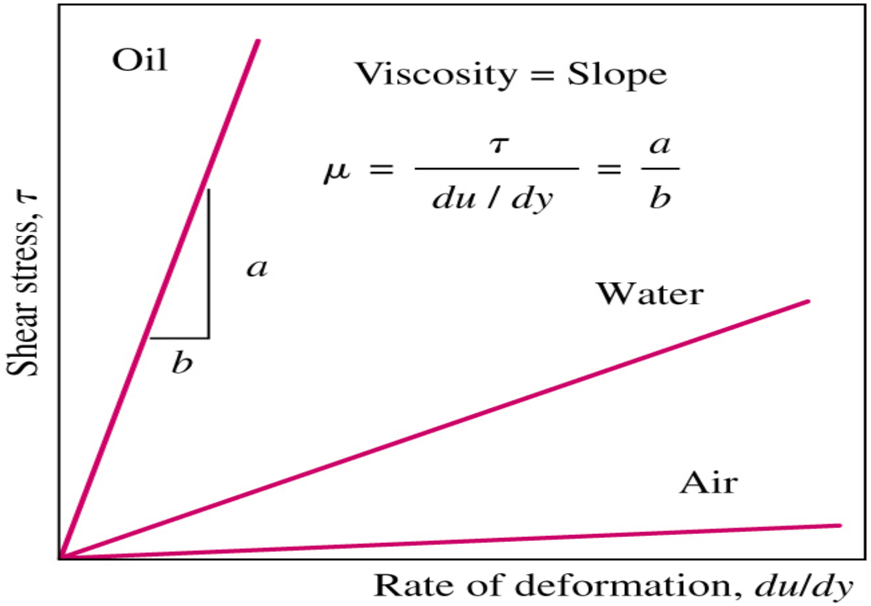


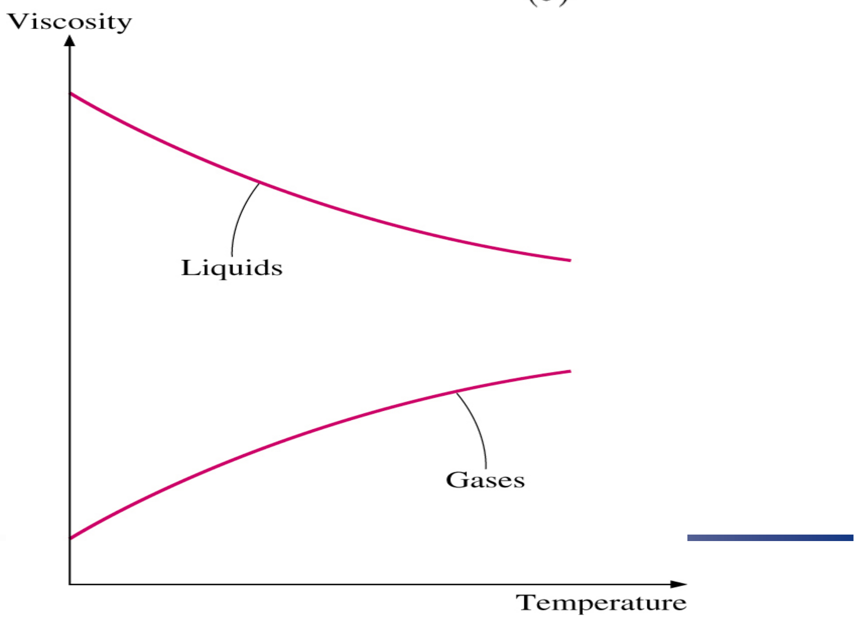
Review: Key Points in Chapter 1

- **Forces on fluids**
  - Body force
  - Surface force
  - Surface tension (Capillary)

Review: Key Points in Chapter 1

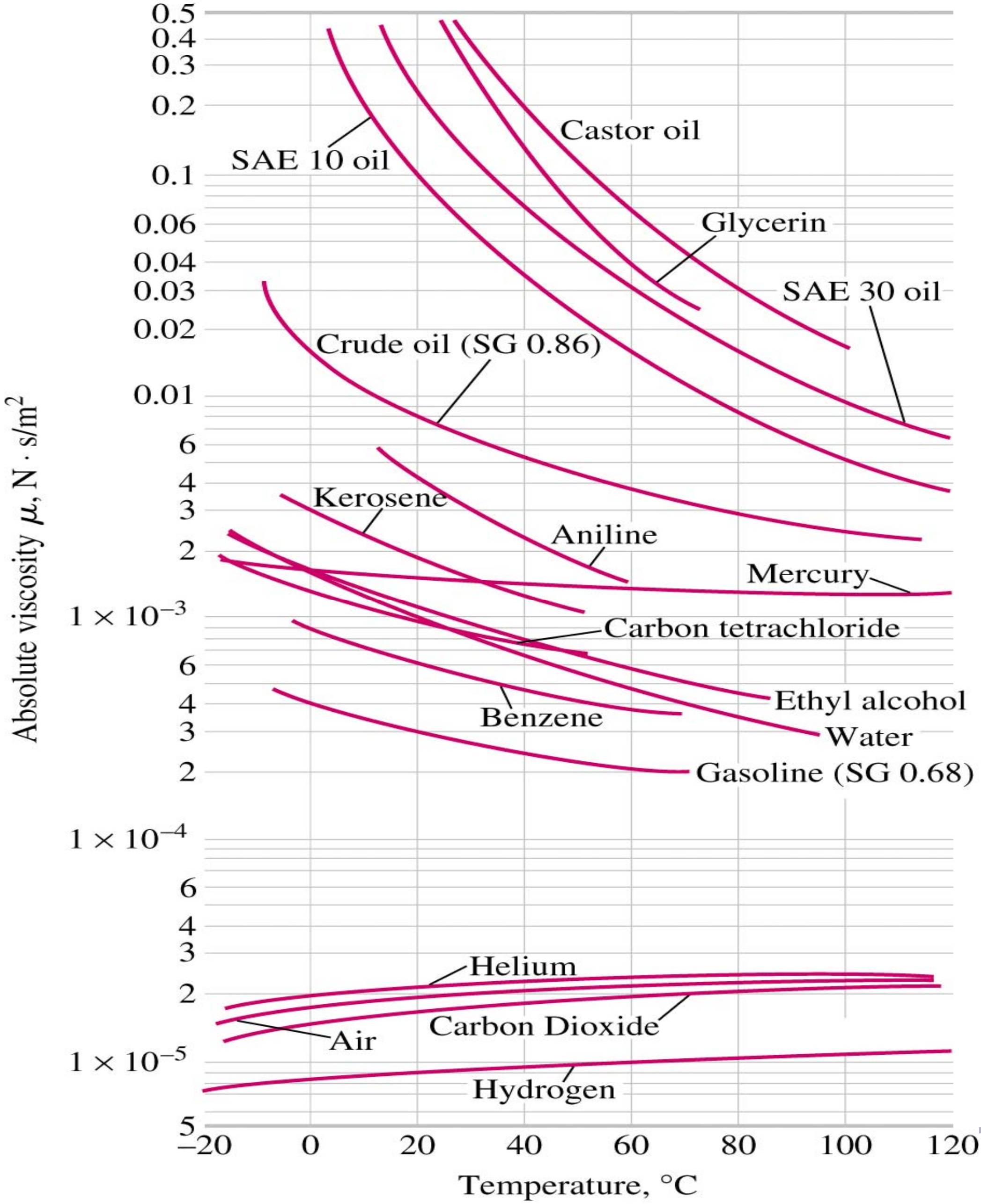






上海交通大學
Shanghai Jiao Ton

Review: Key Points in Chapter 1



- **Fluid kinematics** (Chapter 2)
- **Fluid dynamics** (Chapter 3)
- **Fluid statics** (Chapter 4)

Chapter 2

Fluid kinematics

Fluid kinematics: study of the motion of fluids without considering the forces and moments that cause the motion.

2.1 Descriptions of Fluid Flow

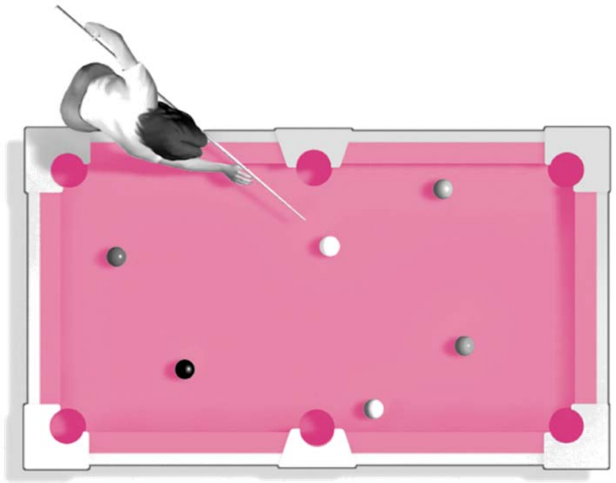
Two ways of describing a fluid flow:

- **Lagrangian** description (particle description): based on **fluid particle**
- **Eulerian** description (field description): based on **fluid field**

2.1 Descriptions of Fluid Flow

Fluid particle: a physical point, taken from the continuum. If the size is small enough, it can be regarded as a point including a large number of molecules with properties (pressure, density, velocity, temperature, etc.)

With a small number of billiard balls on a pool table, individual balls can be tracked



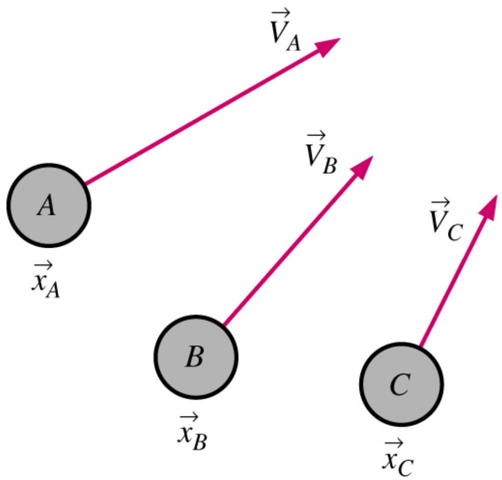
Fluid field: spatial points, denoting the location in space.

Relationships: fluid particles flow in and out through a spatial point in the fluid field. This point is fixed in space but the properties in it are changed.

2.1.1 Lagrangian Description

~ Follows individual fluid particles as they move about, determining how the fluid properties associated with these particles change as a function of time.

Lagrangian description keeps track of the position of individual **fluid particles**. Fluid particles are labeled by their positions. In Cartesian coordinates, for instant, the position of a particle at a reference time ($t=t_0$) is represented by a spatial coordinates (a, b, c). The value of (a, b, c) varies for different particles.



For a particle (a_1, b_1, c_1) moving in space, its motion can be written as:

$$
\begin{aligned}
x &= x(a_1, b_1, c_1, t) \\
y &= y(a_1, b_1, c_1, t) \\
z &= z(a_1, b_1, c_1, t)
\end{aligned}
\qquad (2\text{-}1\text{-}1)
$$

2.1.1 Lagrangian Description

The position of any fluid particle at any time is expressed as a function of these four variables (a, b, c, t), namely:

$$\begin{aligned} x &= x(a,b,c,t) \\ y &= y(a,b,c,t) \\ z &= z(a,b,c,t) \end{aligned} \qquad (2\text{-}1\text{-}2)$$

or in vector form: $\mathbf{r}=\mathbf{r}(a,b,c,t)$

2.1.1 Lagrangian Description

Velocity, acceleration and other variables of fluid particles

Equation (2-1-2) denotes the function of fluid particle and the particle path. By definition, velocity is the rate of change of the position of a particle. For the same particle, (a, b, c) does not change with time t. Therefore, velocity, acceleration and other variables can be derived from (2-1-2).

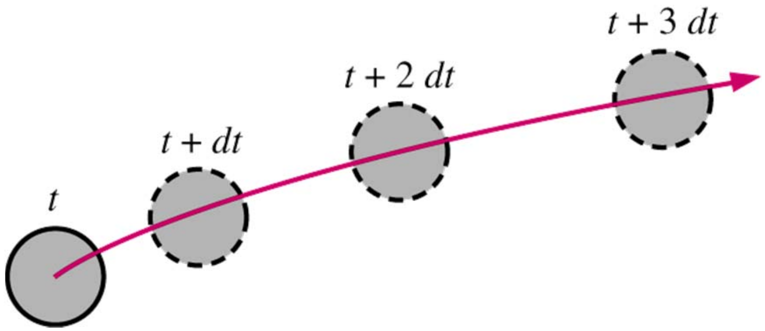
Velocity: first-order temporal derivative (partial time derivative) of particle position

$$
\begin{aligned}
u &= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x(a,b,c,t+\Delta t) - x(a,b,c,t)}{\Delta t} = \frac{\partial x}{\partial t} = u(a,b,c,t) \\
v &= \frac{\partial y}{\partial t} = v(a,b,c,t) \\
w &= \frac{\partial z}{\partial t} = w(a,b,c,t)
\end{aligned} \qquad (2\text{-}1\text{-}3)
$$

2.1.1 Lagrangian Description

Acceleration: the rate at which the velocity of a particle changes with time (second-order temporal derivative of particle position)

$$
\begin{aligned}
a_x &= \frac{\partial v_x}{\partial t} = \frac{\partial^2 x}{\partial t^2} = a_x(a,b,c,t) \\
a_y &= \frac{\partial v_y}{\partial t} = \frac{\partial^2 y}{\partial t^2} = a_y(a,b,c,t) \\
a_z &= \frac{\partial v_z}{\partial t} = \frac{\partial^2 z}{\partial t^2} = a_z(a,b,c,t)
\end{aligned}
\qquad (2\text{-}1\text{-}4)
$$



Similarly, density, pressure and temperature of fluid particles can be expressed as functions of (a, b, c, t):

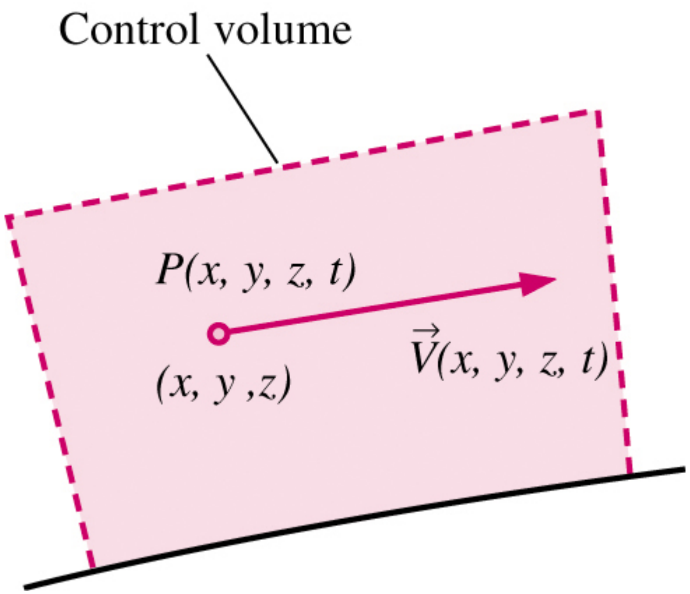
$$
\begin{aligned}
\rho &= \rho(a,b,c,t) \\
p &= p(a,b,c,t) \\
T &= T(a,b,c,t)
\end{aligned}
\qquad (2\text{-}1\text{-}5)
$$

2.1.1 Lagrangian Description

Lagrangian description is simple to be understood as: conservation of mass and Newton's laws apply directly to each fluid particle. However, it is computationally expensive to keep track of the trajectories of all the fluid particles in a flow.

2.1.2 Eulerian Description

Eulerian description focuses on flow field properties at locations or in regions of interest, and involves four independent variables: the three **spatial** coordinates represented by the position (x, y, z), and the time $\boldsymbol{t}$. In other words, it defines all **field variables** at any position (x, y, z) in a control volume and at any instant in time t:



$$u = u(x, y, z, t)$$
$$v = v(x, y, z, t) \qquad (2\text{-}1\text{-}6)$$
$$w = w(x, y, z, t)$$

$$p = p(x, y, z, t,)$$
$$\rho = \rho(x, y, z, t,) \qquad (2\text{-}1\text{-}7)$$
$$T = T(x, y, z, t,)$$

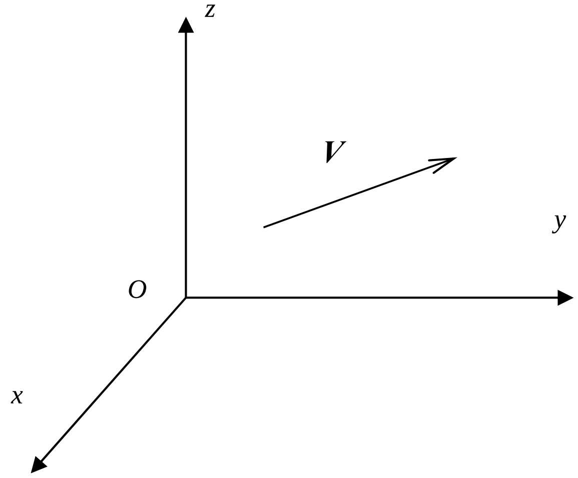
2.1.2 Eulerian Description

Field theory:

$$\boldsymbol{x} = (x, y, z) = (x_1, x_2, x_3) = x_i \quad (i = 1, 2, 3)$$

$$\boldsymbol{V} = (u, v, w) = (u_1, u_2, u_3) = u_i \quad (i = 1, 2, 3)$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right) = \frac{\partial}{\partial x_i} \quad (i = 1, 2, 3)$$



$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

Hamilton operator

Divergence of velocity field

$$\nabla \bullet \boldsymbol{V} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \bullet (u, v, w) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{\partial u_i}{\partial x_i}$$

Curl of velocity field

$$\nabla \times \boldsymbol{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \underbrace{\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)}\vec{i} + \underbrace{\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)}\vec{j} + \underbrace{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}\vec{k}$$

pressure $p(\boldsymbol{x}, t)$ - scalar (0th order tensor)

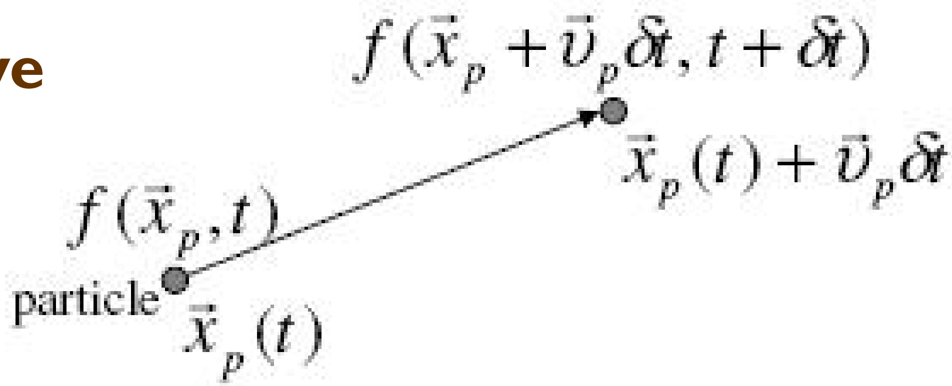
velocity $\boldsymbol{V}(\boldsymbol{x}, t)$ - vector (1st order tensor)

stress $\boldsymbol{\tau}(\boldsymbol{x}, t)$ - 2nd order tensor

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} = \tau_{ij} \quad (i, j = 1, 2, 3)$$

2.1.2 Eulerian Description

Total derivative operator:



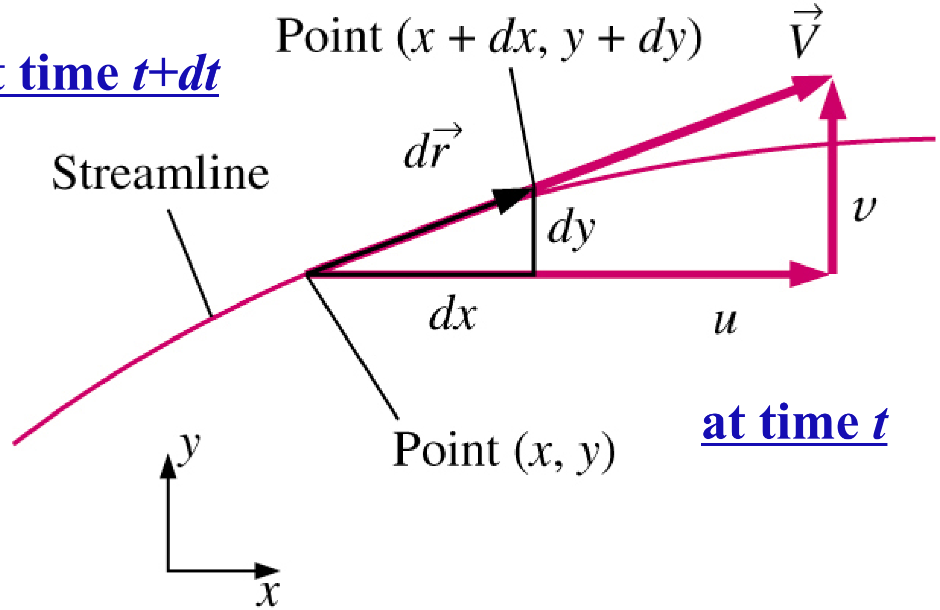
$$\frac{Df(\overrightarrow{x_p}(t), t)}{Dt} = \lim_{\delta t \to 0} \frac{f(\overrightarrow{x_p} + \overrightarrow{v_p}\delta t, t + \delta t) - f(\overrightarrow{x_p}, t)}{\delta t}$$

$$f(\overrightarrow{x_p} + \overrightarrow{v_p}\delta t, t + \delta t) = f(\vec{x}, t) + \delta t \frac{\partial f(\vec{x}, t)}{\partial t} + \delta \vec{x} \cdot \nabla f(\vec{x}, t) + O(\delta^2)$$

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \overrightarrow{v_p} \cdot \nabla f$$

2.1.2 Eulerian Description

Acceleration of a fluid particle:



$$\mathbf{a} = \lim_{\Delta \to t} \frac{\mathbf{V}'(x+\Delta x,\ y+\Delta y, z+\Delta z, t+\Delta t) - \mathbf{V}(x, y, z, t)}{\Delta t}$$

Local acceleration

$$= \frac{\partial \mathbf{V}}{\partial t} + \frac{\partial \mathbf{V}}{\partial x} \cdot u + \frac{\partial \mathbf{V}}{\partial y} \cdot v + \frac{\partial \mathbf{V}}{\partial z} \cdot w$$

$$= \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V}$$

Advective acceleration
(Convective acceleration)

2.1.2 Eulerian Description

Discussion:

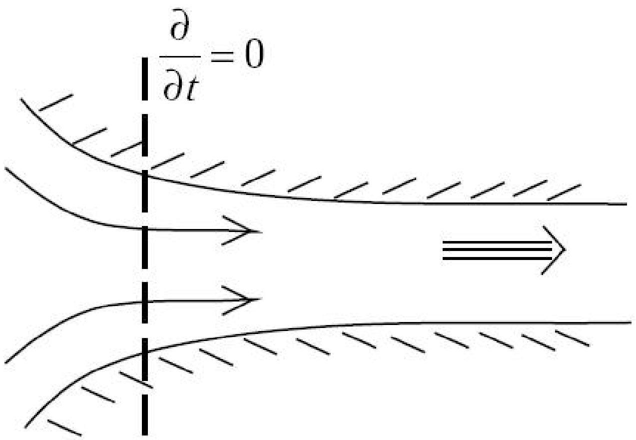
Eulerian description is a **field-based** description, focusing on flow field properties (expressed as functions of space and time, within the control volume), e.g., velocity field (vector field) , pressure field (scalar field), density field (scalar field).

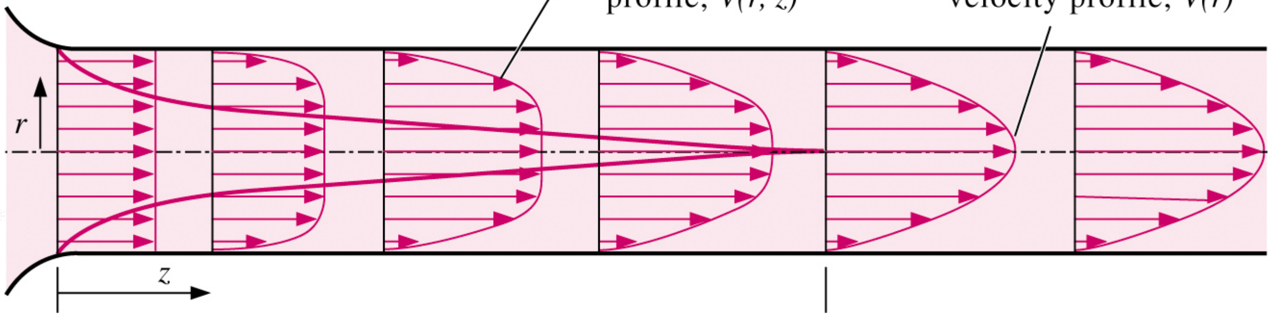
- **If properties in a flow field do not change with time t, i.e., $\frac{\partial()}{\partial t}=0$, the flow field is steady, the flow is thus steady flow; otherwise, the flow field is unsteady, corresponding to unsteady flow.**
- **If properties do not change with spatial position (x, y, z), the flow field is uniform, the flow is thus uniform flow; otherwise, the flow field is non-uniform, corresponding to non-uniform flow.**

2.1.2 Eulerian Description

1) Steady flow: velocity at a given point in space does not vary with time.

If $\frac{\partial}{\partial t} \equiv 0$, the flow is steady flow. It is defined from Eulerian description. Note that when $\frac{D}{Dt} = 0$, it does **not** mean the flow is steady.





2.1.2 Eulerian Description

2) Incompressible flow

If $\frac{D\rho}{Dt} \equiv 0$, the flow is incompressible flow. It is defined from defined from Lagrangian description.

Note that when $\frac{\partial \rho}{\partial t} = 0$, it does **not** mean the flow is incompressible.

In addition, the distinction between $\rho = const$ and $\frac{D\rho}{Dt} \equiv 0$ should be drew. The former denotes incompressible flow of the same fluid, while the latter is the general incompressible flow.

2.1.2 Eulerian Description

From the introduction above, the acceleration of a fluid particle is composed of two parts: advective acceleration (convective acceleration) caused by the convective effects of the flow and local acceleration caused by the unsteady effects of the flow, i.e.,

Total acceleration = Local acceleration + Convective acceleration

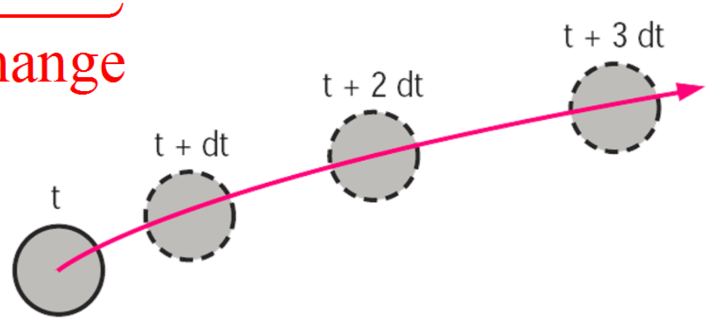
(unsteady effects) (convective effects)

2.1.2 Eulerian Description

Total derivative (material, substantial derivative):

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + (\mathbf{V} \bullet \nabla)(\ )$$

$$= \underbrace{\frac{\partial(\ )}{\partial t}}_{\text{local rate of change}} + \underbrace{u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}}_{\text{convective rate of change}}$$



e.g., local acceleration $= \dfrac{\partial \mathbf{V}}{\partial t}$

convective acceleration $= (\mathbf{V} \bullet \nabla)\mathbf{V} = u\dfrac{\partial \mathbf{V}}{\partial x} + v\dfrac{\partial \mathbf{V}}{\partial y} + w\dfrac{\partial \mathbf{V}}{\partial z}$

2.1.2 Eulerian Description

Similarly, total time derivatives of other flow properties are:

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + (\mathbf{V} \cdot \nabla)\rho$$

$$\frac{Dp}{Dt} = \frac{\partial p}{\partial t} + (\mathbf{V} \cdot \nabla)p$$

上海交通大學
Shanghai Jiao Tong University

2.1.2 Eulerian Description

Example: a three-dimensional velocity field is given by:

$$\mathbf{V} = 3yz^2\mathbf{i} + xz\mathbf{j} + y\mathbf{k}$$

Determine the acceleration field.

Solution: from the given velocity field, $u = 3yz^2,\ v = xz,\ w = y$

Acceleration in the *x*-direction can be obtained from material Derivative:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$= 0 + (3yz^2)(0) + (xz)(3z^2) + (y)(6yz) = 3zx^3 + 6y^2 z$$

Similarly:

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$= 0 + (3yz^2)(z) + (xz)(0) + (y)(x) = 3yx^3 + xy$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$= 0 + (3yz^2)(0) + (xz)(1) + (y)(0) = xz$$

Therefore, the acceleration field is:

$$a = (3yz^3 + 6y^2 z)\mathbf{i} + (3yx^3 + xy)\mathbf{j} + xz\mathbf{k}$$

2.1.3 Differences between Lagrangian and Eulerian Descriptions

Parameters	Lagrangian description	Eulerian description
Independent variable	a, b, c, t	x, y, z, t
Dependent variable	$x, y, z; p, \rho, T$	$\mathbf{V}; \rho, p, T$
Particle derivative	$\frac{\partial}{\partial t}$	$\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla (\ )$

Eulerian description is a field-based description, uses the field concept. Lagrangian description is used occasionally in problems of, e.g., waves and tornados.

In Eulerian description, $\frac{d\boldsymbol{v}}{dt}$ is first-order derivative; In Lagrangian description, $\frac{\partial^2 \boldsymbol{r}}{\partial t^2}$ is second-order derivative. Thus, the Eulerian description is often more convenient for fluid mechanics applications.

2.1.3 Differences between Lagrangian and Eulerian Descriptions

$$\underbrace{\frac{D}{Dt}}_{\text{Lagrangian}} \equiv \underbrace{\frac{\partial}{\partial t} + \overrightarrow{v_p} \cdot \nabla}_{\text{Eulerian}}$$

$$\underbrace{\frac{D\vec{G}}{Dt}}_{\substack{\text{Lagragian} \\ \text{rate of change}}} = \underbrace{\frac{\partial \vec{G}}{\partial t}}_{\substack{\text{Eulerian} \\ \text{rate of change}}} + \underbrace{\vec{v} \cdot \nabla \vec{G}}_{\substack{\text{Convective} \\ \text{rate of change}}}$$

$$\underbrace{\frac{D\vec{v}}{Dt}}_{\substack{\text{Lagragian} \\ \text{acceleration}}} = \underbrace{\frac{\partial \vec{v}}{\partial t}}_{\substack{\text{Eulerian} \\ \text{acceleration}}} + \underbrace{\vec{v} \cdot \nabla \vec{v}}_{\substack{\text{Convective} \\ \text{acceleration}}}$$

2.2 Pathlines and Streamlines

Various ways to visualize flow fields —

- **Pathlines**
- **Streamlines**