第三次作业

四、已知不可压缩流体平面流动的速度分布$v_x = x^2 + 2x - 4y$，$v_y = -2xy - 2y$。试确定流动：(1)是否满足连续性方程；(2)是否有旋；(3)如存在速度势和流函数，则求之。

解： (1)由于 $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = (2x+2) + (-2x-2) = 0$，故该流动满足连续性方程；

(2)因为是平面流动，显然，旋转角速度分量

$$\omega_x = \omega_y = 0$$

而

$$\omega_z = \frac{1}{2}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) = \frac{1}{2}[-2y - (-4)] = 2 - y \neq 0$$

所以流动有旋；

(3)因为流动有旋，所以不存在势函数；

因为流速分量满足不可压缩流体平面流动的连续性方程，所以存在流函数ψ：

$$\partial\psi/\partial x = -v_y, \quad \partial\varphi/\partial y = v_x$$

第三次作业

1)待定函数法求流函数ψ

$$\partial\varphi/\partial y = v_x = x^2 + 2x - 4y$$

对y积分得

$$\psi = \int (x^2 + 2x - 4y)dy$$

$$= (x^2 + 2x)y - 2y^2 + f(x) \qquad (1)$$

因为是偏导数对y的积分，因此积分常数应是自变量x的函数$f(x)$，即待定函数。

而ψ又满足

$$\partial\psi/\partial x = -v_y$$

所以由式(1)

$$\partial\psi/\partial x = (2x+2)y + f'(x) = -(-2x-2)y$$

$$\Rightarrow \quad f'(x) = 0, \quad f(x) = const$$

将解得的待定函数代人式(1)，略去常数，得

$$\psi = x^2y + 2xy - 2y^2$$

2)积分路径无关法求流函数ψ

$$d\psi = -v_y dx + v_x dy$$

$$\psi = \int_L -v_y dx + v_x dy$$

根据积分与路径无关，选择积分路径$L:(0,0)->(x,0)->(x,y)$，则

$$\psi = \int_{(0,0)}^{(x,y)} -v_y dx + v_x dy$$

$$= \int_{(0,0)}^{(x,y)} -(-2xy - 2y)dx + (x^2 + 2x - 4y)dy$$

$$= \int_{(0,0)}^{(x,0)} -(-2x\cdot 0 - 2\cdot 0)dx + \int_{(x,0)}^{(x,y)} (x^2 + 2x - 4y)dy$$

$$= x^2y + 2xy - 2y^2$$

第三次作业

六、不可压平面势流的速度势为 $\varphi = 0.04x^3 + axy^2 + by^3$；$x, y$ 的单位为m，势函数单位为$\mathrm{m^2/s}$。（1）求常数a，b（2）计算 $(0,0)$和 $(3,4)$两点的压力差，设流体密度为$1300\ \mathrm{kg/m^3}$。

解： (1) 求a，b。

$$\begin{cases} v_x = \dfrac{\partial \varphi}{\partial x} = 0.12x^2 + ay^2 \\ v_y = \dfrac{\partial \varphi}{\partial y} = 2axy + 3by^2 \end{cases} \qquad (a)$$

因为存在速度势，必无旋，所以应有

$$\Omega = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = 0$$

验证：

$$\Omega = 2ay - 2ay = 0$$

验证结果自然满足无旋条件，与常数a、b取值无关。

实际流动应满足连续性条件，则应用

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0.24x + (2ax + 6by)$$
$$= (0.24 + 2a)x + 6by = 0$$

由于x、y是欧拉变数——独立自变量，所以必有

$$\begin{cases} 0.24 + 2a = 0 \\ b = 0 \end{cases} \Rightarrow \begin{cases} a = -0.12 \\ b = 0 \end{cases}$$

于是，将a、b值代入(a)式，得

$$\begin{cases} v_x = 0.12x^2 - 0.12y^2 \\ v_y = -0.24xy \end{cases} \qquad (b)$$

第三次作业

(2) 求压差。运用伯努利方程。

平面势流研究的是理想不可压缩流体平面无旋流动，且流动恒定，所以理想流体恒定流动运动微分方程在无旋条件下的积分

$$z+\frac{p}{\rho g}+\frac{v^2}{2g}=c \qquad (g)$$

在流场内成立。

根据式(b)，

在(0,0)点：

$$\begin{cases} v_x=0 \\ v_y=0 \end{cases}$$

$$v^2=0$$

在(3,4)点：

$$\begin{cases} v_x=0.12\times 3^2-0.12\times 4^2 \\ \quad =-0.84\text{m/s} \\ v_y=-0.24\times 3\times 4 \\ \quad =2.88\text{m/s} \end{cases}$$

$$v^2=9\text{m}^2/\text{s}^2$$

根据式(g)

$$\left.(z+\frac{p}{\rho g}+\frac{v^2}{2g})\right|_{(0,0)}=\left.(z+\frac{p}{\rho g}+\frac{v^2}{2g})\right|_{(3,4)}$$

$$0+\frac{p_{(0,0)}}{\rho g}+0=0+\frac{p_{(3,4)}}{\rho g}+\frac{9}{2g}$$

解得

$$\Delta p=p_{(0,0)}-p_{(3,4)}=\rho g\frac{9}{2g}$$

$$=1300\times 9.807\times\frac{9}{2\times 9.807}$$

$$=5850\ \text{Pa}$$

$$=58.5\ \text{kPa}$$

第三次作业

七、已知不可压缩流体平面流动的速度矢量的模为$q=\sqrt{x^2+y^2}$，该流动的流线方程为$y^2-x^2=c$，其中c为常数。试求该流动的速度分布。

解： 令流函数

$$\psi = A(y^2-x^2) \qquad (a)$$

其中A为常数。

则流线方程为

$$\psi = A(y^2-x^2)=c$$

即

$$y^2-x^2=c$$

从而有

$$\begin{cases} v_x=\dfrac{\partial\psi}{\partial y}=2Ay \\ v_y=-\dfrac{\partial\psi}{\partial x}=2Ax \end{cases} \qquad (b)$$

模

$$q=\sqrt{v_x^2+v_y^2}=\sqrt{(2Ay)^2+(2Ax)^2}$$

$$=2|A|\sqrt{x^2+y^2}$$

已知

$$q=\sqrt{x^2+y^2}$$

于是

$$2|A|\sqrt{x^2+y^2}=\sqrt{x^2+y^2}, \qquad 2|A|=1$$

⇨ $$A=\pm\frac{1}{2}$$

代入式(b)，得

$$\begin{cases} v_x=y \\ v_y=x \end{cases} \quad 或 \quad \begin{cases} v_x=-y \\ v_y=-x \end{cases}$$

复习

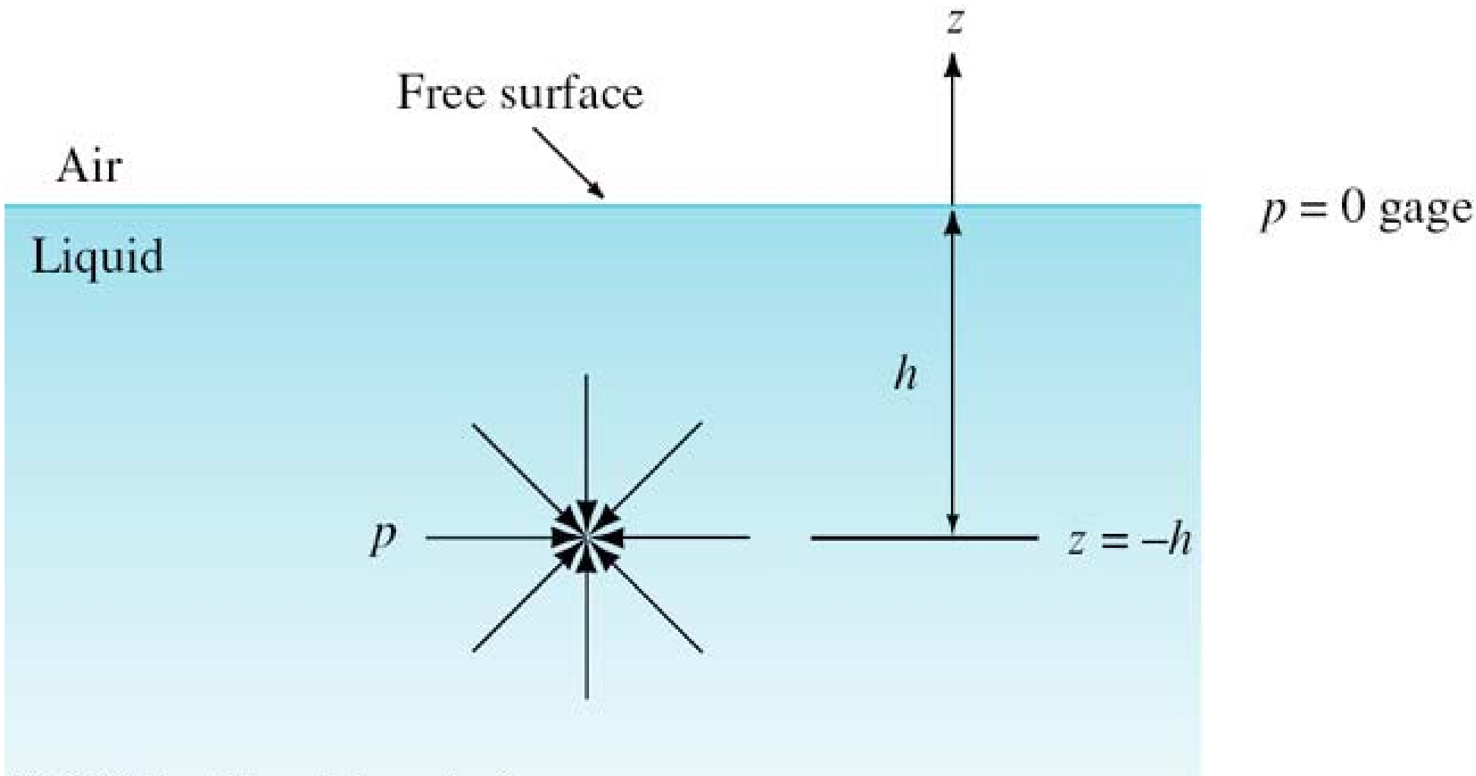
- **Hydrostatics**

1) Pascal's law: **Pressure at a point in a fluid is independent of direction as long as there are no shear stresses present, ie, Pressure at a point has the same magnitude in all directions.**

2) Pressure Variation with Depth: **The pressure is the same at all points with the same depth from the free surface regardless of geometry, while its direction depends on the geometry.**

复习

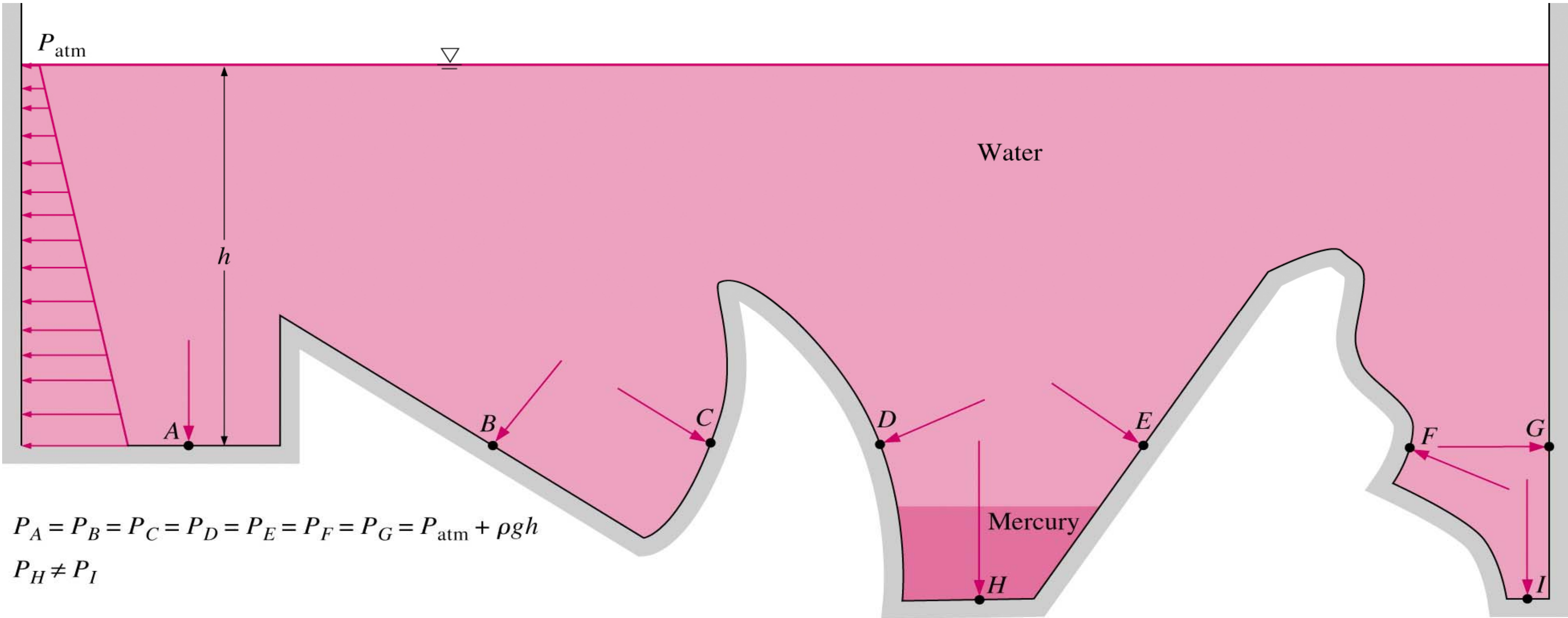
Pressure at a point has the same magnitude in all directions, and is called isotropic.



复习

The pressure is the same at all points with the same depth from the free surface regardless of geometry, provided that the points are interconnected by the same fluid.

However, the thrust due to pressure is perpendicular to the surface on which the pressure acts, and hence its direction depends on the the geometry.



$$P_A = P_B = P_C = P_D = P_E = P_F = P_G = P_{atm} + \rho g h$$

$$P_H \neq P_I$$

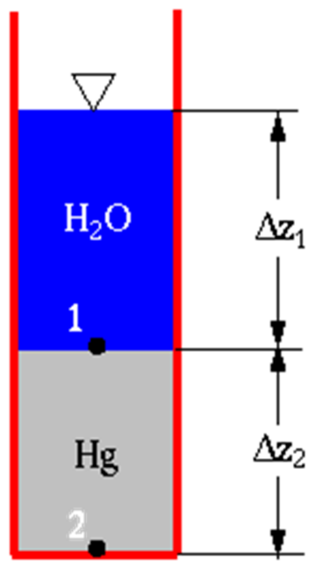
复习

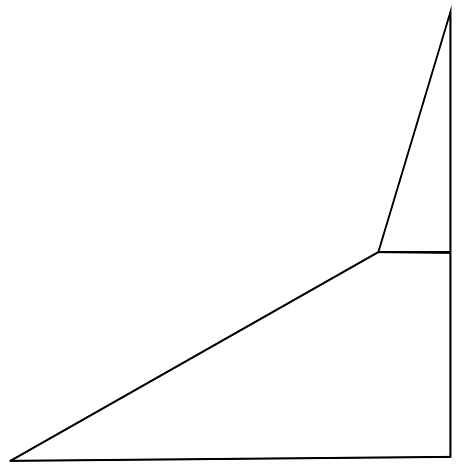
3) Hydrostatics equation: when the density is constant, the pressure in a liquid at rest increases linearly with depth from the free surface.

$$\frac{dp}{dz} = -\rho g$$

$$p = -\rho g z + p_0$$

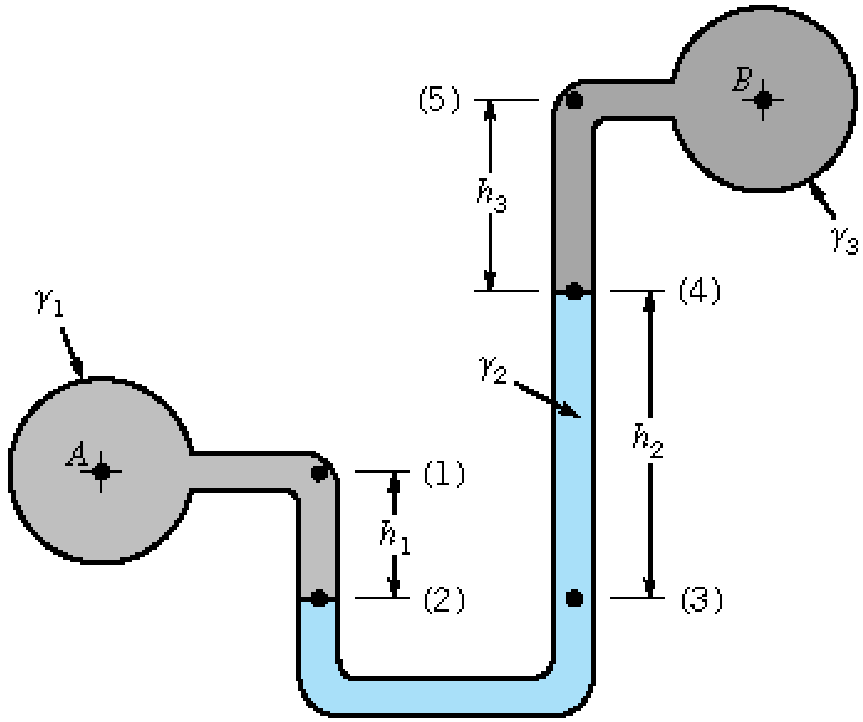
$$p_{\text{below}} = p_{\text{above}} + \rho g \left|\Delta z\right|$$





3) Differential manometer (差压力计)

A differential manometer can be used to measure the difference in pressure between two containers or two points in the same system. Again, on equating the pressures at points labeled (2) and (3), we may get an expression for the pressure difference between *A* and *B*:

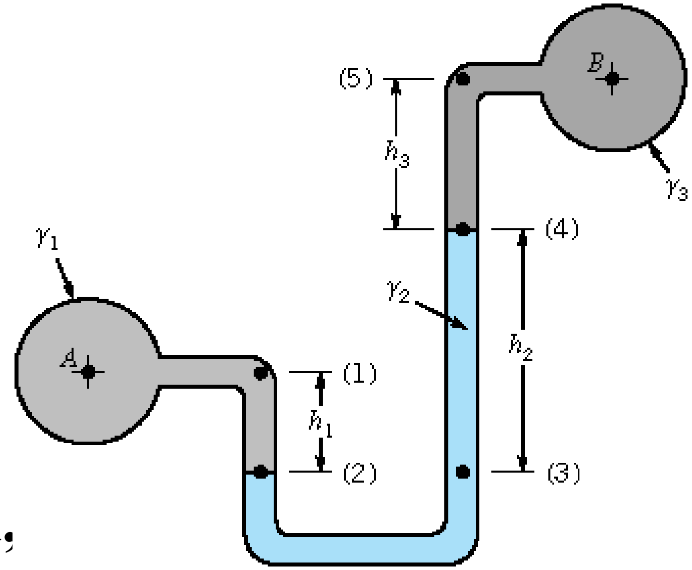


$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

4.6 Pressure Measurement and Manometers

In the common case when *A* and *B* are at the same elevation $(h_1 = h_2 + h_3)$ and the fluids in the two containers are the same $(\gamma_1 = \gamma_3)$ one may show that the pressure difference registered by a differential manometer is given by

$$\Delta p = \left(\frac{\rho_m}{\rho} - 1 \right) \rho g h_2$$



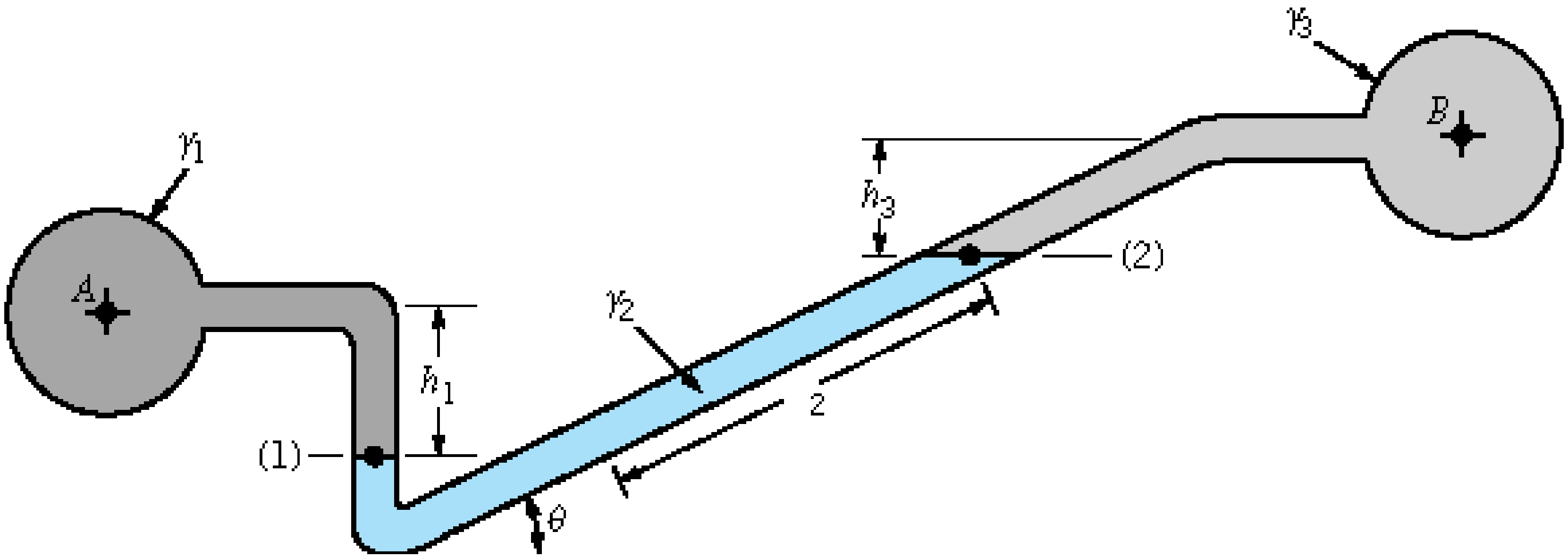
where

ρ_m **is the density of the manometer fluid,**

ρ **is the density of the fluid in the system,**

h **is the manometer differential reading.**

4) Inclined-tube manometer (斜管压力计)



As shown above, the differential reading is proportional to the pressure difference. If the pressure difference is very small, the reading may be too small to be measured with good accuracy. To increase the sensitivity of the differential reading, one leg of the manometer can be inclined at an angle θ .

and the differential reading is measured along the inclined tube. As shown above, $h_2 = \ell_2 \sin\theta$ **and hence**

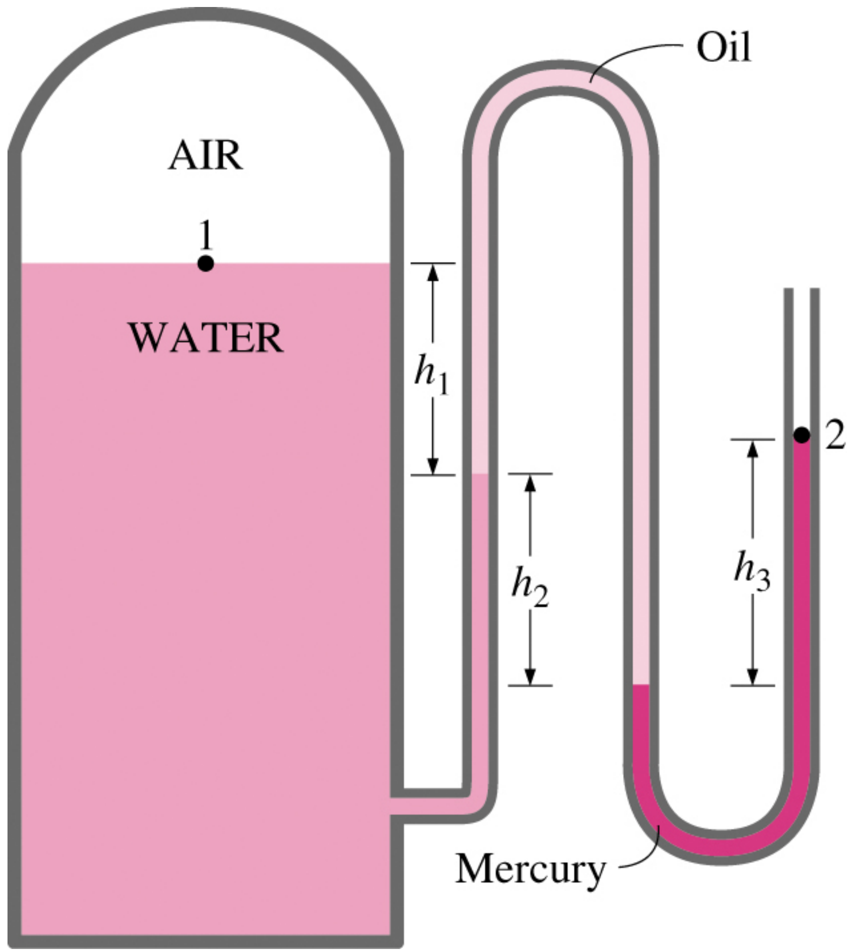
$$p_A - p_B = \gamma_2 \ell_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$

Obviously, the smaller the angle θ **, the more the reading** ℓ_2 **is magnified.**

5) Multifluid manometer (多流体压计管)

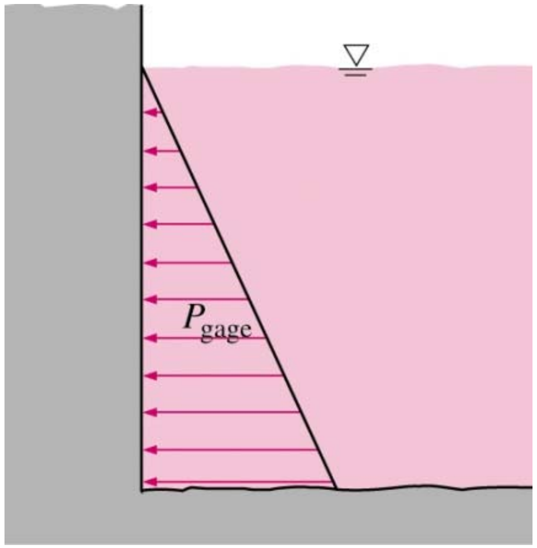
The pressure in a pressurized tank is measured by a multifluid manometer, as is shown in the figure. Show that the air pressure in the tank is given by

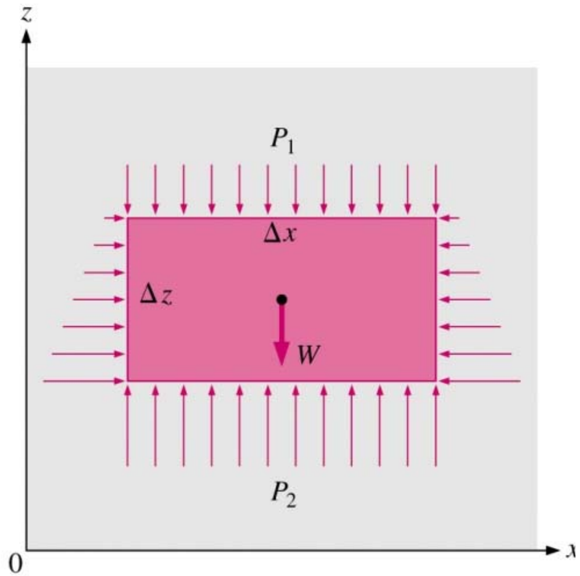
$$P_{\text{air}} = P_{\text{atm}} + g\left(\rho_{\text{mercury}} h_3 - \rho_{\text{oil}} h_2 - \rho_{\text{water}} h_1\right)$$

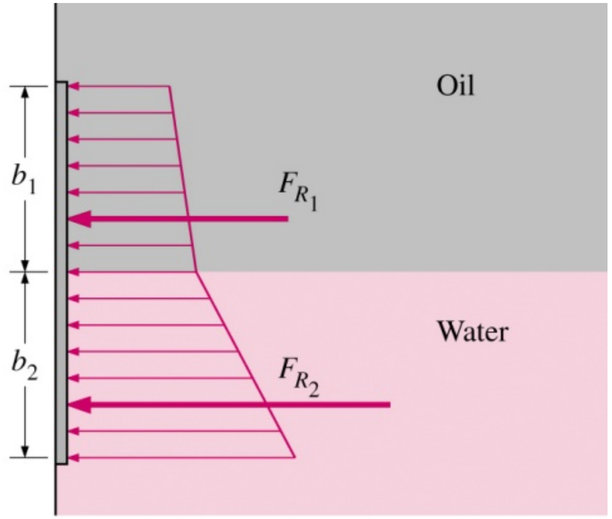


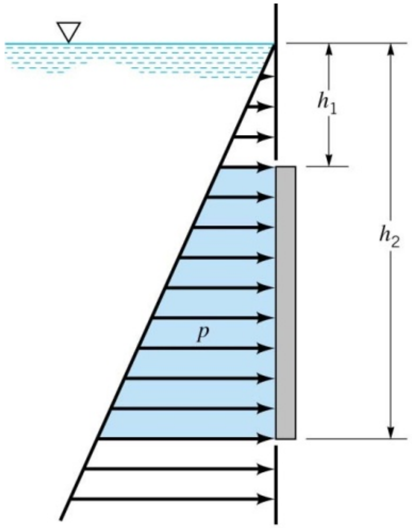
4.7 Pressure Distributions

1) Flat Surfaces

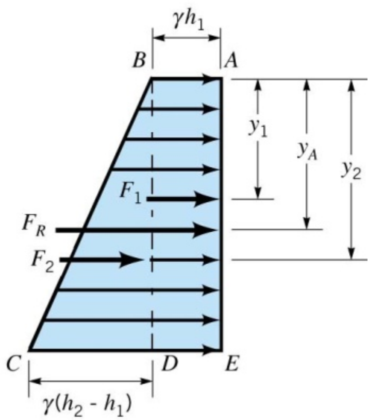




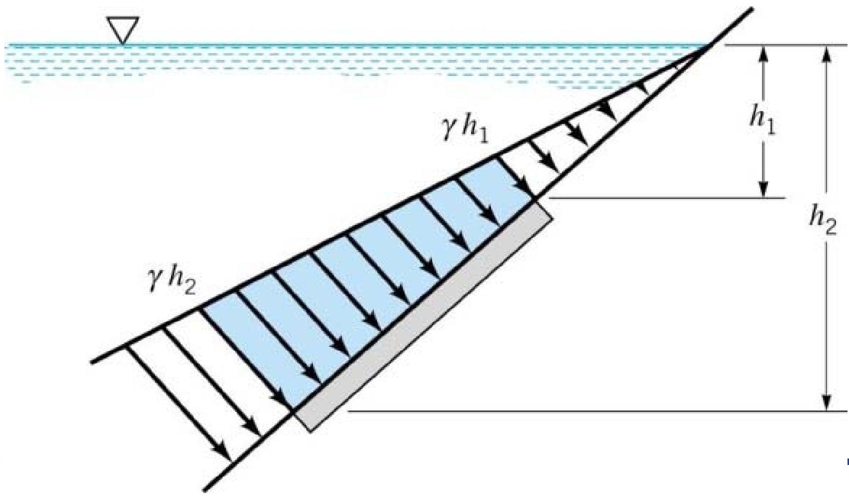




(a)



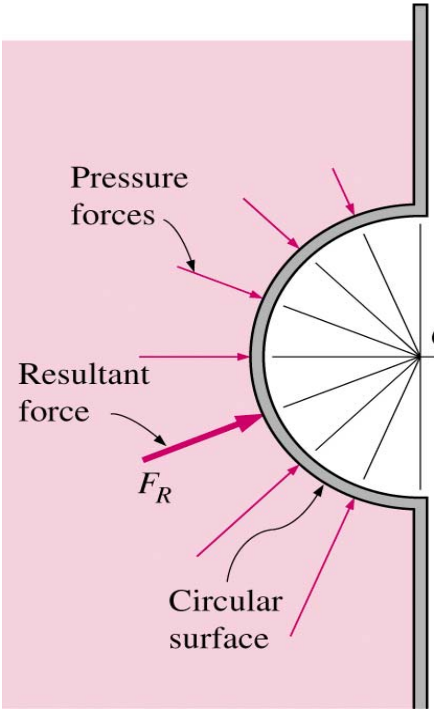
(b)

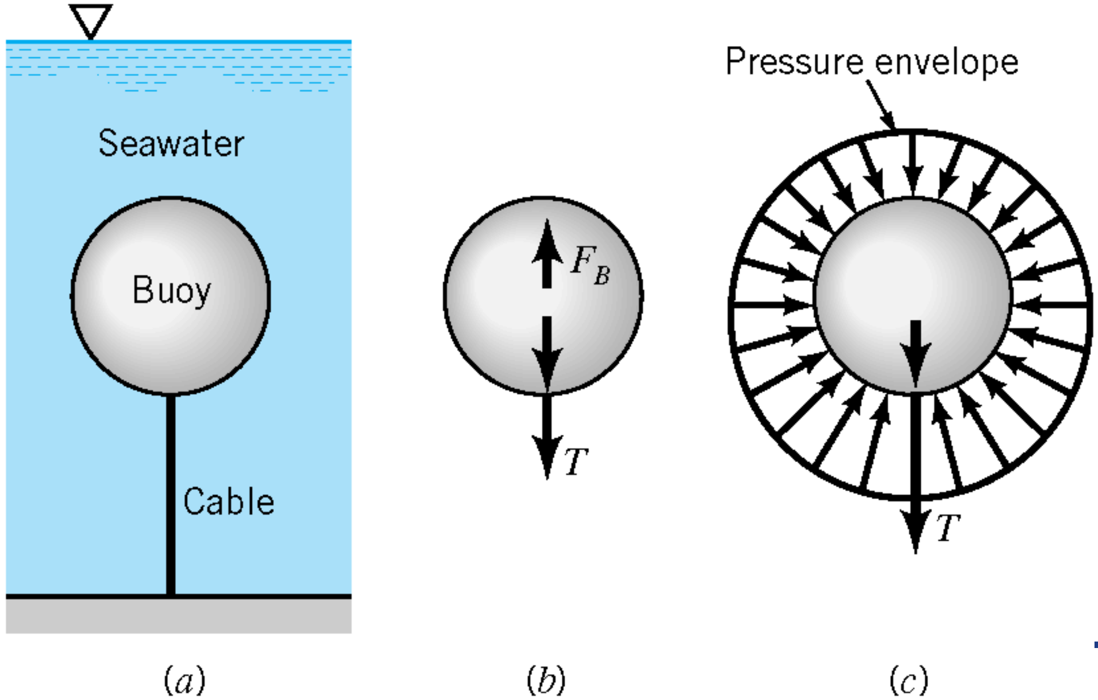


4.7 Pressure Distributions

2) Curved Surfaces

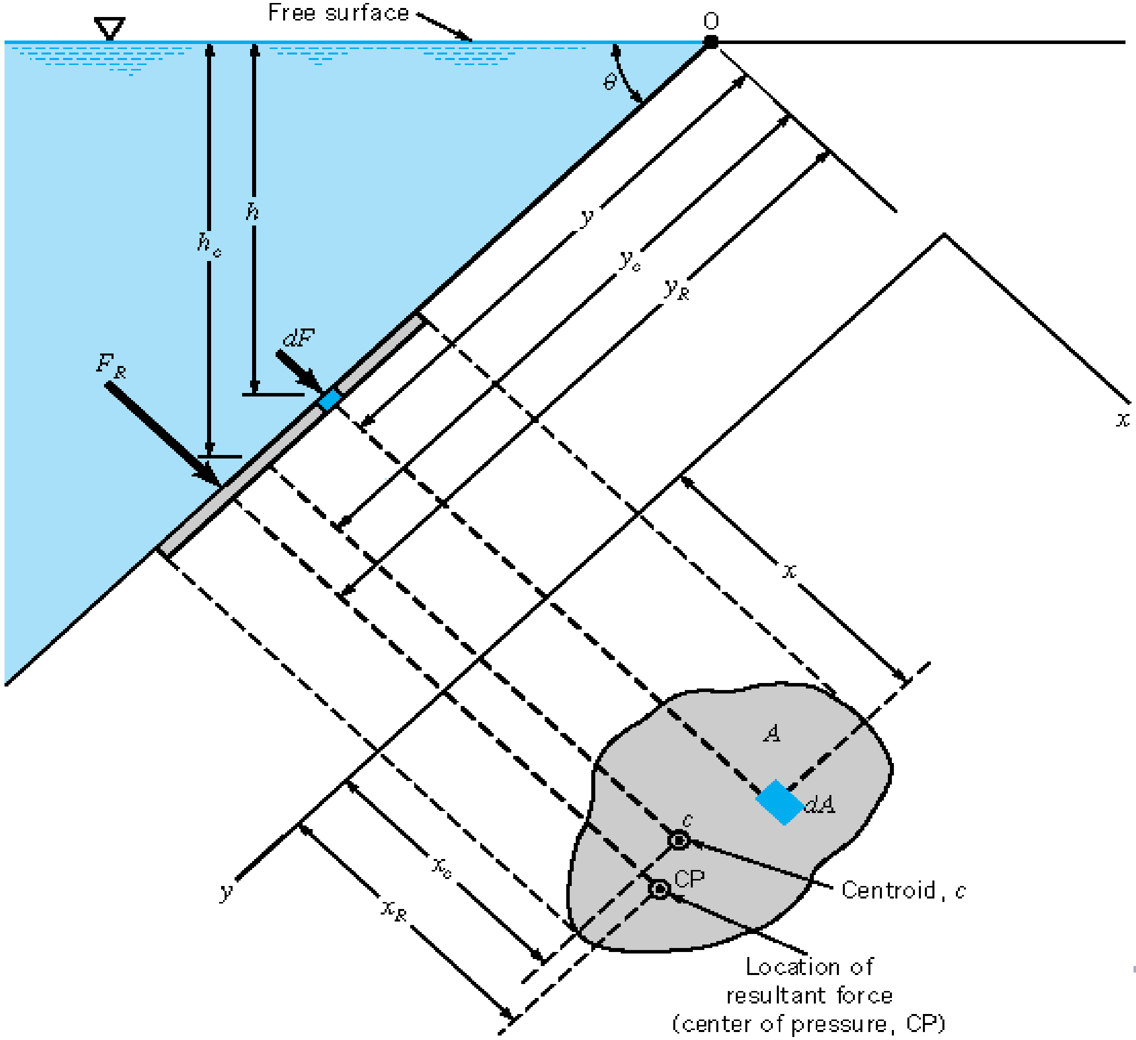
When the curved surface is a *circular arc* (full circle or any part of it), the resultant hydrostatic force acting on the surface always passes through the center of the circle. This is because the elemental pressure forces are normal to the surface, and by the well-known geometrical property all lines normal to the surface of a circle must pass through the center of the circle.



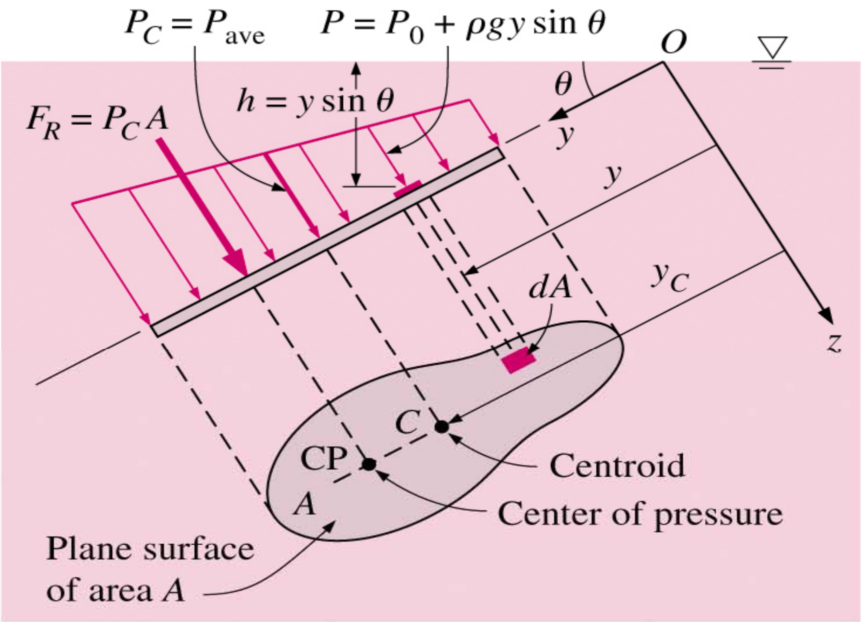


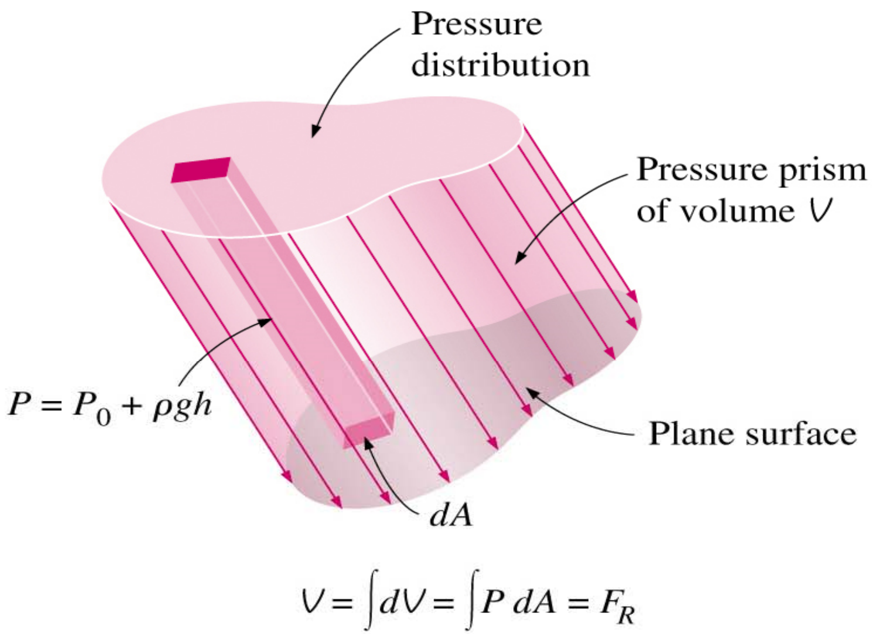
4.8 Hydrostatic Force on a Plane Surface

Suppose a *submerged* plane surface is inclined at an angle θ to the free surface of a liquid.



4.8 Hydrostatic Force on a Plane Surface





A - area of the plane surface

O - the line where the plane in which the surface lies intersects the free surface,

C - centroid (or centre of area) of the plane surface,

CP - center of pressure (point of application of the resultant force on the plane surface),

F_R - magnitude of the resultant force on the plane surface (acting normally),

h_R - vertical depth of the center of pressure CP,

h_c - vertical depth of the centroid C,

y_R - inclined distance from O to CP, y_c - inclined distance from O to C.

4.8 Hydrostatic Force on a Plane Surface

1) Find magnitude of resultant force

The resultant force is found by integrating the force due to hydrostatic pressure on an element *dA* at a depth *h* over the whole surface:

$$F_R = \int_A dF = \int_A \rho g h dA = \rho g \sin\theta \int_A y dA$$

where by the first moment of area $\int_A y dA = y_c A$ **, hence**

$$F_R = \rho g \left(y_c \sin\theta \right) A = \rho g h_c A$$

The resultant force on one side of any plane submerged surface in a uniform fluid is therefore equal to the pressure at the centroid of the surface times the area of the surface, independent of the shape of the plane or the angle θ at which it is slanted.

2) Find location of centre of pressure

Taking moment about *O*,

$$F_R y_R = \int_A y\,dF \Rightarrow \left(\rho g y_c \sin\theta A\right) y_R = \int_A y\left(\rho g y \sin\theta\, dA\right) \Rightarrow \left(y_c A\right) y_R = \int_A y^2\, dA$$

$$\int_A y^2\, dA = I_O = I_c + A y_c^2 \quad \text{by parallel axis theorem}$$

where

I_O **= second moment of area (or moment of inertia) of the surface about *O*.**

I_c **= second moment of area (or moment of inertia) about an axis through the centroid *C* and parallel to the axis through *O*.**

4.8 Hydrostatic Force on a Plane Surface

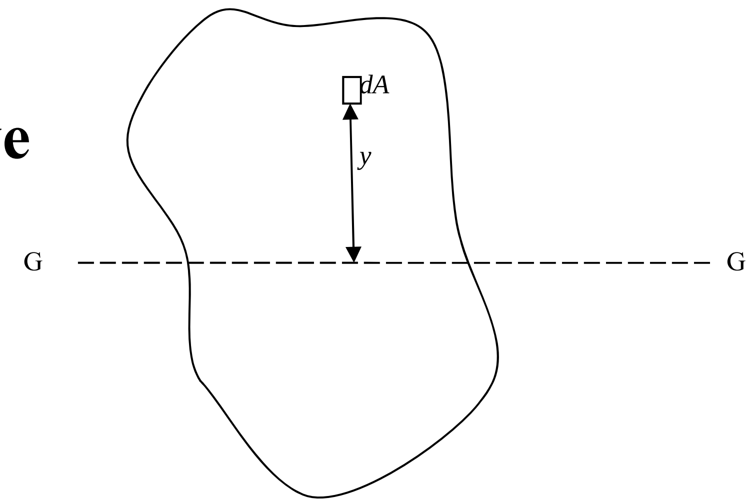
Therefore, on substituting,

$$\left(y_c A\right) y_R = A y_c^2 + I_c$$

$$\Rightarrow \boxed{y_R = y_c + \frac{I_c}{y_c A}} \qquad \text{or} \qquad h_R = h_c + \frac{I_c \sin^2\theta}{h_c A}$$

Now, the depth of the center of pressure depends on the shape of the surface and the angle of inclination, and is always below the depth of the centroid of the plane surface.

Second Moment of Area



For a plane surface of arbitrary shape, we may define the n^{th} (n = 0, 1, 2, 3, ...) moment of area about an axis GG by the integral:

$$\int_A y^n \, dA$$

Then,

- **the zeroth moment of area = total area of the surface,**
- **the first moment of area = 0, if GG passes through the centroid of the surface,**
- **the second moment of area gives the variance of the distribution of area about the axis.**

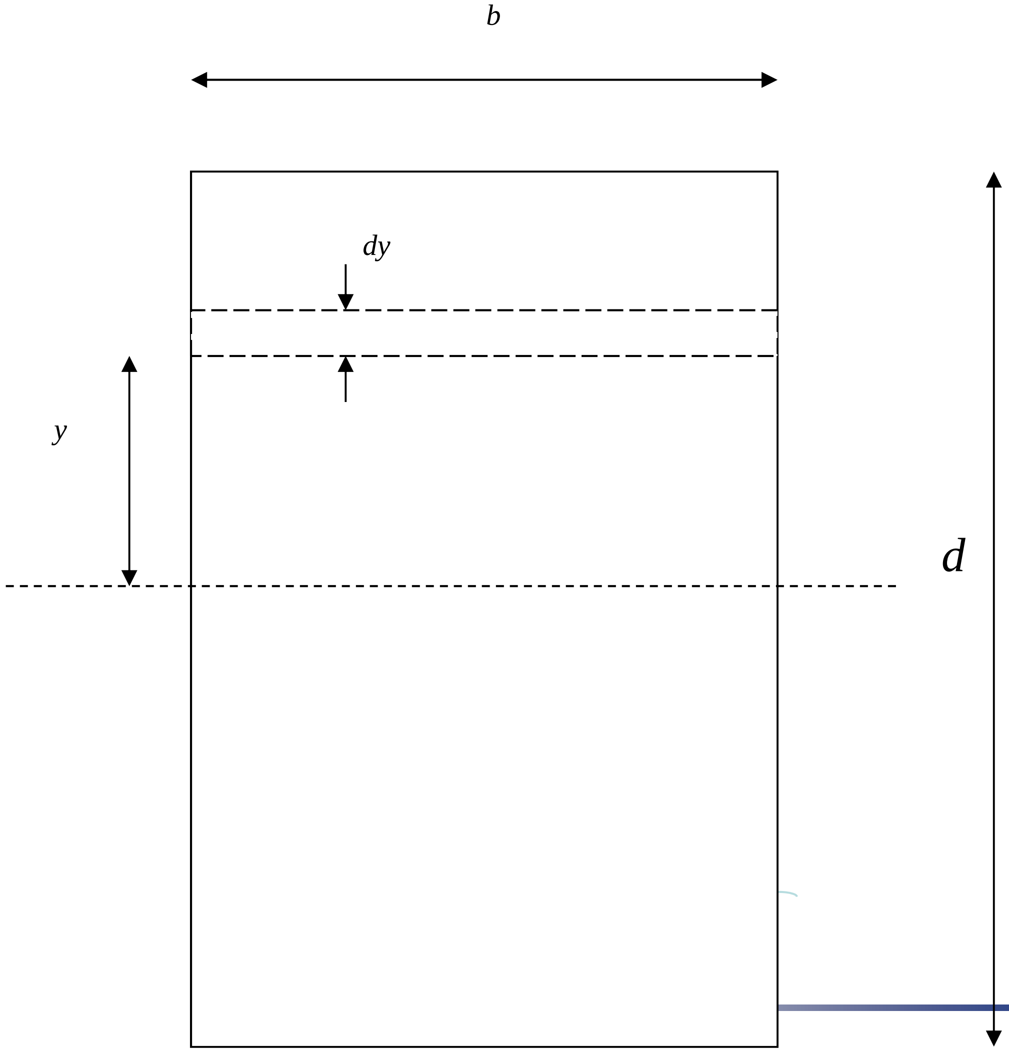
For example, for a rectangular surface, the second moment of area about the axis that passes through the centroid is

$$I_c = \int_A y^2 dA$$

$$= \int_{-d/2}^{d/2} y^2 (bdy)$$

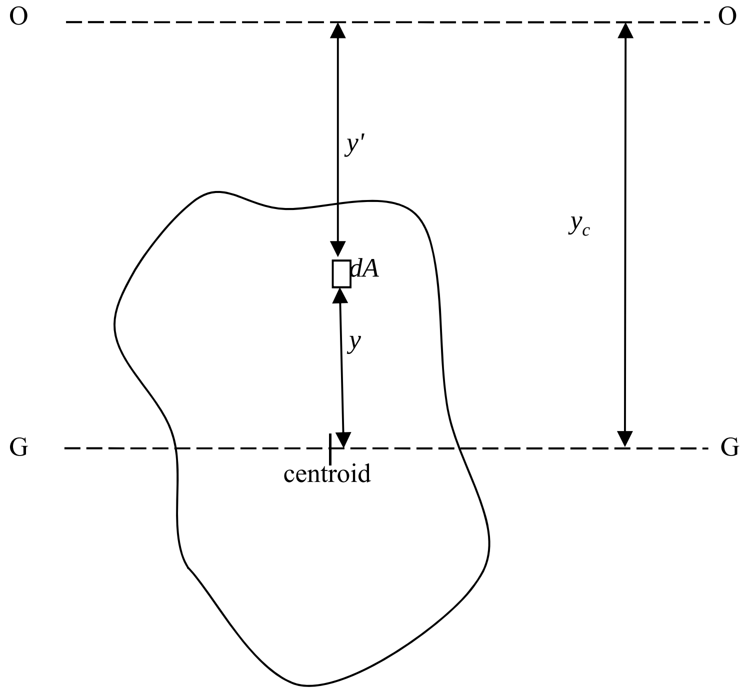
$$= \left[\frac{by^3}{3}\right]_{-d/2}^{d/2}$$

$$= \frac{bd^3}{12}$$



Parallel Axis Theorem

If OO is an axis that is parallel to the axis GG, which passes through the centroid of the surface, then the second moment of area about OO is equal to that about GG plus the square of the distance between the two axes times the total area:

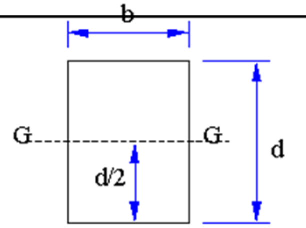
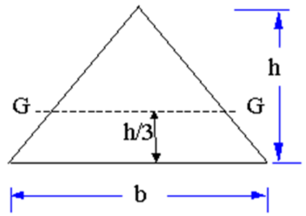
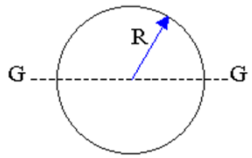
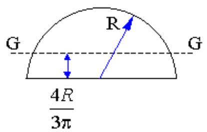


$$I_o = \int_A y'^2 dA = \int_A (y_c - y)^2 dA = \int_A \left(y_c^2 - 2y_c y + y^2\right) dA$$

$$= y_c^2 A - 2y_c \underbrace{\int_A y dA}_{0} + \underbrace{\int_A y^2 dA}_{I_c} = y_c^2 A + I_c$$

Properties for some Common sectional areas

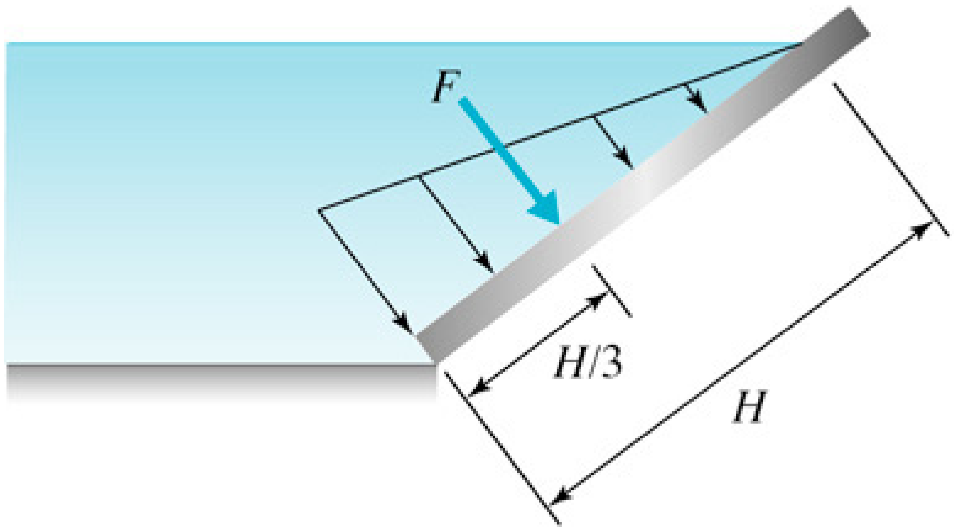
GG is an axis passing through the centroid and parallel to the base of the figure.

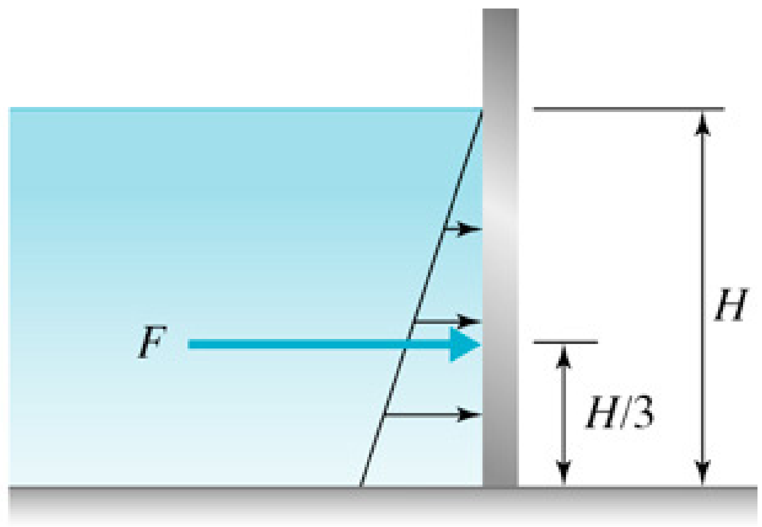
Shape	Dimensions	Area	(moment of inertia about GG) I_c
Rectangle		bd	$\frac{bd^3}{12}$
Triangle		$\frac{bh}{2}$	$\frac{bh^3}{36}$
Circle		πR^2	$\frac{\pi R^4}{4}$
Semi-Circle		$\frac{\pi R^2}{2}$	$0.11R^4$

4.8 Hydrostatic Force on a Plane Surface

3) For a flat surface that pierces through the free surface, and hence triangular pressure distribution:

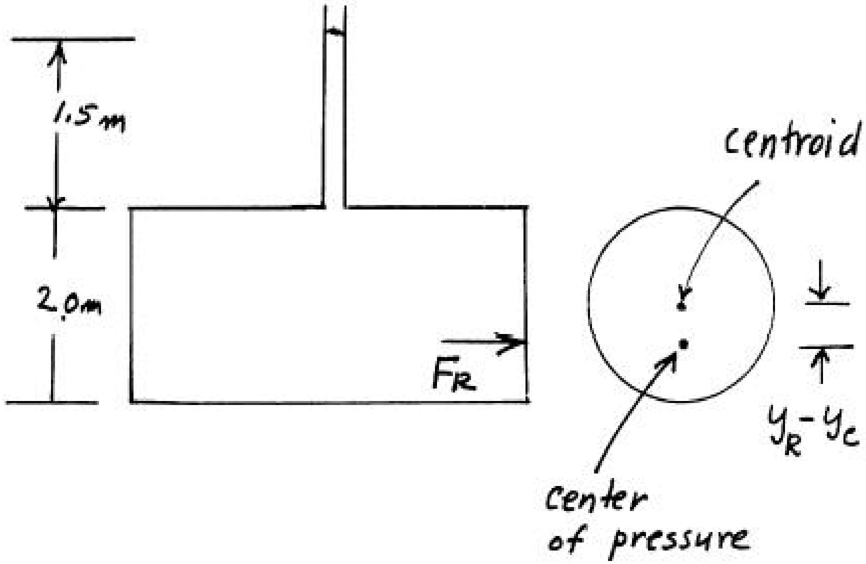
$$A = HB, \quad h_c = \frac{1}{2} H\sin\theta, \quad h_R = \frac{2}{3} H\sin\theta, \quad F = \frac{1}{2}\rho g H^2 B\sin\theta$$





4.8 Hydrostatic Force on a Plane Surface

例子1：如图所示一个水平放置的圆柱桶，直径2.0 m，长4.0 m。在圆柱桶的上部连接一个小圆管，直径0.1 m。在圆柱桶和小圆管内充满比重为7.74 kN/m³的液体，高度达圆柱桶上部的1.5 m。求作用在圆柱桶顶端面的作用力和作用点。



解：

圆柱桶顶端受到水的总压力为：

$$F_R = \gamma h_c A, \quad \text{where} \quad h_c = 1.5\text{m} + 1.0\text{m} = 2.5\text{m}$$

所以有：

$$F_R = \gamma h_c A = \left(7.74\frac{\text{kN}}{\text{m}^3}\right)(2.5\text{m})\left(\frac{\pi}{4}\right)(2.0\text{m})^2 = 60.8\,\text{kN}$$

4.8 Hydrostatic Force on a Plane Surface

圆柱桶顶端受到水的总压力的作用点为:

$$y_R = \frac{I_{xc}}{y_c A} + y_c, \quad \text{where} \quad y_c = h_c$$

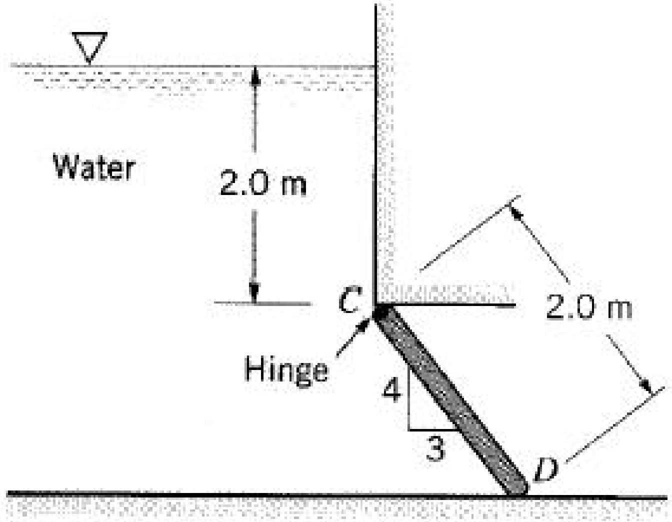
所以有:

$$y_R = \frac{\dfrac{\pi (1\text{m})^4}{4}}{(2.5\text{m})\left(\dfrac{\pi}{4}\right)(2\text{m})^2} + 2.5\text{m} = 2.60\text{m}$$

所以，作用在圆柱桶顶端圆板上的力为**60.8 kN**，作用点在离圆柱桶顶端圆板圆中心点下**0.1 m**，即

$$y_R - y_c = 2.60\text{m} - 2.50\text{m} = 0.100\text{m}$$

例子2： The rectangular gate *CD* shown in the figure is 1.8 m wide and 2.0 m long Assuming the material of the gate to be homogeneous and neglecting friction at the hinge *C*. Determine the weight of the gate necessary to keep it shut until the water level rises to 2.0 m above the hinge

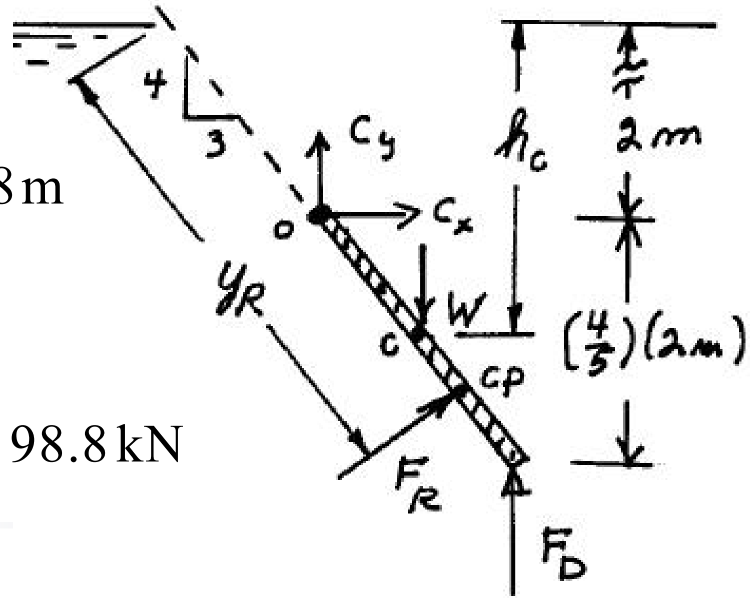


Solution: The force acting on the gate:

$$F_R = \gamma h_c A, \quad \text{where } h_c = 2\,\text{m} + \frac{1}{2}\left[\left(\frac{4}{5}\right)(2\,\text{m})\right] = 2.8\,\text{m}$$

Thus

$$F_R = \gamma h_c A = \left(9.80\,\frac{\text{kN}}{\text{m}^3}\right)(2.8\,\text{m})(1.8\,\text{m} \times 2\,\text{m}) = 98.8\,\text{kN}$$



Also $$y_R = \frac{I_{xc}}{y_c A} + y_c, \quad \text{where} \quad y_c = \frac{2\,\mathrm{m}}{(4/5)} + 1\,\mathrm{m} = 3.5\,\mathrm{m}$$

So that

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\left(\frac{1}{12}\right)(1.8\,\mathrm{m})(2\,\mathrm{m})^3}{(3.5\,\mathrm{m})(1.8\,\mathrm{m} \times 2\,\mathrm{m})} + 3.5\,\mathrm{m} = 3.595\,\mathrm{m}$$

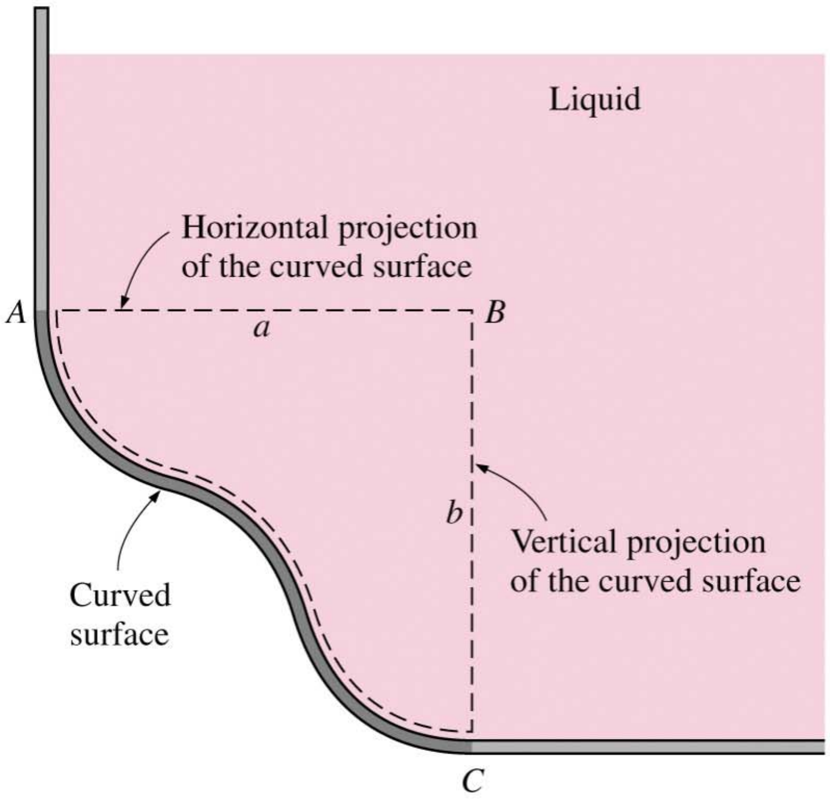
For equilibrium (here set F_D=0 to obtain minimum weight), we have

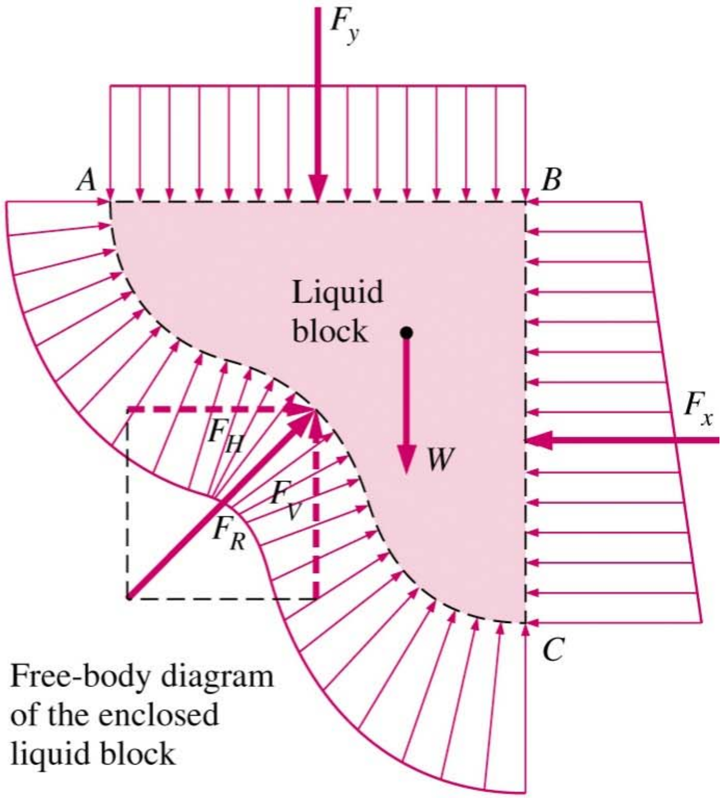
$$\sum M_o = 0 \quad \Rightarrow \quad W\left(\frac{1}{2}\right)\left[\left(\frac{3}{5}\right)(2\,\mathrm{m})\right] - F_R\left(y_R - \frac{2\,\mathrm{m}}{(4/5)}\right) = 0$$

$$\Rightarrow \quad W = \frac{(98.8\,\mathrm{kN})(3.595\,\mathrm{m} - 2.5\,\mathrm{m})}{\left(\frac{1}{2}\right)\left[\left(\frac{3}{5}\right)(2\,\mathrm{m})\right]} = 180\,\mathrm{kN}$$

1) Liquid above surface

Suppose we are required to find the force acting on the upper side of the curved surface AC.



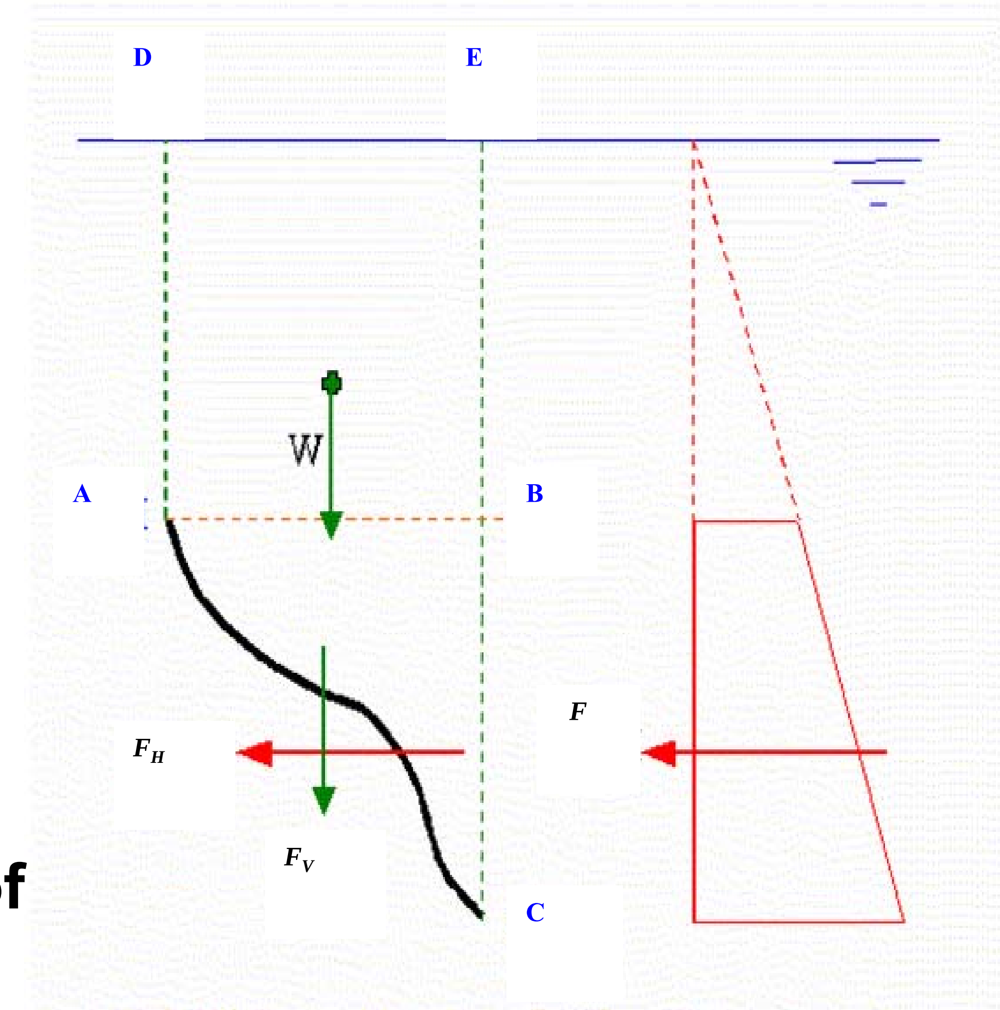


Horizontal component of force on surface:

By considering the equilibrium of the liquid mass contained in ABC,

we get

$F_H = F$ = resultant force of liquid acting on vertically projected area (BC) and acting through the centre of pressure of F.

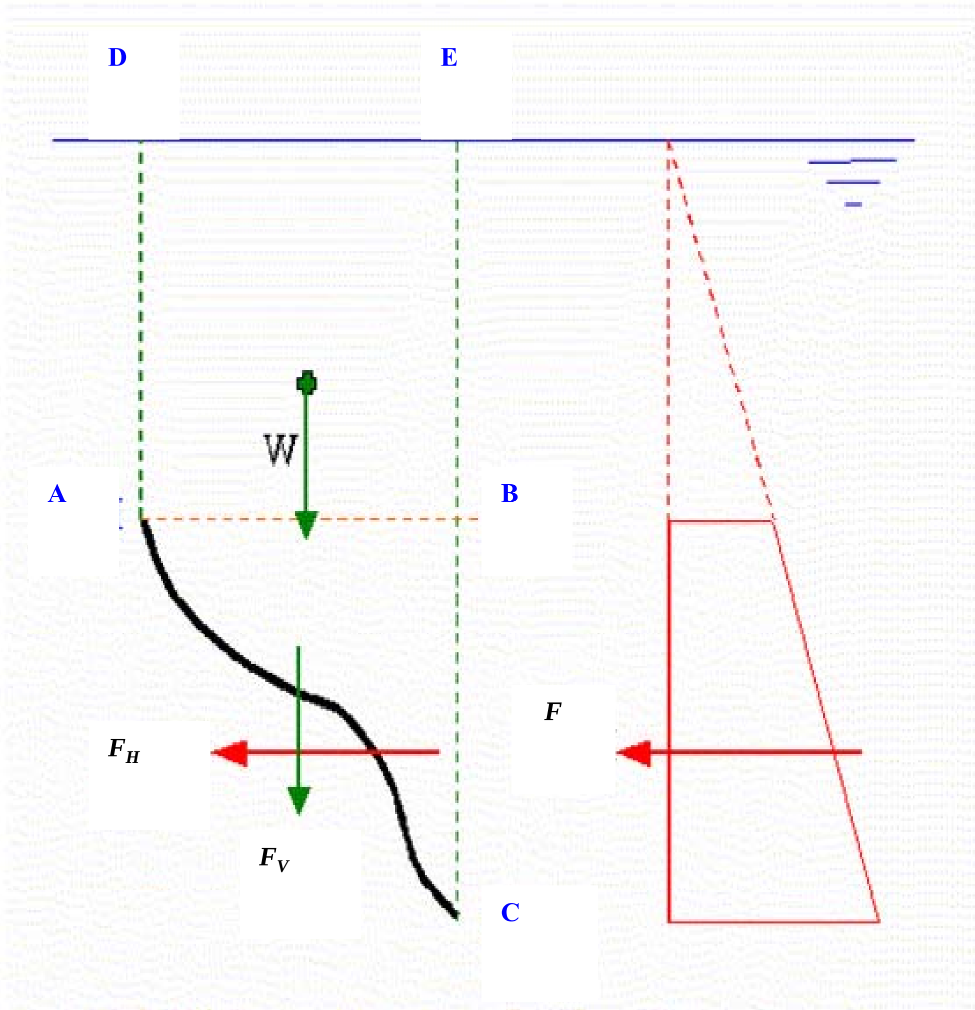


Vertical component of force on surface:

By considering the equilibrium of the liquid mass contained in ADEC,

we get

$F_V = W$ = weight of liquid vertically above the surface (ADEC) and through the centre of gravity of the liquid mass.



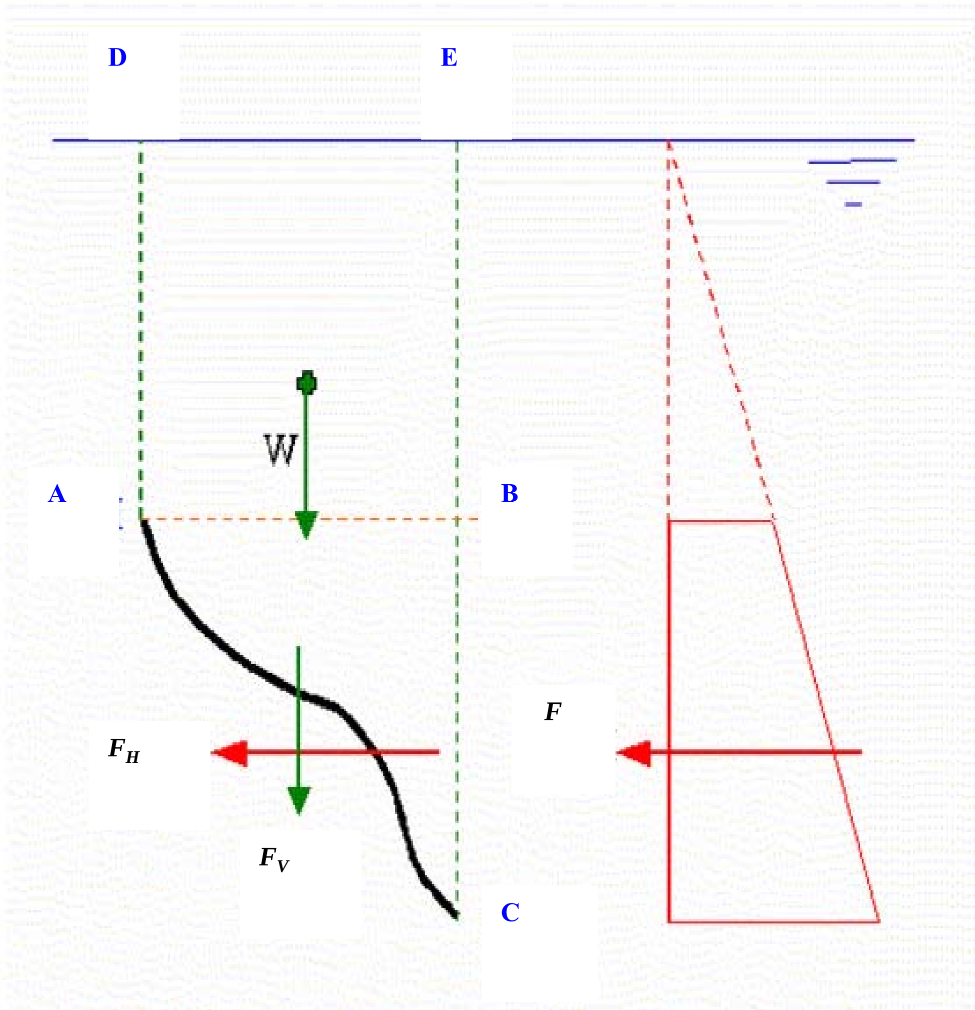
Resultant force:

$$F_R = \sqrt{F_H^2 + F_V^2}$$

pointing downward, and making an angle

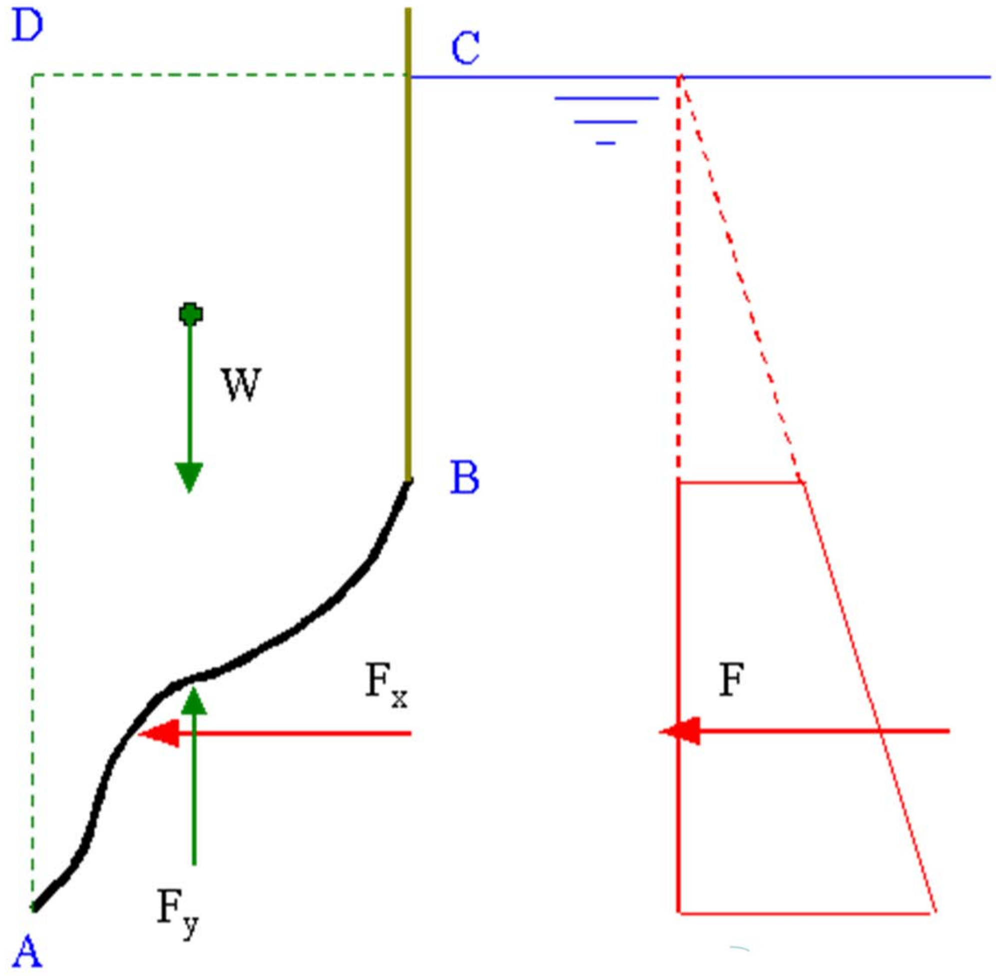
$$\alpha = \tan^{-1}\left(F_V / F_H\right)$$

with the horizontal.

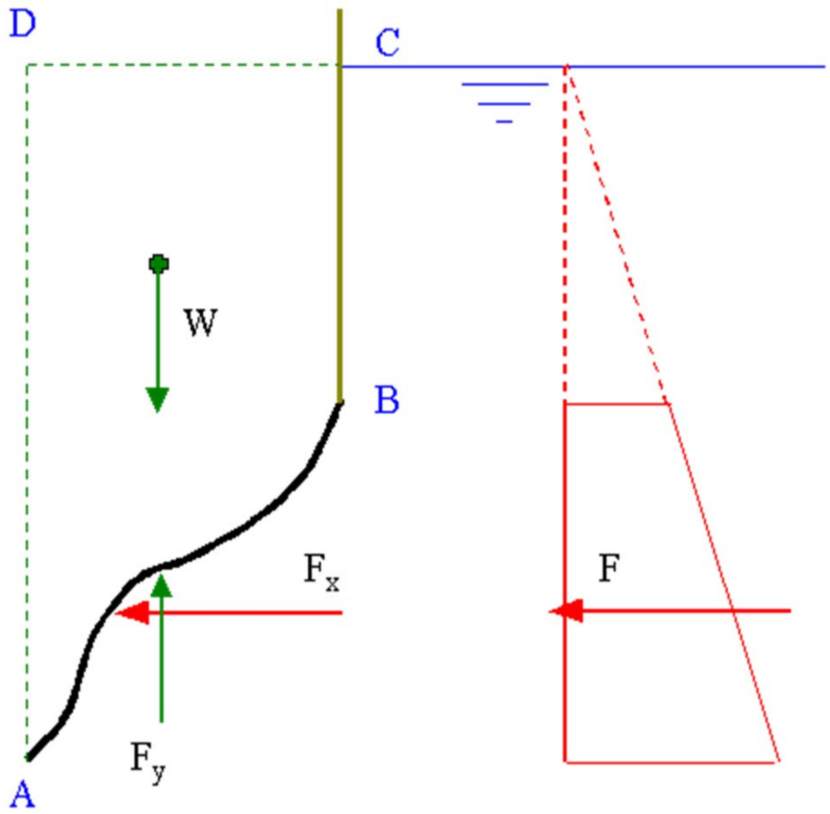


2) Liquid below surface

Suppose we are required to find the force acting on the underside of the curved surface AB. The space above the surface ADCB may be empty or contain other fluid.

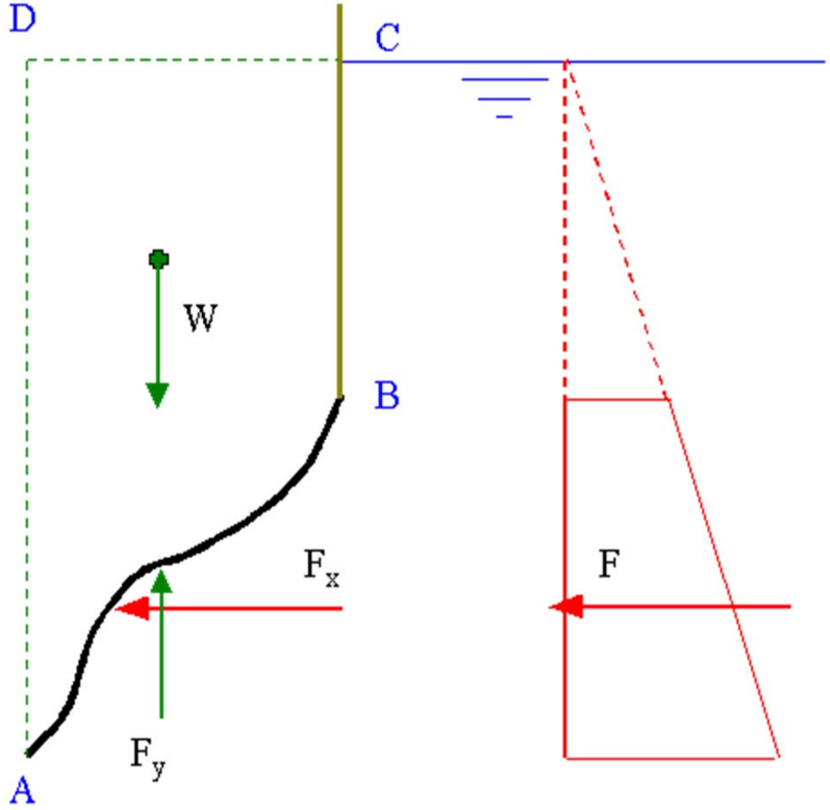


Imagine that the space (ADCB) vertically above the curved surface is occupied with the same fluid as that below it. Then the surface AB could be removed without disrupting the equilibrium of the fluid. That means, the force acting on the underside of the surface would be balanced by that acting on the upper side under this imaginary condition. Therefore we may use the same arguments as in the preceding case:



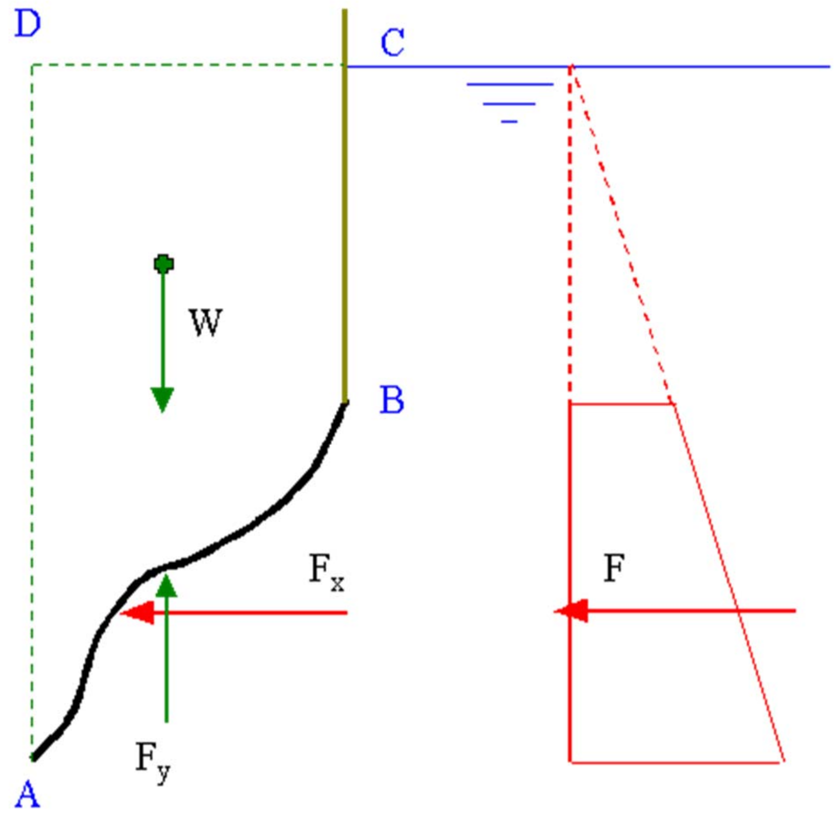
Horizontal component of force on surface:

F_H = F = resultant force of liquid acting on vertically projected area (AB) and acting through the centre of pressure of F.



Vertical component of force on surface:

$F_V = W$ = **weight of imaginary liquid (i.e., same liquid as on the other side of the surface) vertically above the surface (ADCB) and through the centre of gravity of the liquid mass.**



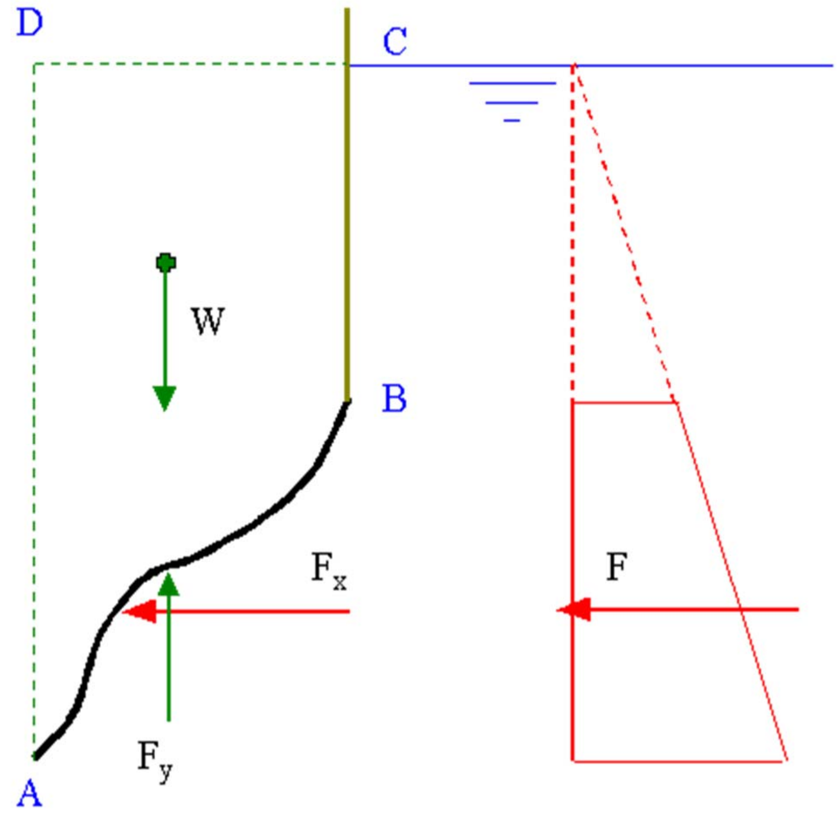
Resultant force:

$$F_R = \sqrt{F_H^2 + F_V^2}$$

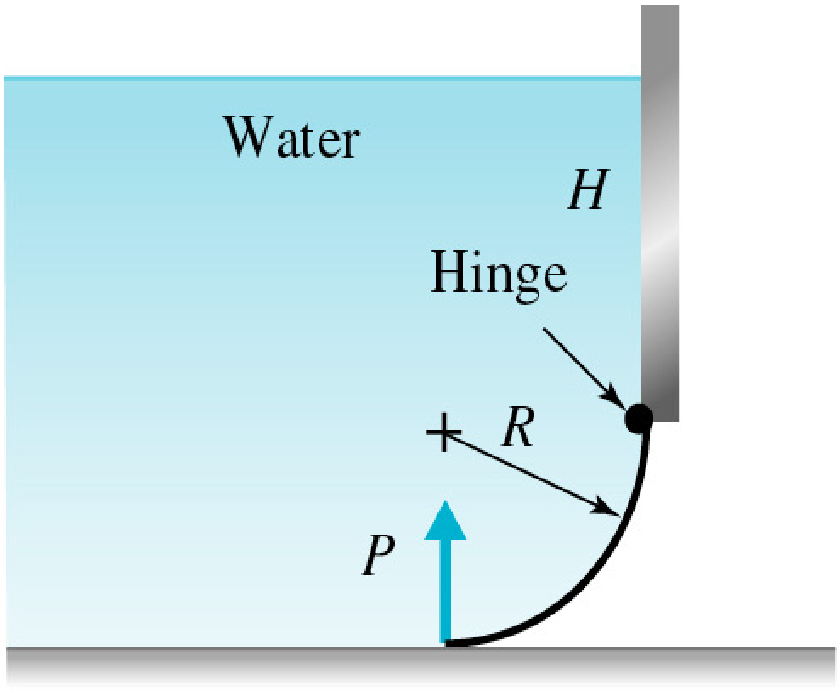
which points upward, and makes an angle

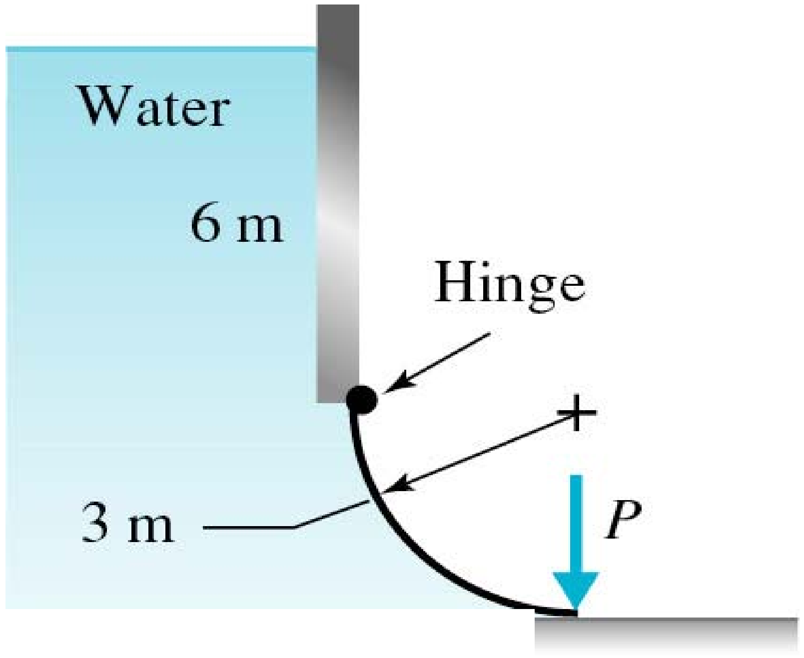
$$\alpha = \tan^{-1}\left(F_V / F_H\right)$$

with the horizontal.

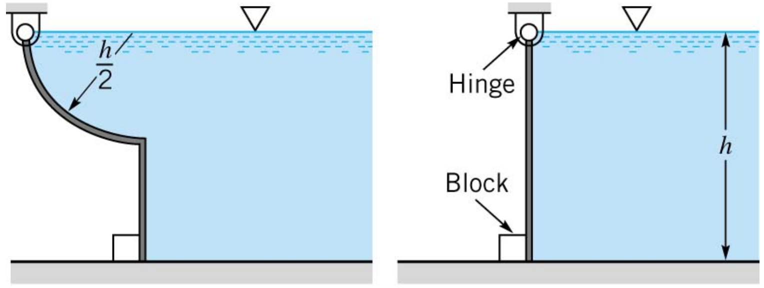


4.9 Hydrostatic Force on Submerged Curved Surfaces

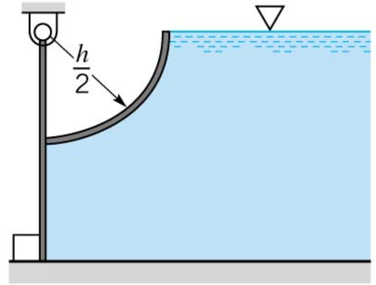




例子2：如图所示三个挡水的闸门，闸门的宽度为b，不考虑闸门的重量，已知图(b)中的闸门左下角的挡板块受到的力为R，请问图(a)和(b)的闸门左下角的挡板块受到的力分别为多少？



(a) (b)



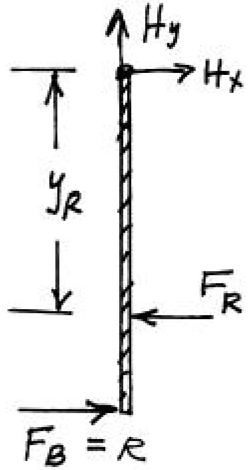
(c)

解：对图(b)，闸门受到水的总压力为：

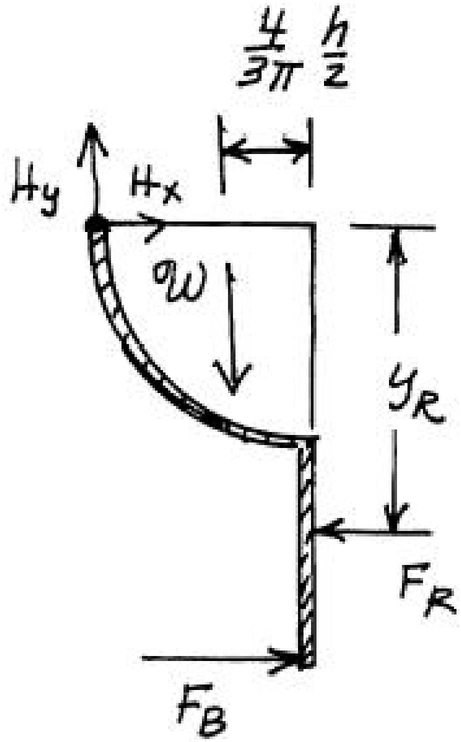
$$F_R = R = \gamma h_c A = \gamma\left(\frac{h}{2}\right)(h\cdot b) = \frac{\gamma h^2 b}{2}, \qquad y_R = \frac{2}{3}h$$

根据力拒平衡原理，有：

$$F_B h = F_R y_R = \left(\frac{\gamma h^2 b}{2}\right)\left(\frac{2}{3}h\right) \Rightarrow F_B = R = \frac{\gamma h^2 b}{3}$$



对图(a)，闸门受到水的总压力仍为：



$$F_R=\gamma h_c A=\gamma\left(\frac{h}{2}\right)(h\cdot b)=\frac{\gamma h^2 b}{2},\qquad y_R=\frac{2}{3}h$$

四分之一半圆的流体重量：

$$W=\gamma\times V=\gamma\left[\frac{\pi(h/2)^2}{4}b\right]=\frac{\pi\gamma h^2 b}{16}$$

根据力拒平衡原理，有：

$$F_B h=F_R\left(\frac{2}{3}h\right)+W\left(\frac{h}{2}-\frac{4h}{6\pi}\right)\Rightarrow\frac{\pi\gamma h^2 b}{16}\left(\frac{h}{2}-\frac{4h}{6\pi}\right)+\frac{\gamma h^2 b}{2}\left(\frac{2}{3}h\right)=F_B h$$

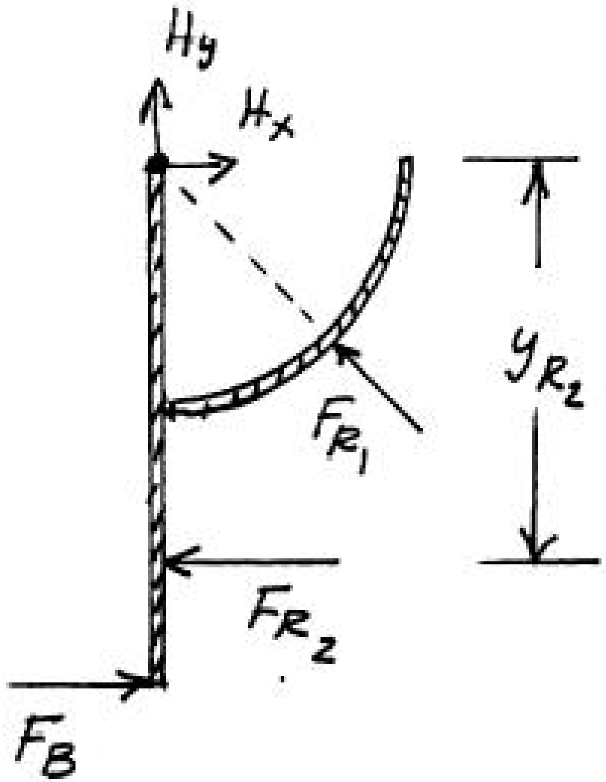
所以有：

$$F_B=\gamma h^2 b(0.390)=3R(0.390)=1.17R$$

对图(c)，由于圆弧部分受力通过铰链支点，对铰链支点的力拒没有贡献。对于水下直板部分的闸门受到水的总压力为：

$$F_{R_2}=\gamma h_c A=\gamma\left(\frac{3h}{4}\right)\left(\frac{h}{2}\times b\right)=\frac{3\gamma h^2 b}{8},$$

$$y_{R_2}=\frac{I_{xc}}{y_c A}+y_c=\frac{\frac{1}{12}b\left(h/2\right)^3}{\left(\frac{3h}{4}\right)\left(\frac{h}{2}b\right)}+\frac{3h}{4}=\frac{28}{36}h$$



根据力拒平衡原理，有：

$$F_B h=F_{R_2}\left(\frac{28}{36}h\right)$$

所以有：$F_B=\dfrac{3\gamma h^2 b}{8}\left(\dfrac{28}{38}\right)=\dfrac{7}{24}\gamma h^2 b=\dfrac{7}{8}R=0.875\ R$

第四章要点

- 流体静力学

1) Pascal's law: Pressure at a point in a fluid is independent of direction as long as there are no shear stresses present, ie, Pressure at a point has the same magnitude in all directions.

2) Pressure Variation with Depth: The pressure is the same at all points with the same depth from the free surface regardless of geometry, while its direction depends on the geometry.

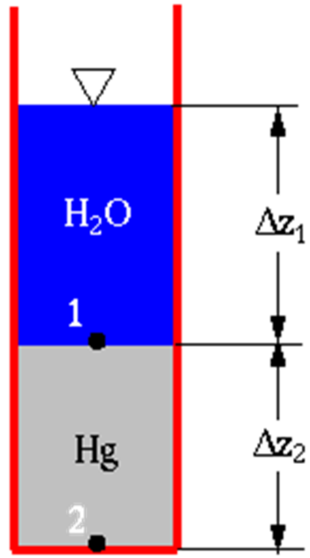
第四章要点

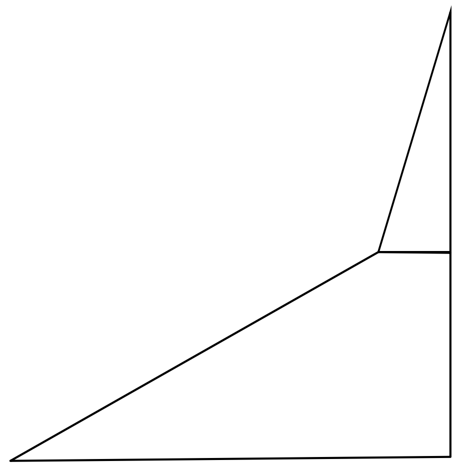
3) Hydrostatics equation: when the density is constant, the pressure in a liquid at rest increases linearly with depth from the free surface.

$$\frac{dp}{dz} = -\rho g$$

$$p = -\rho g z + p_0$$

$$p_{\text{below}} = p_{\text{above}} + \rho g \left|\Delta z\right|$$





第四章要点

• Hydrostatic Force on a Plane Surface

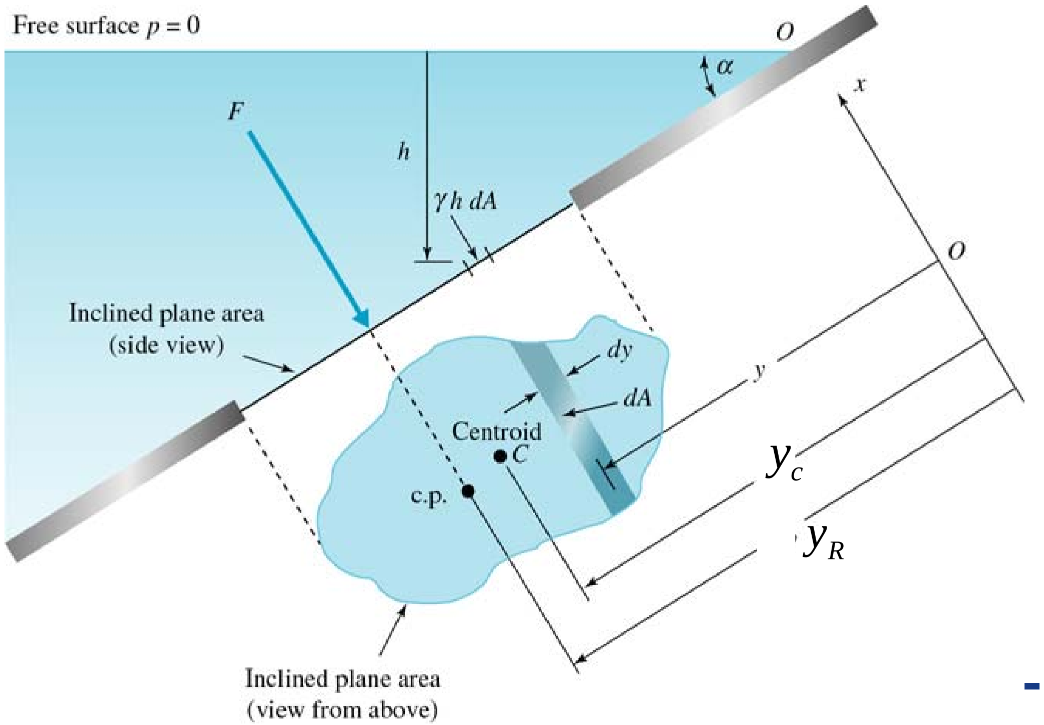
Magnitude of resultant force： **equal to the pressure at the centroid of the surface times the area of the surface.**

$$F_R = \rho g\,(y_c \sin\alpha)\,A = \rho g h_c A$$

Location of centre of pressure： **always below the depth of the centroid of the plane surface.**

$$y_R = y_c + \frac{I_c}{y_c A}$$

or $$h_R = h_c + \frac{I_c \sin^2\alpha}{h_c A}$$



第四章要点

- **Hydrostatic Force on Submerged Curved Surfaces**

Magnitude of resultant force： Horizontal and Vertical

Location of centre of pressure： $\alpha = \tan^{-1}\left(F_V / F_H\right)$