第四次作业

- 作业题目在交大 public网站上：

  目录名：船舶流体力学作业2015

  文件名：Exercise2015-04.pdf

- 共 7 题

- 在4月20日提交作业。

复习

- 不可压缩流体表面力与应变率关系(本构方程)

$$\sigma_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

- 不可压缩流体运动的动量方程(动量守恒)

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V}\right) = -\nabla p + \mu\nabla^2\mathbf{V} + \rho\mathbf{g}$$

- 不可压缩流体运动的基本控制方程(Navier-Stokes方程)

$$\begin{cases} \nabla\cdot\mathbf{V} = 0 \\ \dfrac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V} = -\dfrac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{V} + \mathbf{g} \end{cases}$$

复习

NS方程是针对真实流体的运动，为了简化问题，可以考虑理想流体，即流体不存在粘性，粘性系数为0，$\mu=\nu=0$，这样NS方程可以简化为：

$$\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{V}=-\frac{1}{\rho}\nabla p+\mathbf{f}, \qquad \nabla\cdot\mathbf{V}=0$$

上面的控制方程称为理想流体的Euler方程。

由于 $\mathbf{V}\cdot\nabla\mathbf{V}=\nabla\left(\frac{V^2}{2}\right)-\mathbf{V}\times(\nabla\times\mathbf{V})=\nabla\left(\frac{V^2}{2}\right)-\mathbf{V}\times\mathbf{\Omega}=\nabla\left(\frac{V^2}{2}\right)-2\mathbf{V}\times\boldsymbol{\omega}$

所以Euler方程可改写为：

$$\frac{\partial \mathbf{V}}{\partial t}+\nabla\left(\frac{V^2}{2}\right)-2\mathbf{V}\times\boldsymbol{\omega}=-\frac{1}{\rho}\nabla p+\mathbf{f},$$

这种形式的Euler方程称为Lamb方程。

复习

- Bernoulli方程

理想不可压流体作定常流动， Bernoulli函数在同一条流线或涡线上为常数：

$$\frac{V^2}{2}+\frac{p}{\rho}+gz=C_l=\mathrm{const}$$

$$\frac{V^2}{2g}+\frac{p}{\rho g}+z=C_l=\mathrm{const}$$

速度水头	+	压力水头	+	位置水头	=	常数

复习

如果我们考虑：

- 理想流体 (ideal fluid, Inviscid flow)；
- 不可压流体 (constant density, incompressible flow)；
- 无旋流动 (irrotational flow)；
- 体积力为重力 (gravity)；

这样Lamb方程可以改写为：

$$\frac{\partial \mathbf{V}}{\partial t}+\nabla\left(\frac{V^2}{2}\right)-2\mathbf{V}\times\boldsymbol{\omega}=-\frac{1}{\rho}\nabla p+\mathbf{f}$$

$$\nabla\phi=\mathbf{V},\quad \boldsymbol{\omega}=0$$

$$\nabla\left(\frac{\partial\phi}{\partial t}\right)+\nabla\left(\frac{V^2}{2}\right)+\nabla\left(\frac{p}{\rho}\right)+\nabla\left(gz\right)=0$$

复习

$$\nabla\left(\frac{\partial \phi}{\partial t}\right)+\nabla\left(\frac{V^{2}}{2}\right)+\nabla\left(\frac{p}{\rho}\right)+\nabla(gz)=0$$

$$\nabla\left(\frac{\partial \phi}{\partial t}+\frac{V^{2}}{2}+\frac{p}{\rho}+gz\right)=0$$

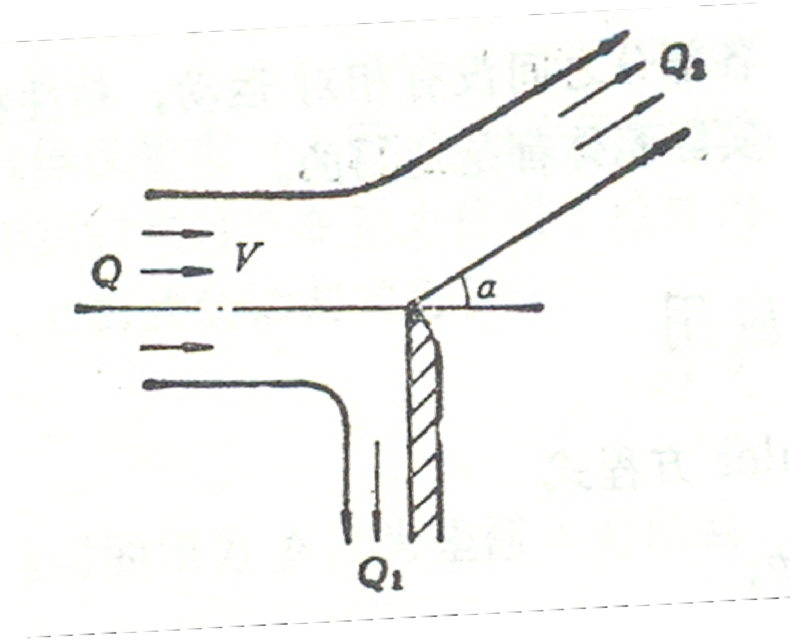
$$\nabla H=0,\quad H=\frac{\partial \phi}{\partial t}+\frac{V^{2}}{2}+\frac{p}{\rho}+gz=const$$

这里H为Bernoulli函数。

势流的Bernoulli函数每时刻在全场为常数。

3.7 Bernoulli方程和动量方程例子

例子7： 将一平板伸到水柱内，板面垂直于水柱的轴线，水柱被截后的流动如图所示。已知水柱的流量 Q = 0.036 m³/s，水柱的来流速度 V = 30 m/s，如被截取的流量 Q_1 = 0.012 m³/s，试确定水柱作用在板上的合力R和水流的偏转角 α (略去水的重量及粘性)。



解：

不考虑重力，入口和出口压力都为大气压力，根据Bernoulli方程，可以得到： $V = V_1 = V_2 = 30\ \text{m/s}$。

由质量守恒，可以得到出口2的流量为：

$$Q_2 = Q - Q_1 = 0.036 - 0.012 = 0.024\ \text{m}^3/\text{s}$$

3.7 Bernoulli方程和动量方程例子

设平板对射流的作用力为 ***R***，由动量定理知道，动量的变化等于流体所受的力，即：

在*x*方向： $-\rho QV + \rho Q_2 V_2 \cos\alpha = -R$ (1)

在*y*方向： $-\rho Q_1 V_1 + \rho Q_2 V_2 \sin\alpha = 0$ (2)

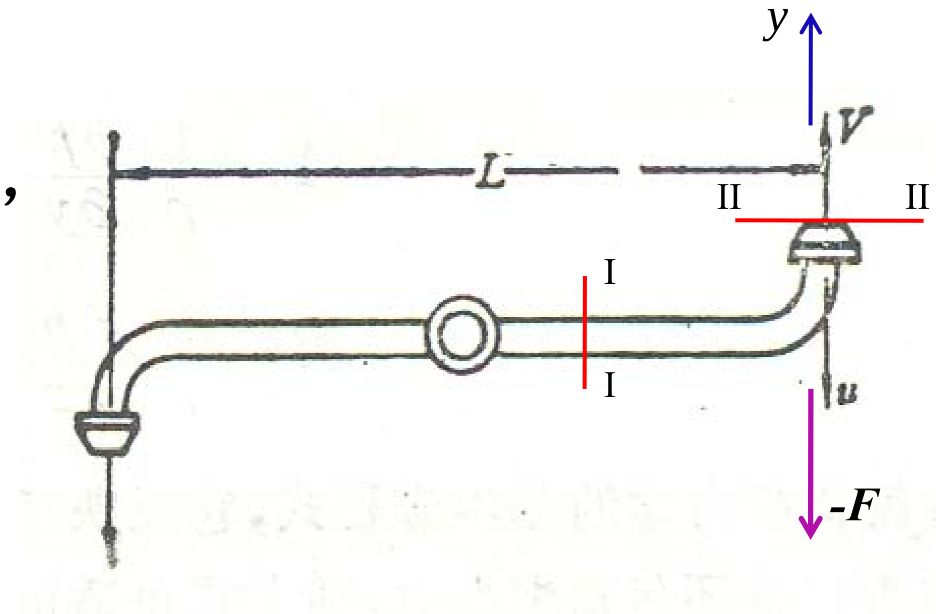
由(2)式得： $\sin\alpha = \frac{Q_1}{Q_2} = \frac{0.012}{0.024} = 0.5 \Rightarrow \alpha = 60^0$

由(1)式得：

$$R = \rho\left(QV - Q_2 V_2 \cos\alpha\right) = 1000\times\left(0.036\times30 - 0.024\times30\times0.867\right) = 455.76\,\mathrm{N}$$

水柱对平板冲击力： $R' = -R$ $R' = 0.456\,\mathrm{kN}$

3.7 Bernoulli方程和动量方程例子



例子8： 水从长 $l=0.6$ m的喷管两端喷出，支承点在喷管的中心，相对喷管的出流速度$V=6$ m/s，喷管直径$d=12.5$ mm。试求 (1) 转臂不动时的转动力拒；(2) 转臂以周向速度u旋转时装置的功率表达式；(3) $V=6$ m/s时使功率为最大时的u值。

解： (1) 转臂不动时

取I-II两个断面之间的流体为控制体，根据动量定理，管咀在y方向的作用力：

$$F=\left(\rho V\frac{\pi}{4}d^2\right)V=\frac{\pi}{4}\rho d^2V^2$$

流体给管咀的反作用力为-F。转动力拒：

$$T=-F\cdot l=-\frac{\pi}{4}\rho d^2V^2l=-\frac{\pi}{4}\times 1000\times(0.0125)^2\times 6^2\times 0.6=-2.65\ \mathrm{N\cdot m}$$ 顺时针

3.7 Bernoulli方程和动量方程例子

(2) 转臂以周向速度u转动时，仍取I-II两个断面之间的流体为控制体，此时进出控制体的流量仍为 $\rho V\frac{\pi}{4}d^2$，但喷出的绝对速度为V-u，根据动量定理，管咀在y方向的作用力：

$$F=\left(\rho V\frac{\pi}{4}d^2\right)(V-u)=\frac{\pi}{4}\rho d^2V(V-u)$$

流体给管咀的反作用力为-F。转动力矩：

$$T=-F\cdot l=-\frac{\pi}{4}\rho d^2V(V-u)l$$ 若$V>u$则T为顺时针。

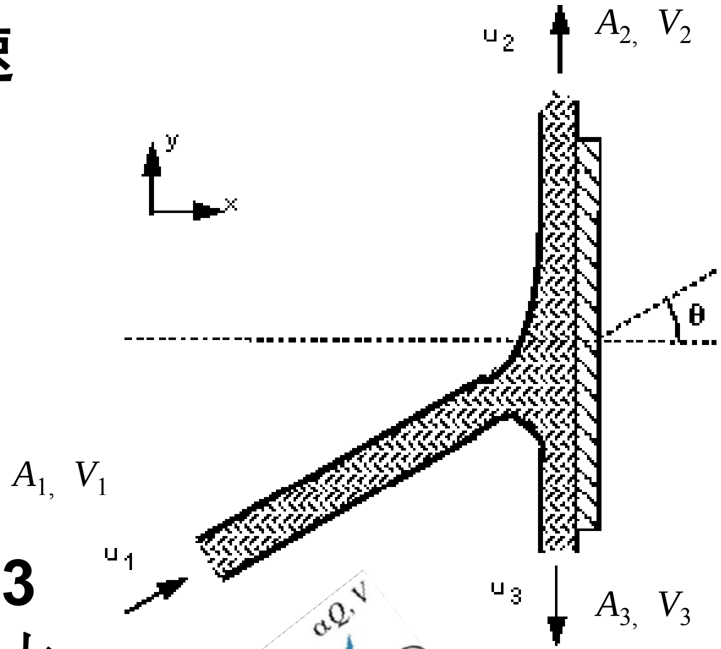
功率： $$N=T\cdot\omega=T\cdot\frac{u}{l/2}=-\frac{\pi}{2}\rho d^2V(V-u)u$$

(3) 使功率为最大的u值

$$\frac{dN}{du}=-\frac{\pi}{2}\rho d^2V^2+\pi\rho d^2Vu=0\quad\Rightarrow\quad u=\frac{V}{2}=3\,\mathrm{m/s}$$

3.7 Bernoulli方程和动量方程例子

例子9：在一个平面上，一个水柱以速度V与水平方向成θ角喷射到一块平板上，喷射流量为Q_1，不考虑水粘性，求作用在平板上的力，以及射流喷射到平板后分成两股水流动的速度和流量。



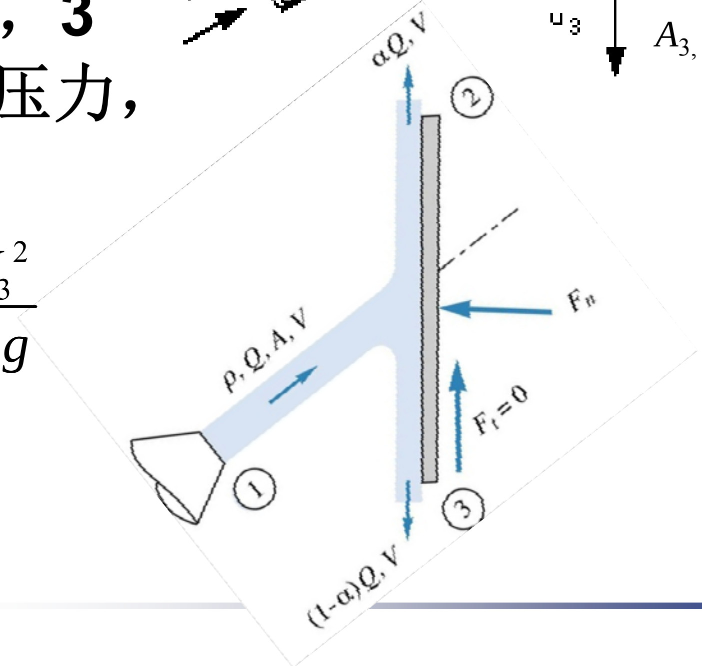
解：

如图取控制体和三个点，点**1，2，3**处于同一水平面，都受到是大气压力，根据**Bernoulli**方程，有：

$$\frac{p_1}{\rho g}+z_1+\frac{V_1^{\ 2}}{2g}=\frac{p_2}{\rho g}+z_2+\frac{V_2^{\ 2}}{2g}=\frac{p_3}{\rho g}+z_3+\frac{V_3^{\ 2}}{2g}$$

得到：

$$V_1=V_2=V_3=V$$



3.7 Bernoulli方程和动量方程例子

由连续方程： $Q_1 = Q_2 + Q_3 \Rightarrow V_1A_1 = V_2A_2 + V_3A_3 \Rightarrow A_1 = A_2 + A_3$

由动量方程，得*x*方向的流体受到的力：

$$\sum F_x = \rho\left[\left(Q_2V_{2x} + Q_3V_{3x}\right) - Q_1V_{1x}\right]$$

因为 $V_{2x} = V_{3x} = 0$ ， $V_{1x} = V\cos\theta$，所以有：

$$\sum F_x = -\rho Q_1 V\cos\theta$$

所以作用在平板上的力为：

$$\sum F_x = -F_n = -\rho Q_1 V\cos\theta \Rightarrow F_n = \rho Q_1 V\cos\theta$$

3.7 Bernoulli方程和动量方程例子

在y方向的流体受到的力为0，由动量方程有：

$$V_{1y}=V\sin\theta,\ V_{2y}=V,\ V_{3y}=-V,$$

$$\sum F_y=\rho\left[\left(Q_2V_{2y}+Q_3V_{3y}\right)-Q_1V_{1y}\right]=\rho V\left[Q_2-Q_3-Q_1\sin\theta\right]=\rho V^2\left[A_2-A_3-A_1\sin\theta\right]=0$$

所以有： $0=A_2-A_3-A_1\sin\theta$

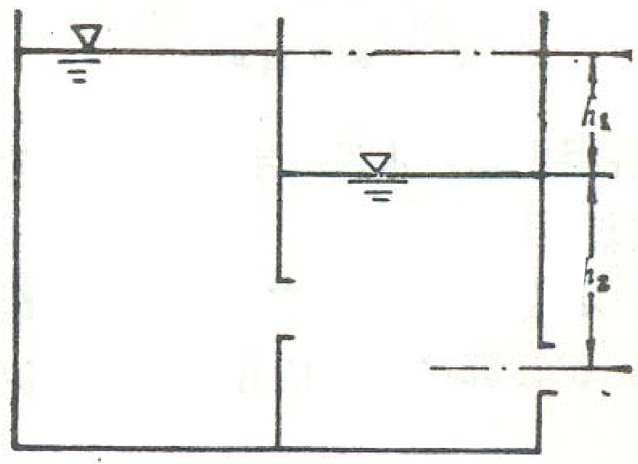
由连续方程有：$A_1=A_2+A_3$ ，得到：

$$A_2=A_3\left(\frac{1+\sin\theta}{1-\sin\theta}\right),\quad A_2=\frac{1}{2}A_1\left(1+\sin\theta\right),\quad A_3=\frac{1}{2}A_1\left(1-\sin\theta\right)$$

因此两股流动的流量分别为：

$$\frac{Q_2}{Q_1}=\alpha=\frac{1}{2}\left(1+\sin\theta\right),\qquad \frac{Q_3}{Q_1}=1-\alpha=\frac{1}{2}\left(1-\sin\theta\right)$$

例子 10 两个紧靠的水箱逐级放水，放水孔的截面积分别为A_1和A_2，试问h_1和h_2成什么关系时流动处于定常状态？这时需在左边水箱补充多大的流量？



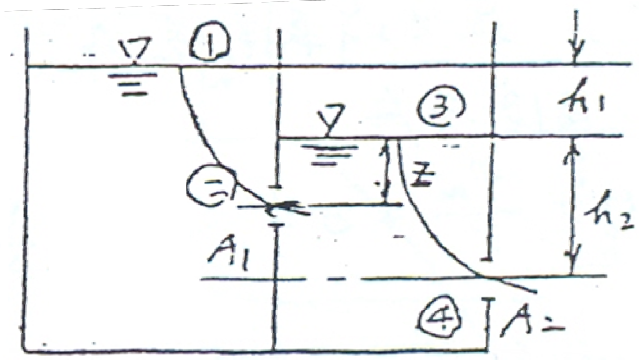
解： 以4处为基准，对3和4建立Bernoulli方程：

$$z_3+\frac{P_3}{\gamma}+\frac{V_3^{\ 2}}{2g}=z_4+\frac{P_4}{\gamma}+\frac{V_4^{\ 2}}{2g}$$

由于 $z_3=h_2$，$z_4=0$，$V_3\approx 0$，$P_3=P_4=P_0$，

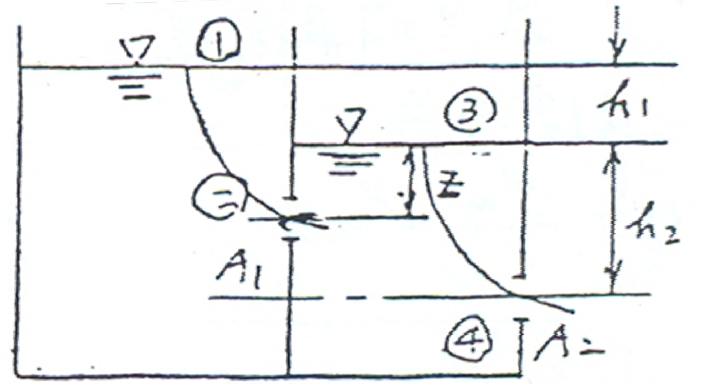
得到

$$V_4=\sqrt{2gh_2}$$



3.7 Bernoulli方程和动量方程例子

以2处为基准，对1和2建立Bernoulli方程：



$$z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

由于 $z_1 = h_1 + z$，$z_2 = 0$，$P_1 = P_0$，$P_2 = P_0 + \gamma z$，$V_1 \approx 0$，得到 $V_2 = \sqrt{2gh_1}$

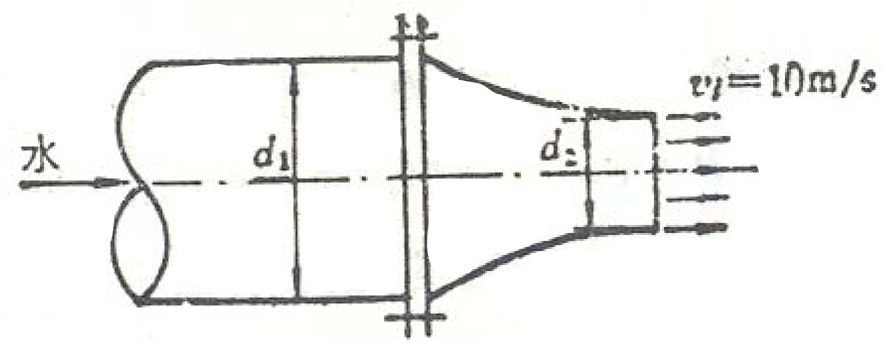
流动为定常，应有 $Q_1 = Q_2$，即 $V_2 A_1 = V_4 A_2$，或 $A_1\sqrt{2gh_1} = A_2\sqrt{2gh_2}$

$$\frac{h_1}{h_2} = \left(\frac{A_2}{A_1}\right)^2$$

左箱需补充的流量为： $Q_1 = A_1 V_2 = A_1\sqrt{2gh_1}$

例子 11

水以**10 m/s**的速度从内径为**50 mm**的喷管中喷出，喷管的一端则用螺栓固定在内径为**100 mm**水管的法兰上，如不计损失，试求作用在连接螺栓上的拉力。

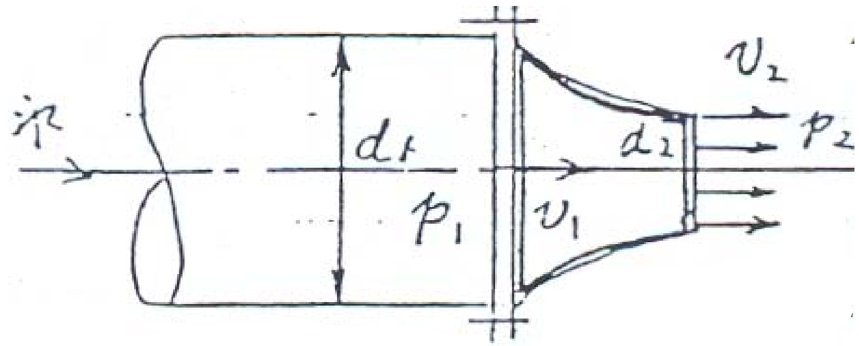


解： 取如图的控制体，由连续方程

$$V_1 \frac{\pi}{4} d_1^2 = V_2 \frac{\pi}{4} d_2^2$$

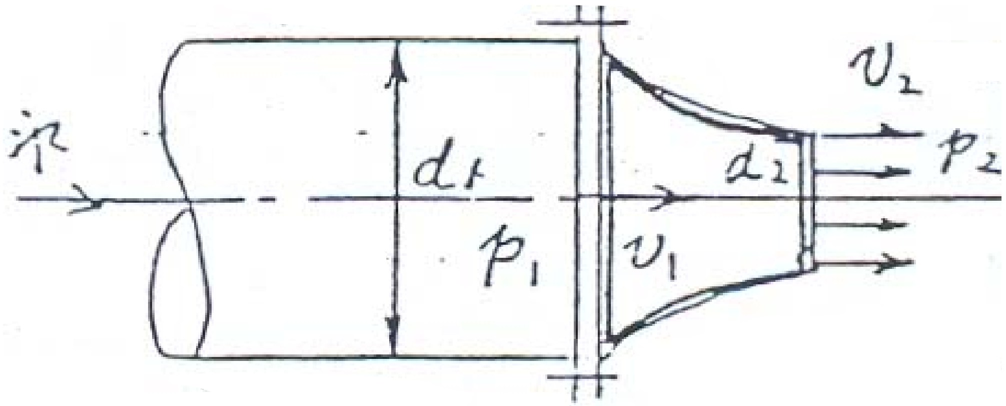
得到：

$$V_1 = (\frac{d_2}{d_1})^2 \cdot V_2 = (\frac{50}{100})^2 \times 10 = 2.5 m/s$$



3.7 Bernoulli方程和动量方程例子

对喷管的入口及出口建立Bernoulli方程：



$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}$$

压力用表压表示，P_2=0，因此得到：

$$P_1=\frac{1}{2}\rho(V_2^2-V_1^2)=0.5\times 999.1\times(100-6.25)=46833N/m^2$$

对喷嘴应用动量定理，设喷嘴对流体作用力为***R***，则有

$$\rho A_2V_2^2-\rho A_1V_1^2=-R+P_1A_1-P_2A_2$$

3.7 Bernoulli方程和动量方程例子

由于：

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(0.1)^2 = 7.85\times10^{-3} m^2$$

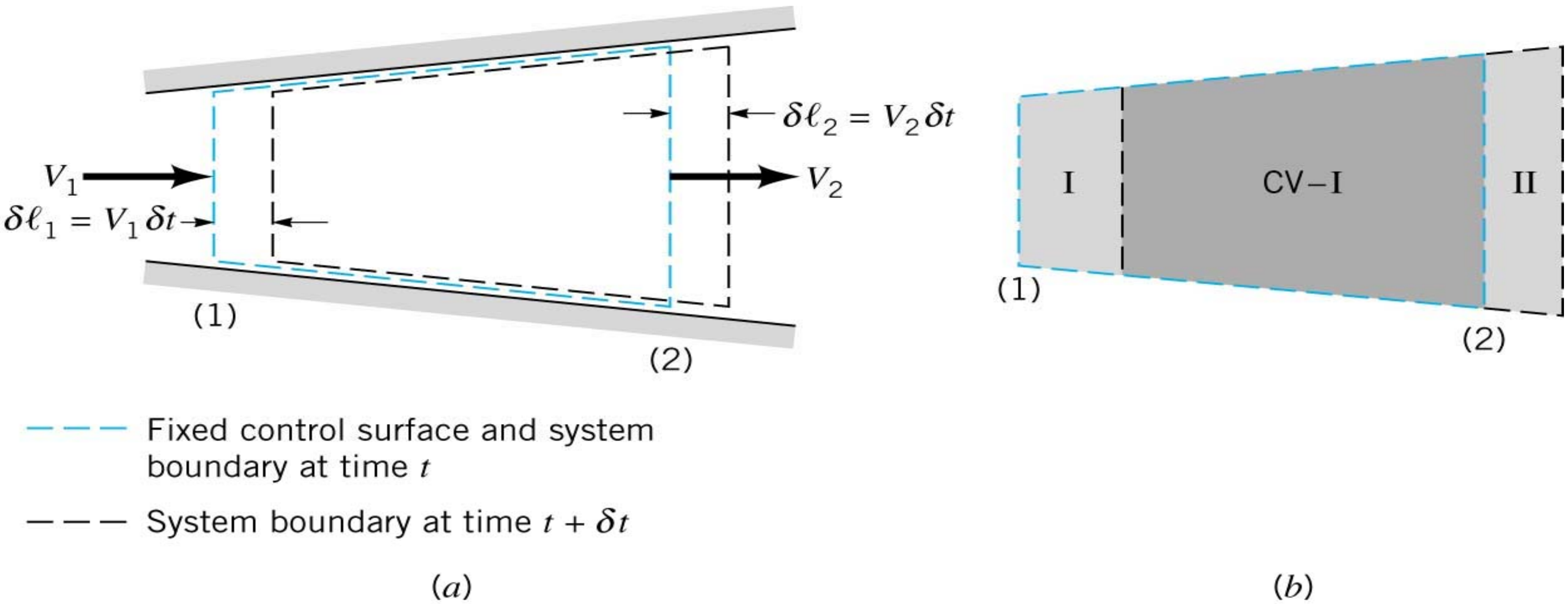
$$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(0.05)^2 = 1.96\times10^{-3} m^2$$

所以

$$R = \rho(A_1 V_1^2 - A_2 V_2^2) + P_1 A_1$$
$$= 999.1\times(0.0491 - 0.196) + 367.6 = 220.8N$$

第三章要点

- 系统与控制体

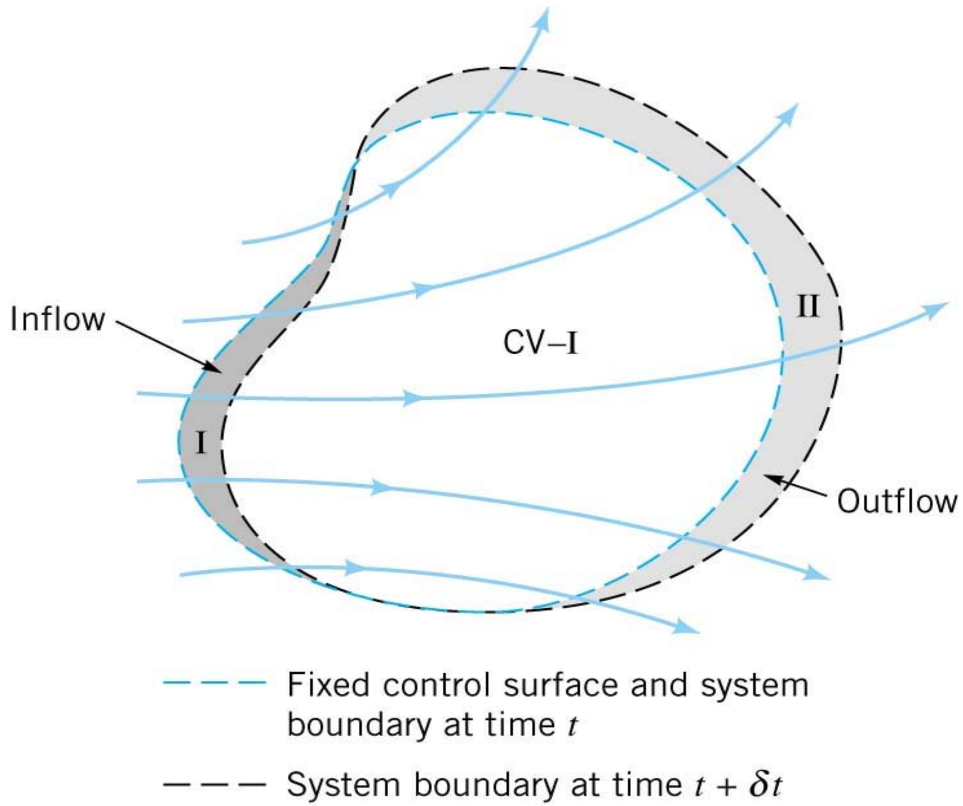


第三章要点

- 雷诺输运定理(RTT)

$$\frac{d}{dt}\iiint_{MV} G d\forall$$

$$= \frac{\partial}{\partial t}\iiint_{CV} G d\forall + \iint_{CS} G\mathbf{V}\cdot\mathbf{n} dA$$



第三章要点

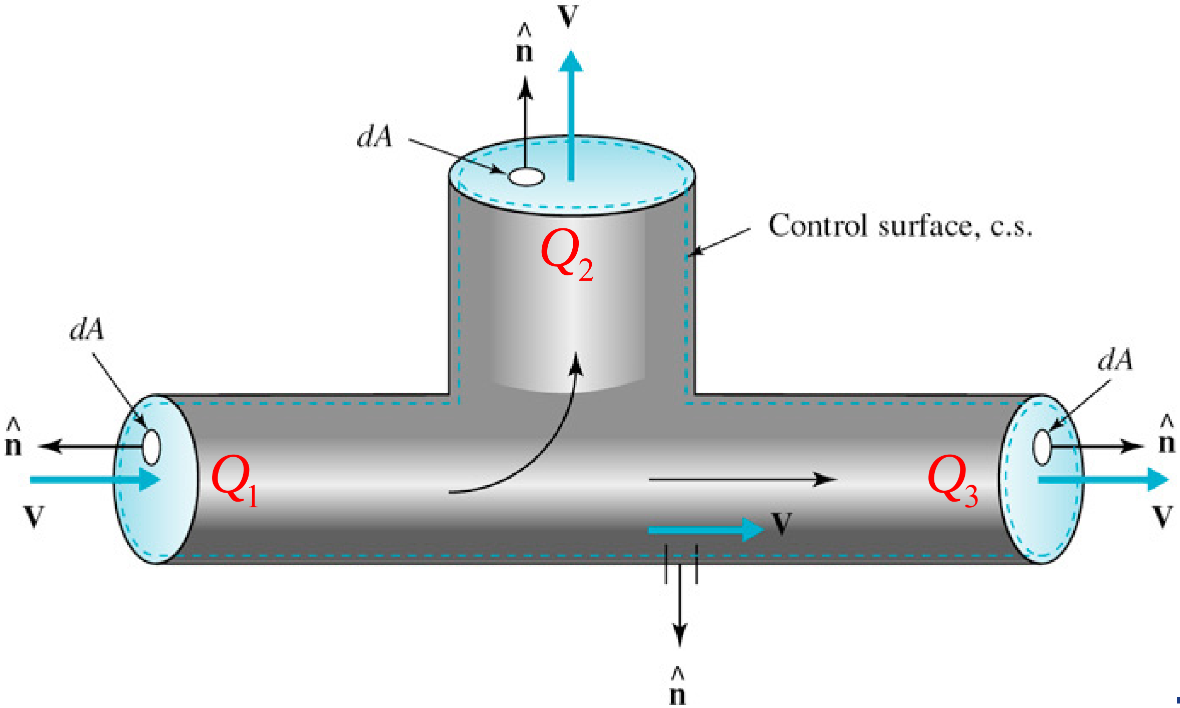
- **连续方程(质量守恒方程)**

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho \mathbf{V})=0 \qquad \frac{D\rho}{Dt}+\rho\nabla\cdot\mathbf{V}=0$$

不可压缩流体： $\nabla\cdot\mathbf{V}=0$

$$\oiint_{CS}\rho\mathbf{V}\cdot\mathbf{n}\,dA=0$$

$$Q_1=Q_2+Q_3$$



第三章要点

- **流函数：** 二维不可压流体的流动，均存在流函数。

$$\psi = \int -v dx + u dy \quad \longleftrightarrow \quad \begin{cases} \dfrac{\partial \psi}{\partial x} = -v \\ \dfrac{\partial \psi}{\partial y} = u \end{cases}$$

- **动量方程(动量守恒)**

$$\iiint_{MV} \rho \frac{d\mathbf{V}}{dt} d\mathcal{V} = \iiint_{MV} (\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g}) d\mathcal{V}$$

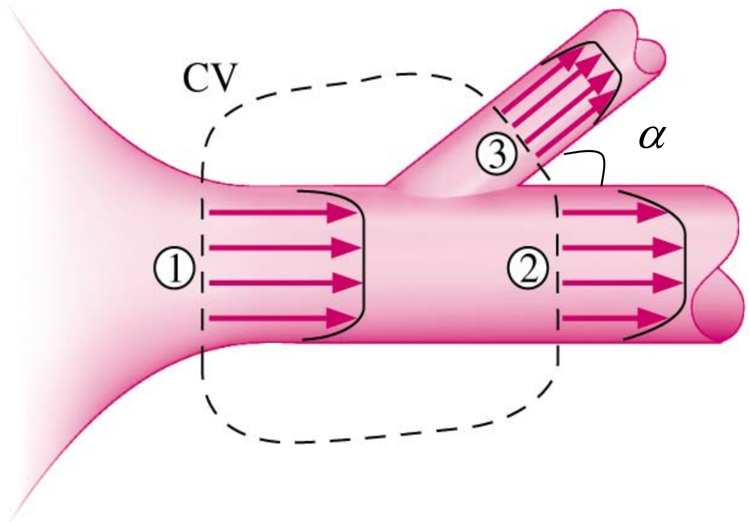
积分形式的动量方程

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g}$$

微分形式的动量方程

第三章要点

在实际工程问题中，可以推出简单实用的动量方程。



如图，根据**动量守恒**：控制体的动量变化率等于作用在控制体上的力，即：

$$F_x = -Q_1V_1 + Q_2V_2 + Q_3V_3\cos\alpha$$

$$F_y = Q_3V_3\sin\alpha$$

Q：流量
V：速度
S：截面面积
F：作用在CV流体上的力

根据**质量守恒**：

$$Q_1 = Q_2 + Q_3 \quad \Rightarrow \quad \rho S_1V_1 = \rho S_2V_2 + \rho S_3V_3$$

第三章要点

- 不可压缩流体表面力与应变率关系(本构方程)

$$\sigma_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

- 不可压缩流体运动的动量方程(动量守恒)

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V}\right) = -\nabla p + \mu\nabla^2\mathbf{V} + \rho\mathbf{g}$$

- 不可压缩流体运动的基本控制方程(Navier-Stokes方程)

$$\begin{cases} \nabla\cdot\mathbf{V} = 0 \\ \dfrac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V} = -\dfrac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{V} + \mathbf{g} \end{cases}$$

第三章要点

- 不可压理想流体运动的动量方程(Euler方程)

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{1}{\rho} \nabla p + \mathbf{f}$$

- 不可压理想流体运动的动量方程(Lamb方程)

$$\frac{\partial \mathbf{V}}{\partial t} + \nabla \left(\frac{V^2}{2} \right) - 2\mathbf{V} \times \boldsymbol{\omega} = -\frac{1}{\rho} \nabla p + \mathbf{f}$$

第三章要点

- **Bernoulli方程**

理想不可压流体作定常流动， Bernoulli函数在同一条流线或涡线上为常数：

$$\frac{V^2}{2} + \frac{p}{\rho} + gz = C_l = \text{const}$$

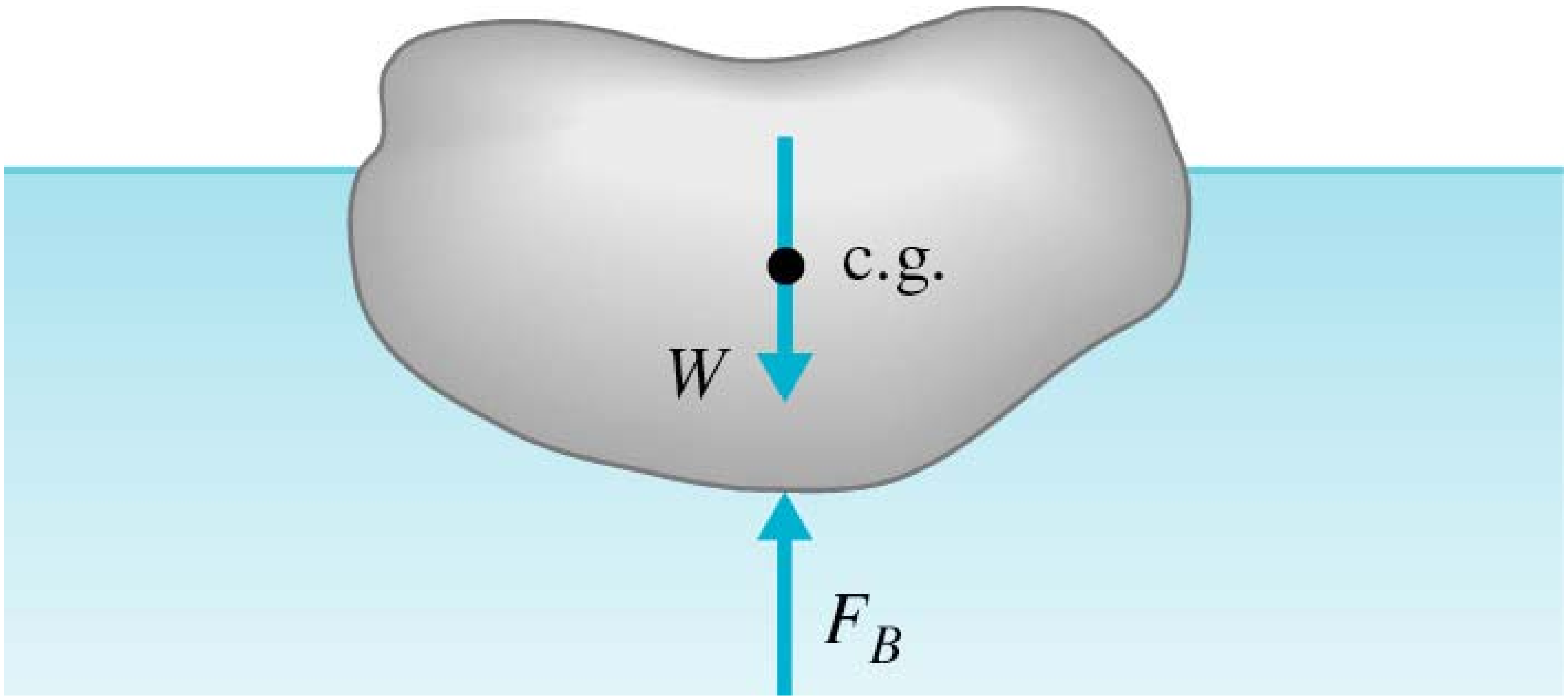
$$\frac{V^2}{2g} + \frac{p}{\rho g} + z = C_l = \text{const}$$

速度水头	＋	压力水头	＋	位置水头	＝	常数

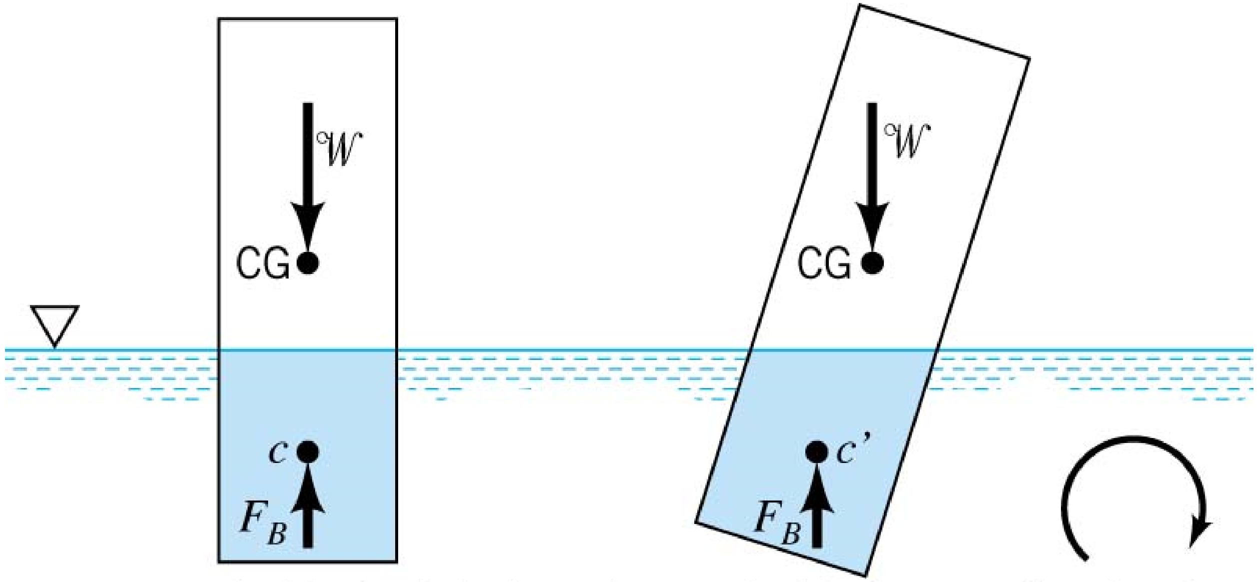
第四章　流体静力学

流体静力学是研究船舶浮性、稳性、抗沉性的主要理论基础

4.1 Indroduction to Hydrostatics



4.1 Indroduction to Hydrostatics

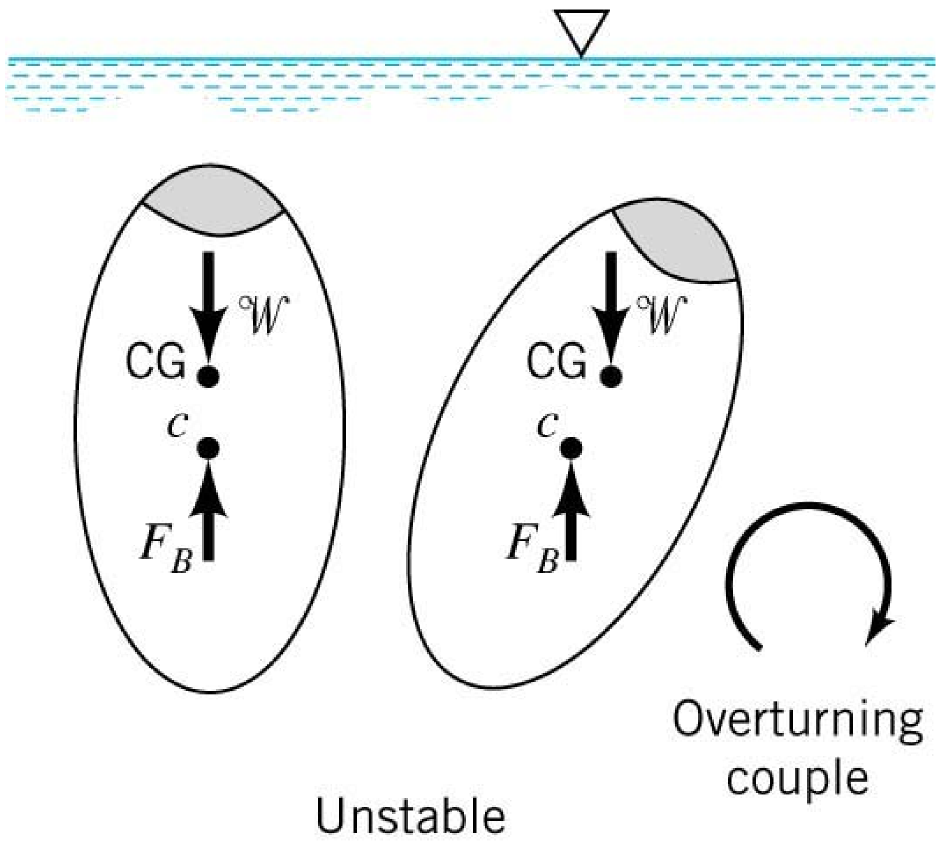


c = centroid of original displaced volume

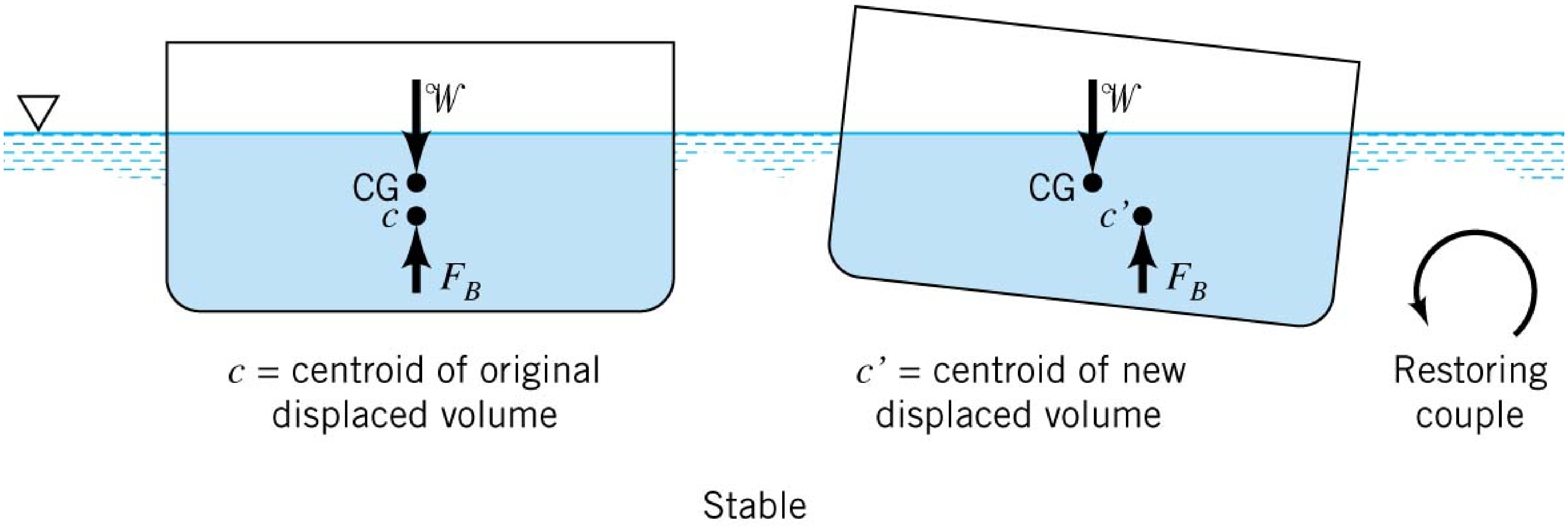
c' = centroid of new displaced volume

Unstable

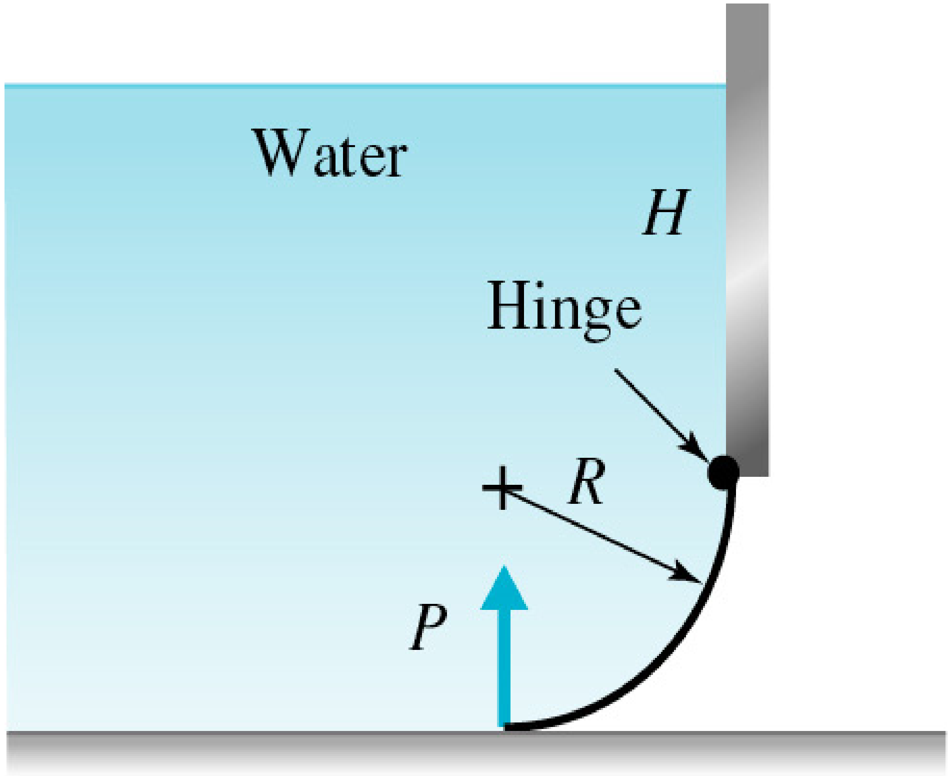
4.1 Indroduction to Hydrostatics

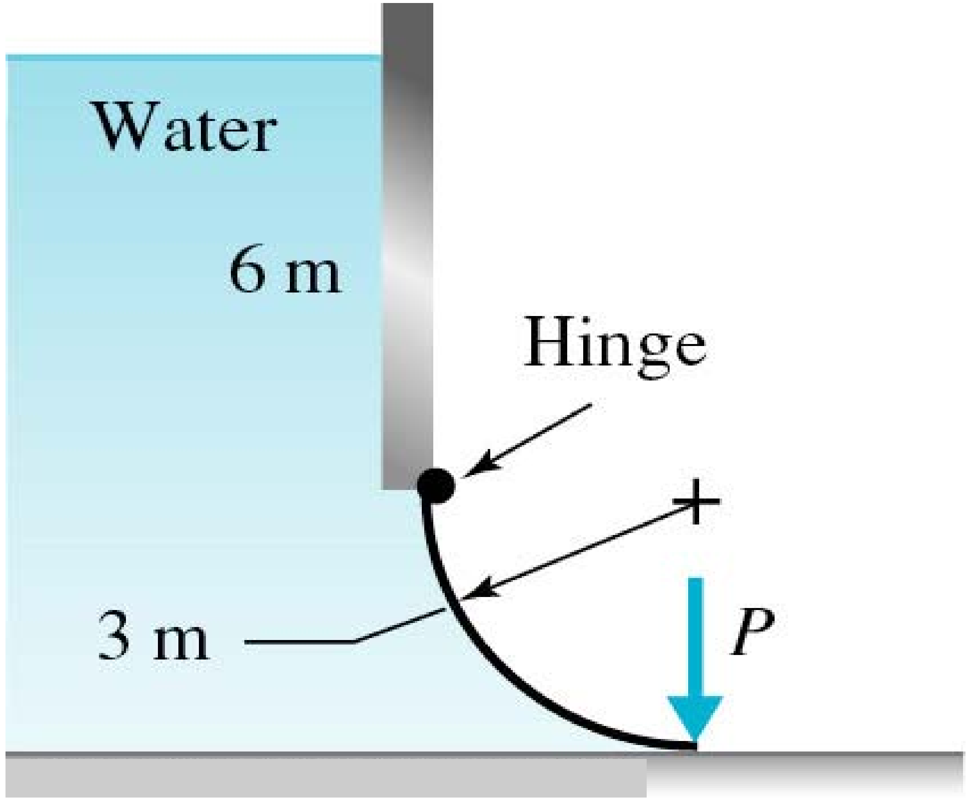


4.1 Indroduction to Hydrostatics



4.1 Indroduction to Hydrostatics



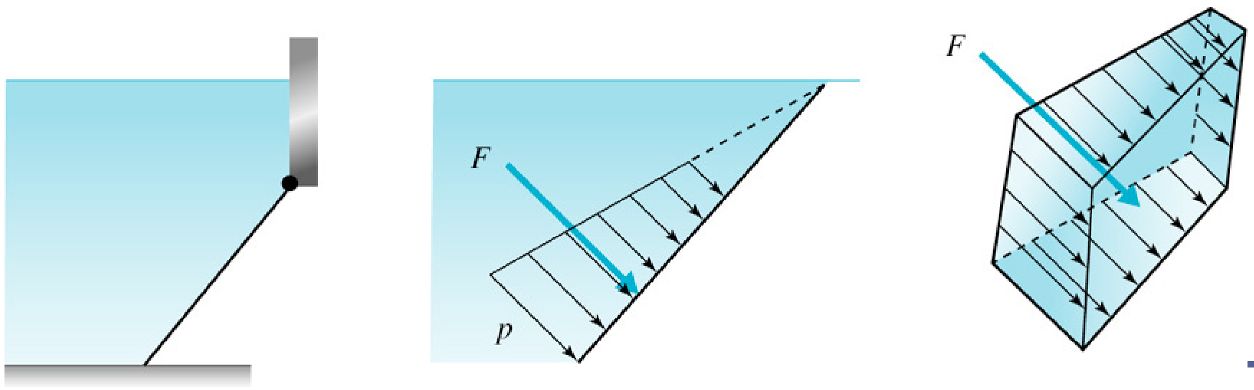


4.1 Indroduction to Hydrostatics

Hydrostatics is the study of pressures throughout a fluid **at rest** and the pressure forces on finite surfaces.

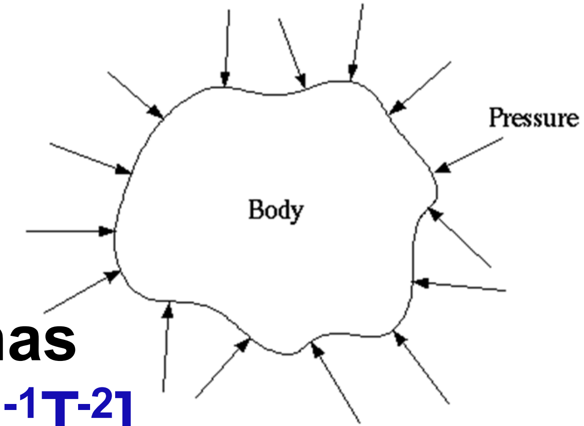
As the fluid is at rest, there are **no shear stresses** in it. Hence the pressure at a point on a plane surface always acts normal to the surface, and all forces are independent of viscosity.

The pressure variation is due only to the weight of the fluid.



(a) (b) (c)

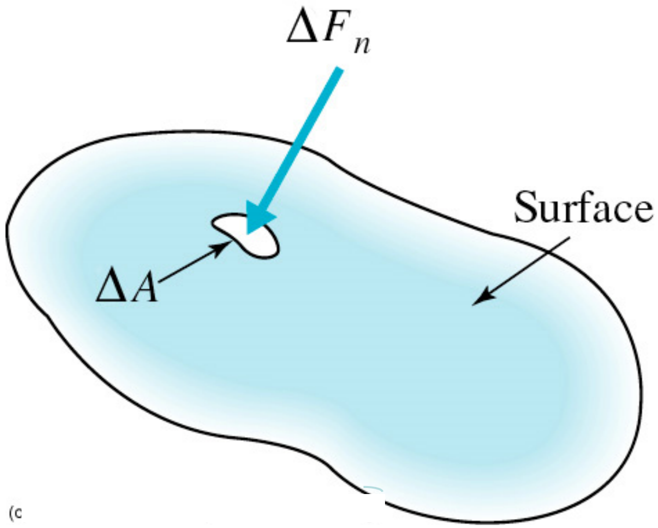
4.2 Indroduction to Pressure



Pressure always acts inward normal to any surface.

Pressure is a normal stress, and hence has dimensions of force per unit area, or [$ML^{-1}T^{-2}$]. In the Metric system of units, pressure is expressed as "pascals" (Pa) or N/m^2.

Standard atmospheric pressure is 101.3 kPa.



Pressure is formally defined to be

$$p = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A}$$

where

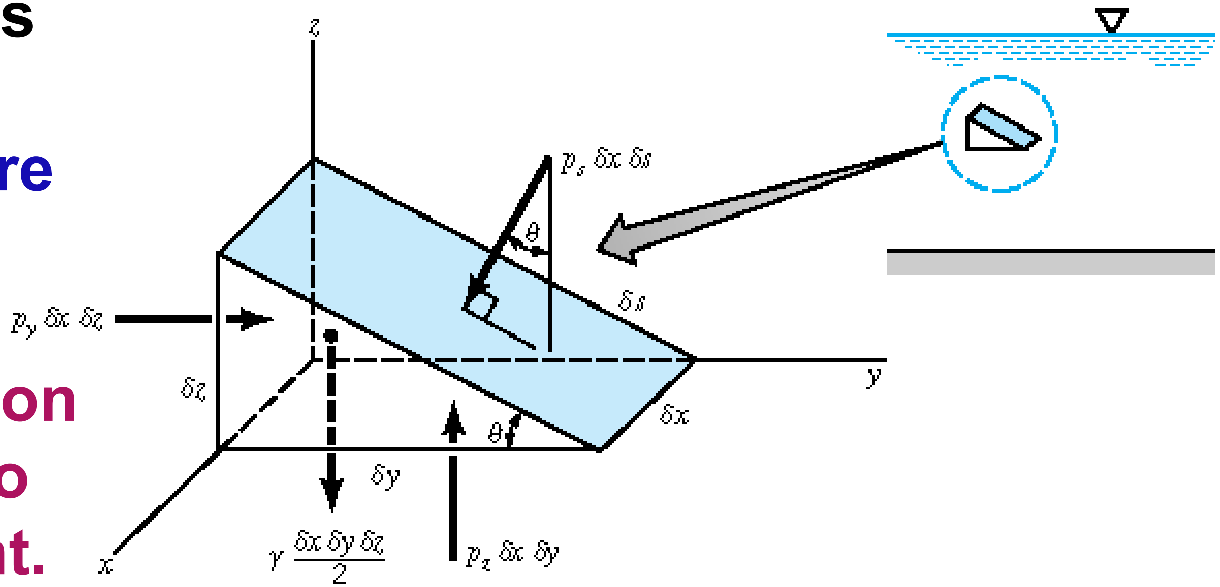
ΔF_n **is the normal compressive force acting on an infinitesimal area** ΔA .

4.3 Pressure at a Point

By considering the equilibrium of a small triangular wedge of fluid extracted from a static fluid body, one can show that for *any* wedge angle *θ*, the pressures on the three faces of the wedge are equal in magnitude:

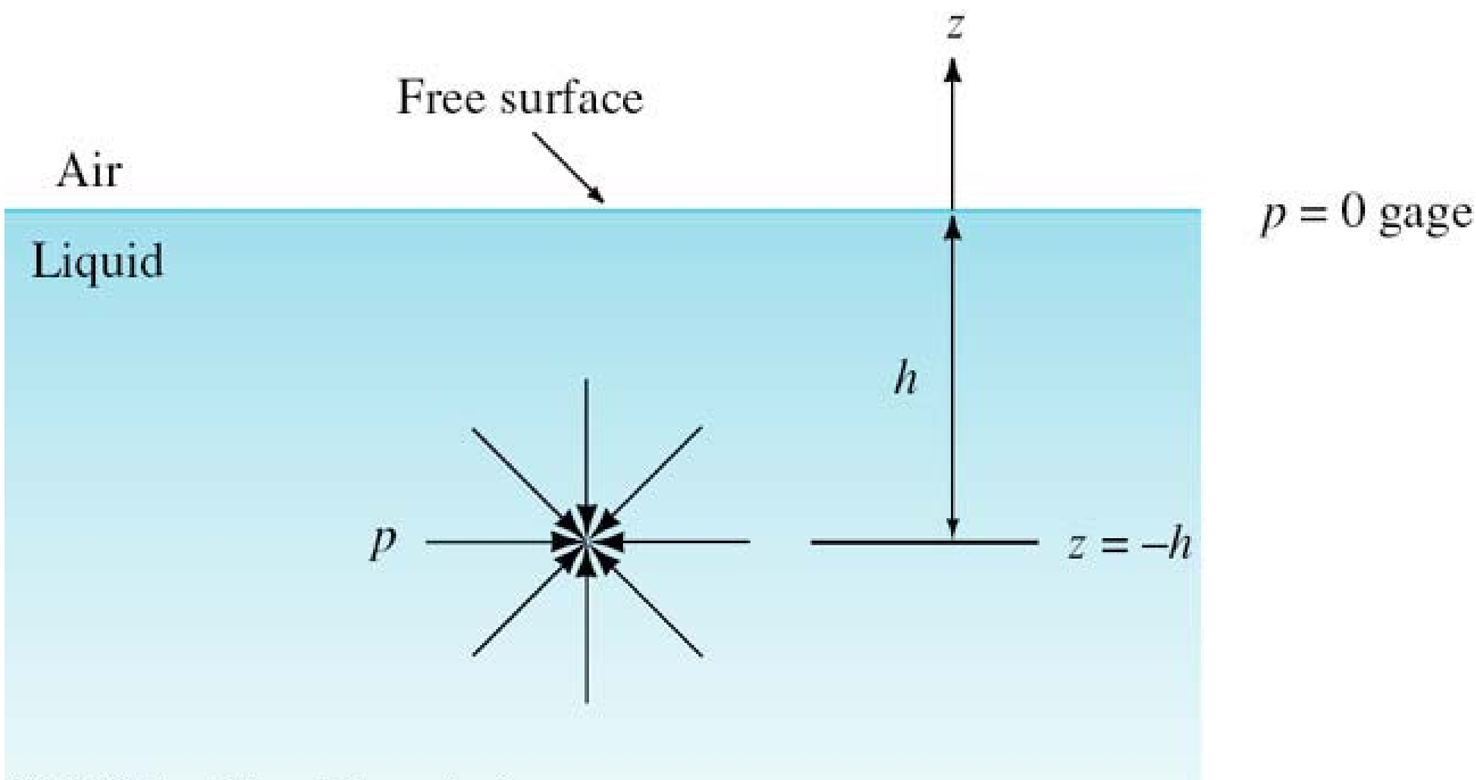
$$p_s = p_y = p_z \qquad \text{independent of } \theta$$

This result is known as Pascal's law, which states that the pressure at a point in a fluid at rest, or in motion, is independent of direction as long as there are no shear stresses present.



4.3 Pressure at a Point

Pressure at a point has the same magnitude in all directions, and is called isotropic.



4.4 Pressure Variation with Depth

Consider a small vertical cylinder of fluid in equilibrium, where *positive z is pointing vertically upward*. Suppose the origin $z = 0$ **is set at the free surface of the fluid. Then the pressure variation at a depth *z = -h* below the free surface is governed by**

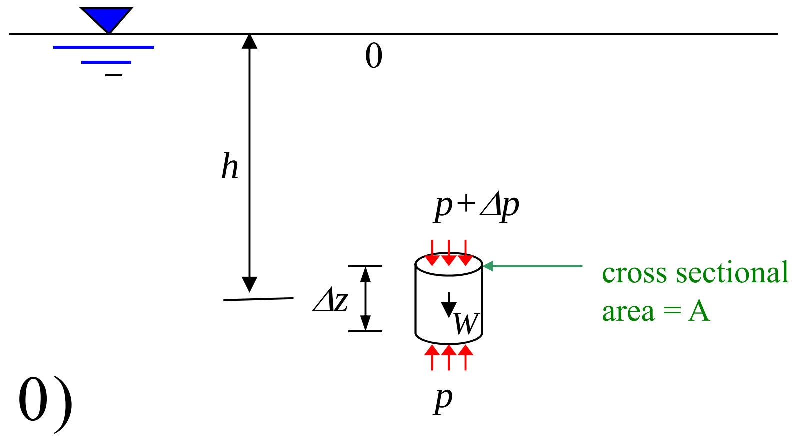
$$(p + \Delta p) A + W = pA$$

$$\Rightarrow \quad \Delta p A + \rho g A \Delta z = 0$$

$$\Rightarrow \quad \Delta p = -\rho g \Delta z$$

$$\Rightarrow \quad \frac{dp}{dz} = -\rho g$$

or $$\frac{dp}{dz} = -\gamma \qquad (\text{as } \Delta z \to 0)$$



Therefore, the hydrostatic pressure increases linearly with depth at the rate of the specific weight $\gamma \equiv \rho g$ of the fluid.

4.4 Pressure Variation with Depth

Example: **Find the relationship between pressure and altitude in the atmosphere near the Earth's surface. For simplicity, neglect the vertical temperature gradient. Let temperature T = 288 K (15°C) and pressure p_0 = 1 atm at the surface. The average molecular weight of air is M_g = 28.8 g/mol. The Universal gas constant is R_g = 8.3 J/mol× K. Assume that air is a perfect gas, its density varies with pressure according to** $\rho = P\frac{M_g}{R_g T}$.

Solution: **Let the altitude above the Earth's surface be denoted by z, then**

$$\frac{dp}{dz} = -\rho g$$

4.4 Pressure Variation with Depth

Since $\rho = p \dfrac{M_g}{R_g T}$

$$\frac{dp}{dz} = -p\frac{M_g g}{R_g T} \Rightarrow \frac{dp}{p} = -\frac{M_g g}{R_g T}dz \Rightarrow \int_{p_0}^{p}\frac{dp}{p} = -\int_0^z \frac{M_g g}{R_g T}dz$$

$$\Rightarrow \ln\frac{p}{p_0} = -\frac{M_g g}{R_g T}z \Rightarrow p = p_0 \exp\left[-\left(\frac{M_g g}{R_g T}\right)z\right]$$

Neglecting temperature variation, the exponential decay rate for pressure with height is,

$$\frac{M_g g}{R_g T} = \frac{28.8\times10^{-3}\times9.81}{8.3\times288} = 1.18\times10^{-4} \quad \text{per meter of rise}$$

Say, at 2000 ft or 610 m above the Earth's surface, the pressure is

$$p = (1\ \text{atm})\exp\left[-1.18\times10^{-4}\times610\right] = 0.93\ \text{atm}$$

That is, for such a high elevation, the pressure drops only by 7%.

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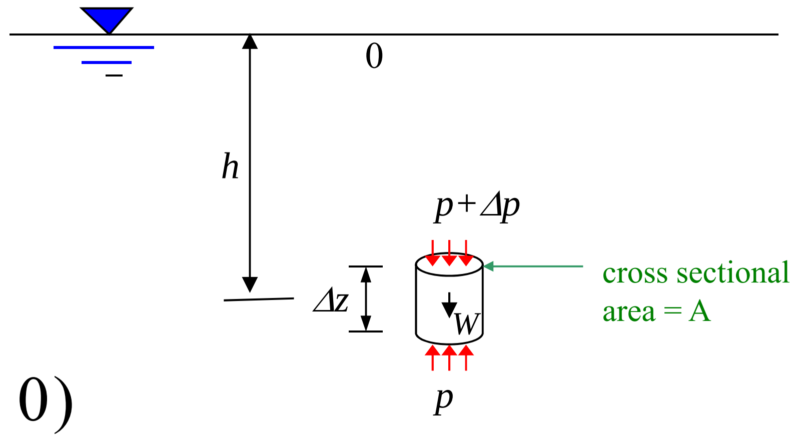


Therefore, the hydrostatic pressure increases linearly with depth at the rate of the specific weight $\gamma \equiv \rho g$ of the fluid.

4.4 Pressure Variation with Depth

Homogeneous fluid: ρ is constant.

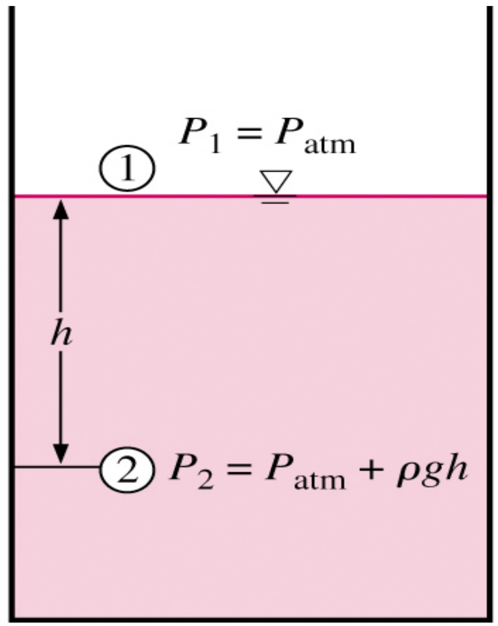
By simply integrating the above equation:

$$\int dp = -\int \rho g dz \qquad \Rightarrow \qquad p = -\rho g z + C$$

where C is an integration constant. When $z = 0$ (on the free surface), $p = C = p_0$ (the atmospheric pressure). Hence,

$$p = -\rho g z + p_0$$

The equation derived above shows that when the density is constant, the pressure in a liquid at rest increases linearly with depth from the free surface.

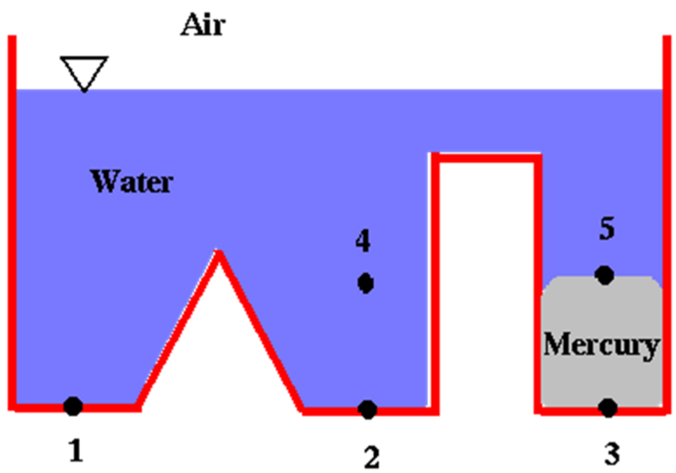


For a fluid with constant density,

$$p_{\text{below}} = p_{\text{above}} + \rho g \left| \Delta z \right|$$

As a diver goes down, the pressure on his ears increases. So, the pressure "below" is greater than the pressure "above."

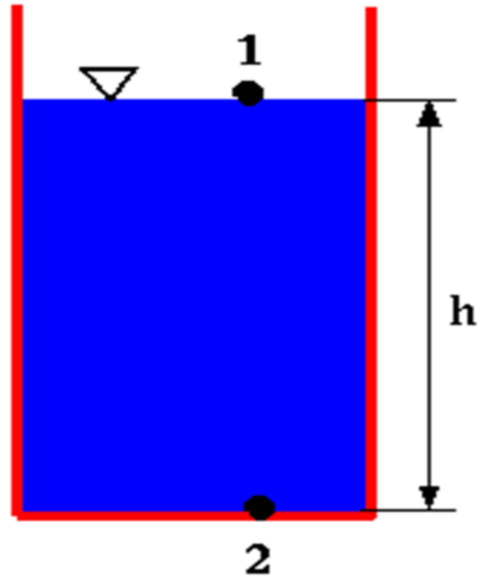
There are several "rules" or comments which directly result from the above equation:



1) If you can draw a continuous line through the same fluid from point 1 to point 2, then $p_1 = p_2$ if $z_1 = z_2$.

For example, consider the oddly shaped container. By this rule, $p_1 = p_2$ and $p_4 = p_5$ since these points are at the same elevation in the same fluid. However, p_2 does not equal p_3 even though they are at the same elevation, because one cannot draw a line connecting these points through the same fluid. In fact, p_2 is less than p_3 since mercury is denser than water.

2) Any free surface open to the atmosphere has atmospheric pressure, p_0.

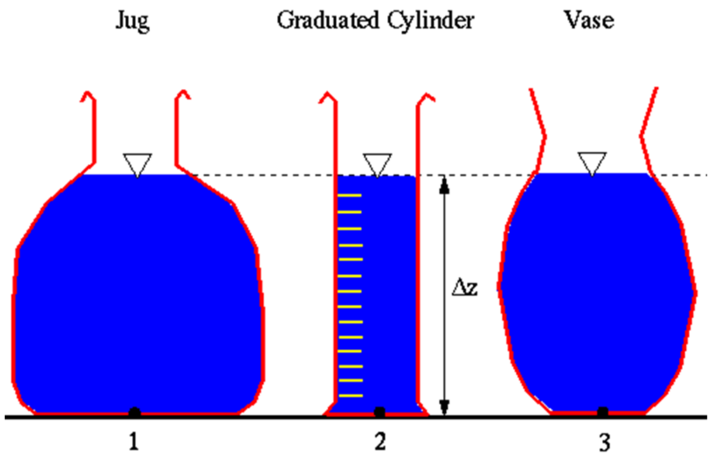


(This rule holds not only for hydrostatics, but for any free surface exposed to the atmosphere, whether the surface is moving, stationary, flat, or mildly curved.)

Consider the hydrostatics example of a container of water: The little upside-down triangle indicates a free surface, and means that the pressure there is atmospheric pressure, p_0. In other words, in this example, $p_1 = p_0$. To find the pressure at point 2, our hydrostatics equation is used: $p_2 = p_0 + \rho g h$ (absolute pressure) or $p_2 = \rho g h$ (gauge pressure).

3) The shape of a container does not matter in hydrostatics.

(Except of course for very small diameter tubes, where surface tension becomes important.) Consider the three containers in the figure below:

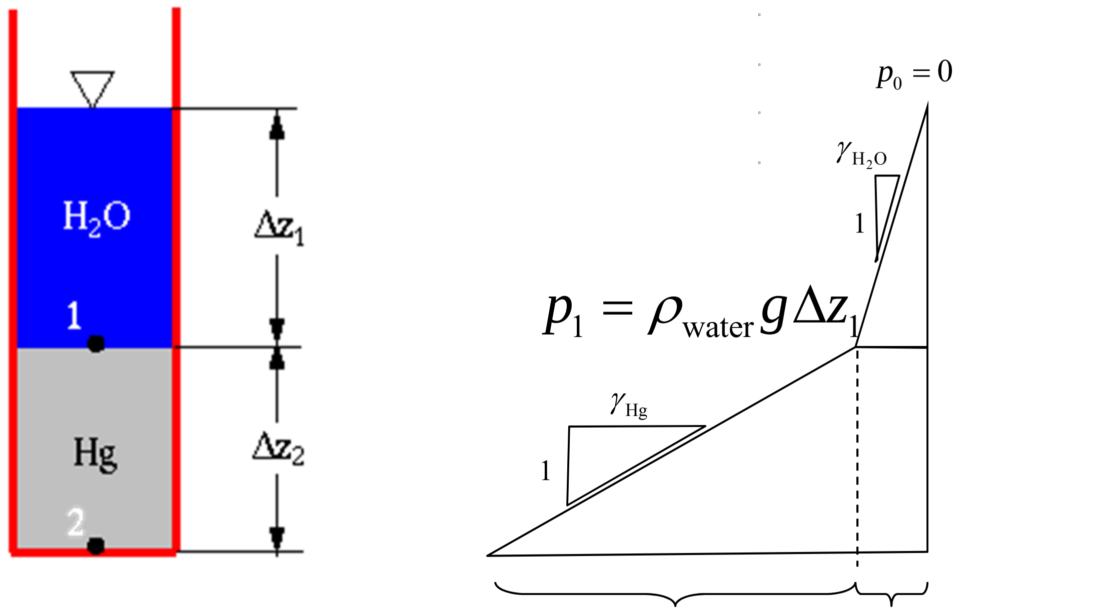


At first glance, it may seem that the pressure at point 3 would be greater than that at point 1 or 2, since the weight of the water is more "concentrated" on the small area at the bottom, but in reality, all three pressures are identical. Use of our hydrostatics equation confirms this conclusion, i.e.

$$p_{\text{below}} = p_{\text{above}} + \rho g \left|\Delta z\right| \Rightarrow p_1 = p_2 = p_3 = p_0 + \rho g \Delta z$$

4) Pressure in layered fluid.

For example, consider the container in the figure below, which is partially filled with mercury, and partially with water:



$$p_1 = \rho_{water} g \Delta z_1$$

$$p_2 = \rho_{mercury} g \Delta z_2 + p_1$$

In this case, our hydrostatics equation must be used twice, once in each of the liquids

$$p_{\text{below}} = p_{\text{above}} + \rho g |\Delta z|$$

$$\Rightarrow p_1 = p_0 + \rho_{\text{water}} g \Delta z_1 \qquad \text{and} \qquad p_2 = p_1 + \rho_{\text{mercury}} g \Delta z_2$$

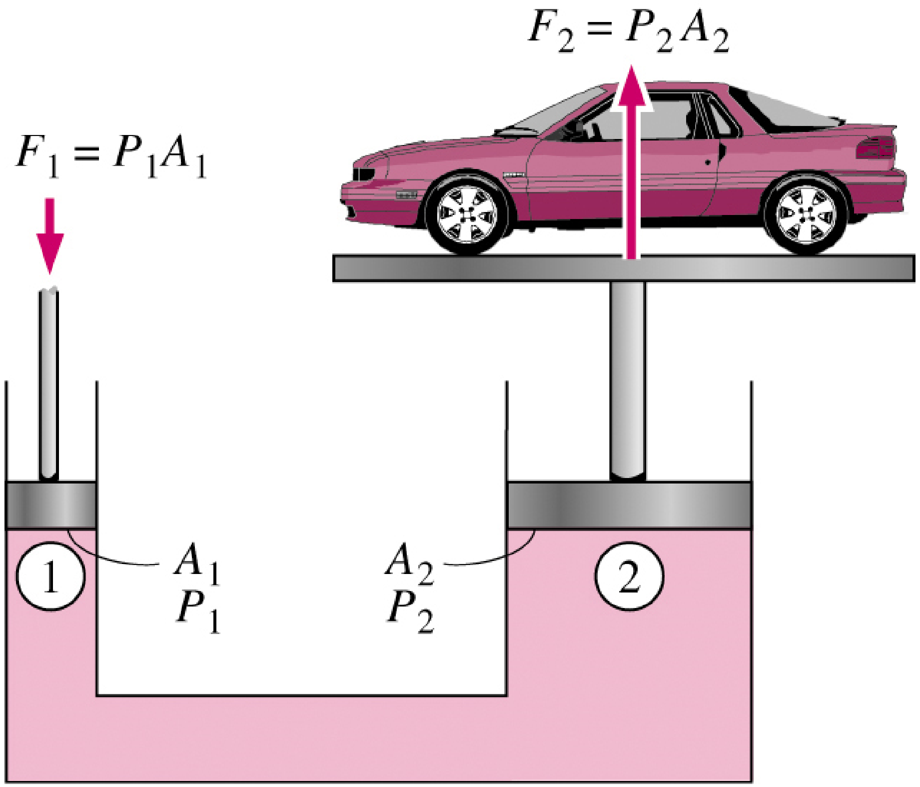
Combining,

$$p_2 = p_0 + \rho_{\text{water}} g \Delta z_1 + \rho_{\text{mercury}} g \Delta z_2$$

Shown on the right side of the above figure is the distribution of pressure with depth across the two layers of fluids, where the atmospheric pressure is taken to be zero $p_0 = 0$.

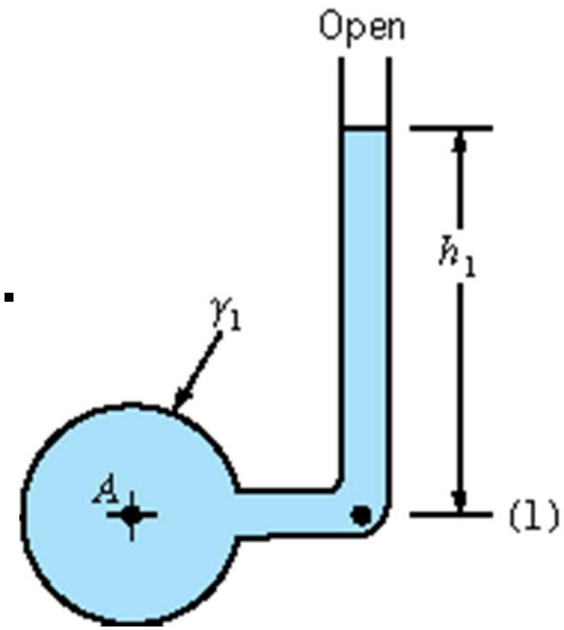
The pressure is continuous at the interface between water and mercury. Therefore, p_1, which is the pressure at the bottom of the water column, is the starting pressure at the top of the mercury column.

The fact that the pressure applied to a confined fluid increases the pressure throughout the fluid by the same amount has important applications, such as in the hydraulic lifting of heavy objects:



$$P_1 = P_2 \Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \frac{F_1}{F_2} = \frac{A_1}{A_2}$$

1) Piezometer tube (匀压计管)

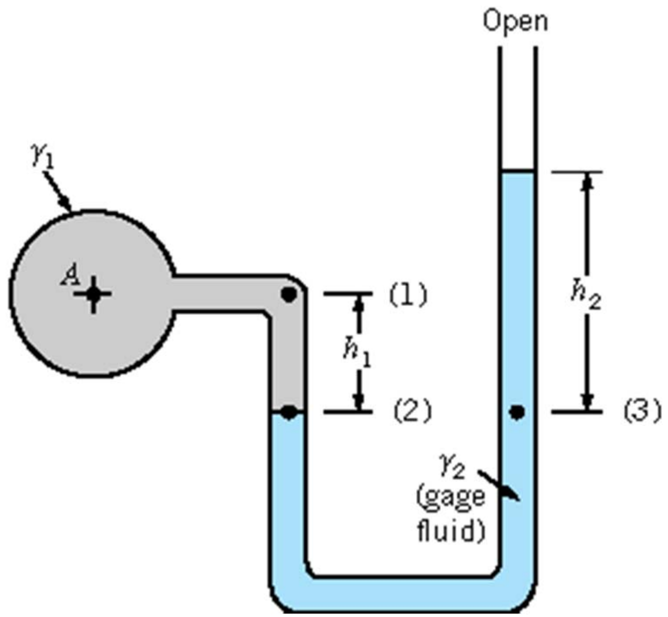


The simplest manometer is a tube, open at the top, which is attached to a vessel or a pipe containing liquid at a pressure (higher than atmospheric) to be measured. This simple device is known as a piezometer tube. As the tube is open to the atmosphere the pressure measured is relative to atmospheric so is gauge pressure:

$$p_A = \gamma_1 h_1$$

This method can only be used for liquids (i.e. not for gases) and only when the liquid height is convenient to measure. It must not be too small or too large and pressure changes must be detectable.

2) U-tube manometer (U形测压计)



This device consists of a glass tube bent into the shape of a "U", and is used to measure some unknown pressure.

For example, consider a U-tube manometer that is used to measure pressure p_A in some kind of tank or machine.

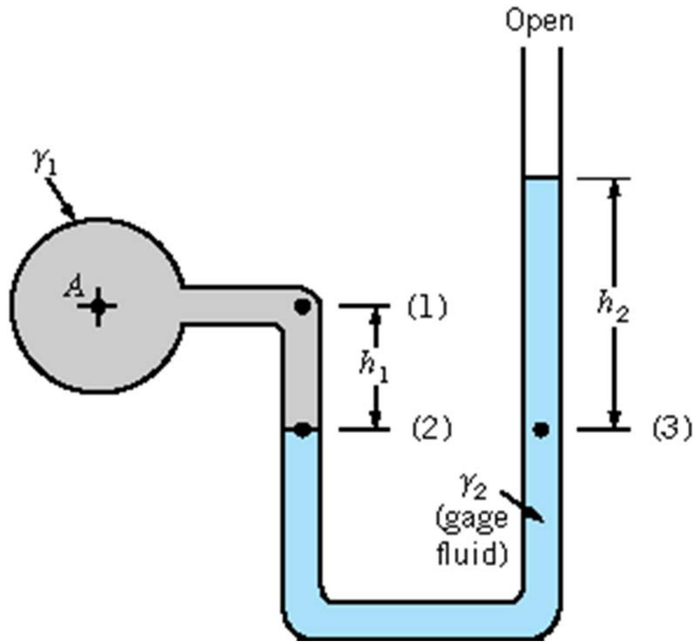
Consider the left side and the right side of the manometer separately:

$$p_2 = p_1 + \gamma_1 h_1 = p_A + \gamma_1 h_1$$

$$p_3 = \gamma_2 h_2$$

Since points labeled (2) and (3) in the figure are at the same elevation in the same fluid, they are at equivalent pressures, and the two equations above can be equated to give

$$p_A = \gamma_2 h_2 - \gamma_1 h_1$$



Finally, note that in many cases (such as with air pressure being measured by a mercury manometer), the density of manometer fluid 2 is much greater than that of fluid 1. In such cases, the last term on the right is sometimes neglected.