上海交通大学
SHANGHAI JIAO TONG UNIVERSITY
CMHL SJTU COMPUTATIONAL MARINE HYDRODYNAMICS LAB
上海交大船舶与海洋工程计算水动力学研究中心

Class-1

NA26018

Finite Element Analysis of Solids and Fluids

万德成

dcwan@sjtu.edu.cn， http://dcwan.sjtu.edu.cn/

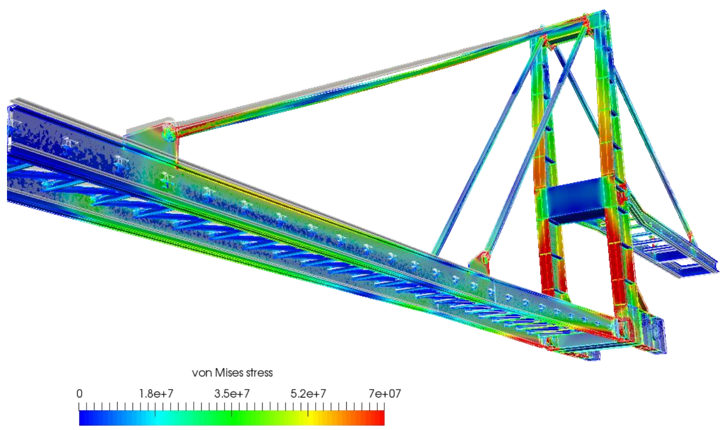
上海交通大学

船舶海洋与建筑工程学院
海洋工程国家重点实验室

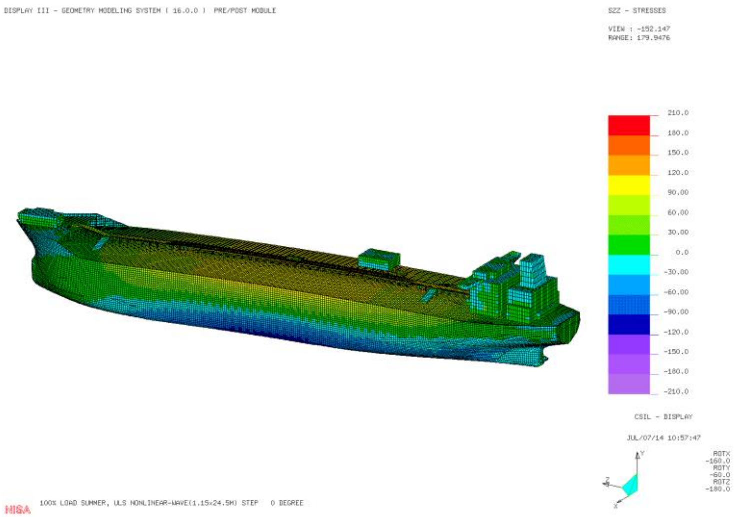
2020年

Introduction

- **The finite element method (FEM) is one of the most widely used method for solving problems of engineering and mathematical models** (Typical finite element application of interest include the traditional fields of structural analysis, heat transfer, fluid flow, mass transport, and electromagnetic potential, et al.)



Bridge stress analysis

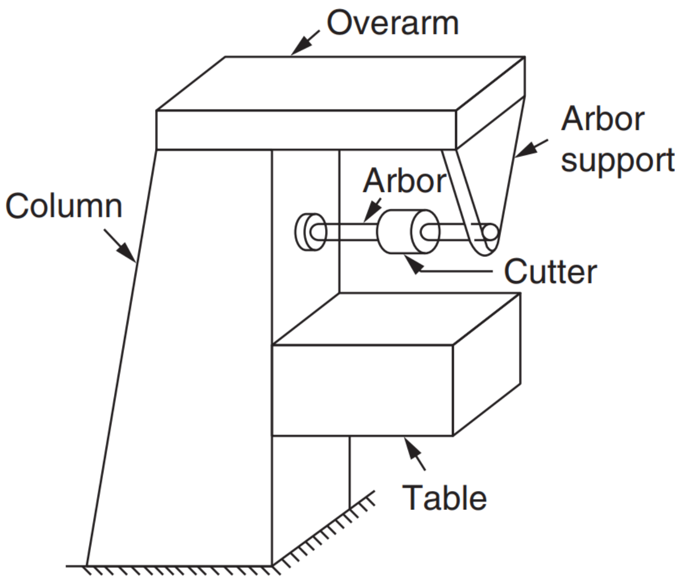


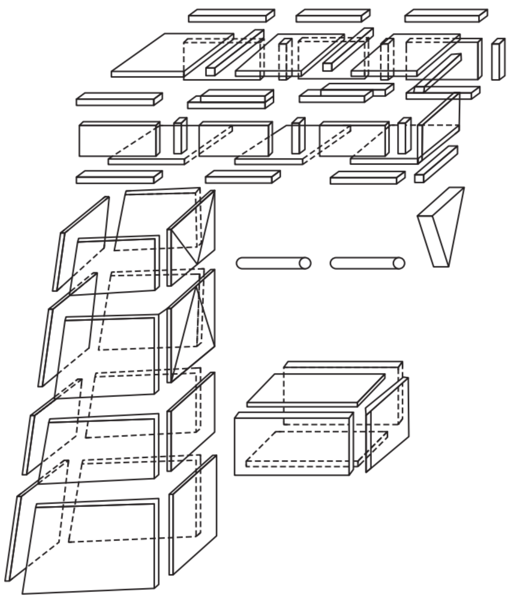
Ship stress analysis

Introduction

Why we need Numerical Method/FEM?

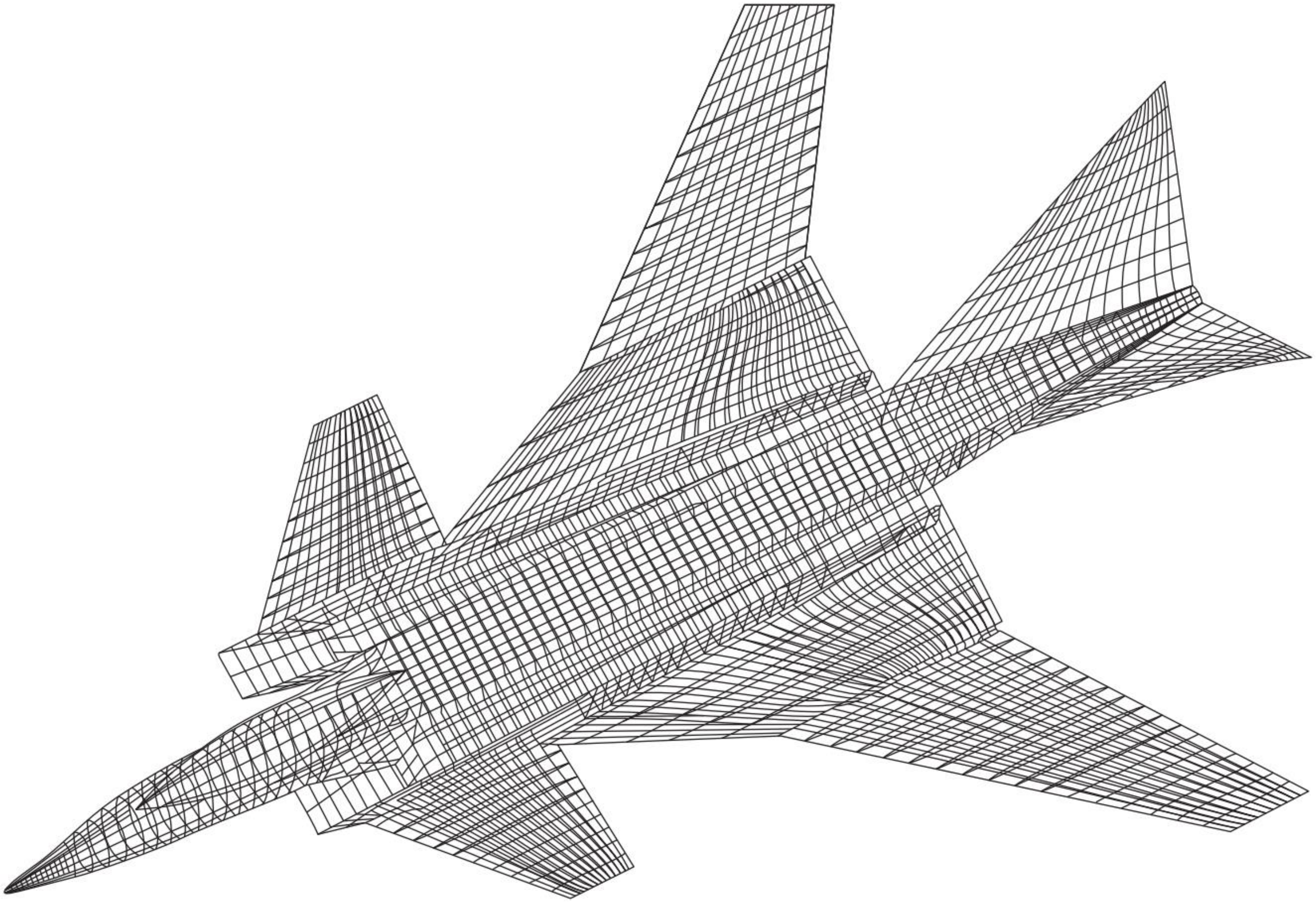
- **The existing mathematical tools will not be sufficient to find the exact solution (and sometimes, even an approximate solution) of most of the practical problems**
- **So the actual problem is often replaced by a simpler one in finding the solution, thus we will be able to find an approximate solution rather than the exact solution**





(A) Milling machine structure (B) Finite element idealization

Introduction



Finite element mesh of a fighter aircraft (from Anamet, Inc.)

Introduction

- **Physical phenomena**
  - **Mathematical formulation of the physical process**
  - **Numerical analysis of the mathematical model**

- **Mathematical model -> Differential equation**
  - **Analytically**
  - **Numerically**
    - **Finite difference method**
    - **Finite element method**
    - **Boundary element method**
    - **Spectrum method**

- **Finite element method (FEM)/finite element analysis(FEA)**

A computational technique used to obtain approximate solutions of boundary value problems in Engineering and science

Basic idea of finite element modeling

- Finite element analysis is started by a discretization in the space dimensions, which is implemented by the construction of a mesh of the object

- The variational methods from the calculus of variations are adopted to obtain "weak form" of the original partial differential equations for a specific problem with boundary conditions

- The finite element formulation of the specific problem finally results in a system of algebraic equations over each element

- The simple equations that model these finite elements are then assembled into a larger system of equations that models the entire problem

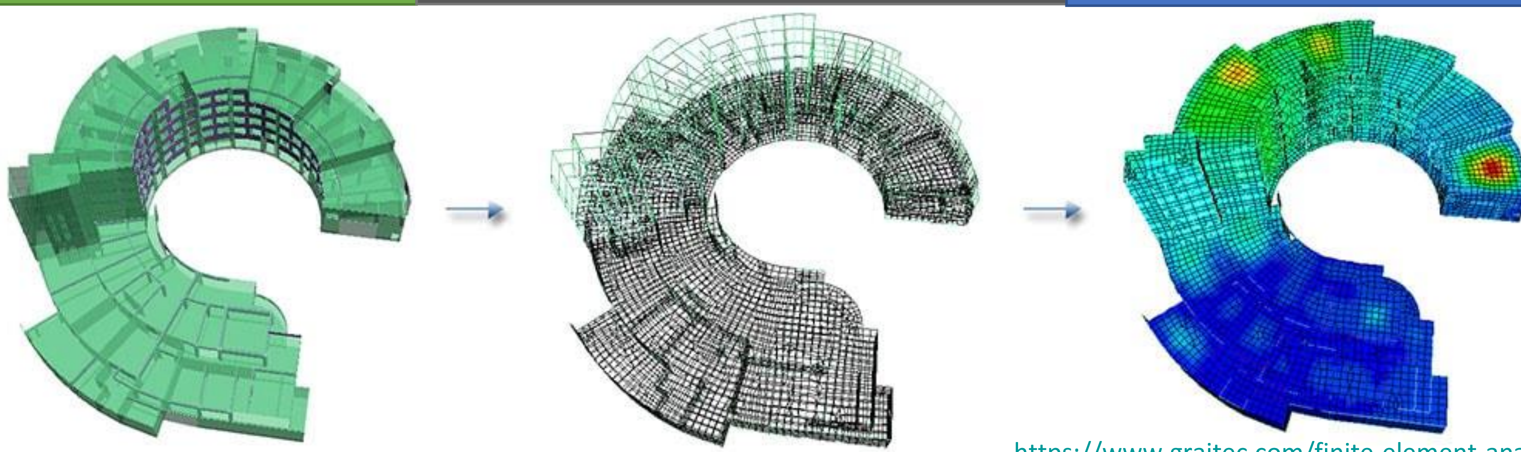
Introduction

Steps for a typical finite element analysis (by FE software)

Preprocessing:
Geometry, material properties, loads (actions), and boundary conditions are given as input data. In-built automatic mesh generation modules develop the finite element mesh with minimal input from the analyst on the type of elements and mesh density to be used

Solving:
The software automatically generates the element characteristic matrices and characteristic vectors, assembles them to generate the system equations, implements the specified boundary conditions, solves the equations to find the nodal values of the field variable, and computes the element

Postprocessing:
The solution of the problem, such as nodal displacements and element stresses, can be displayed either numerically in tabular form or graphically (two- or three-dimensional plots of deformed shape or stress variation). The analyst can choose the mode of display for the results

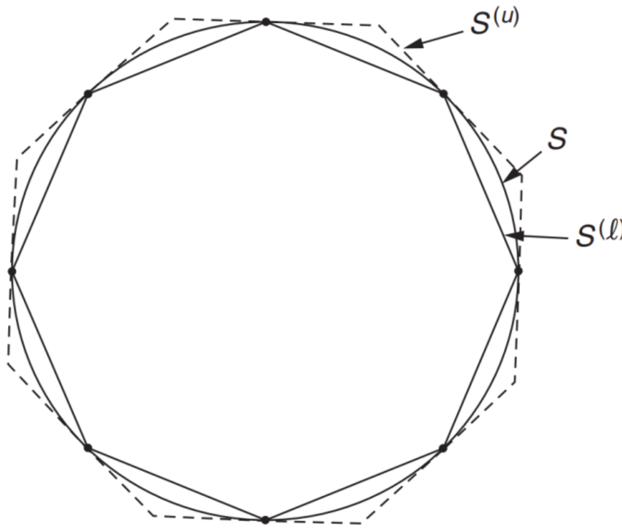


https://www.graitec.com/finite-element-analysis-fea/

Historical Background

Although the name of the finite element method was introduced by Turner, Clough, Martin, and Topp in 1956 in the context of the analysis of aircraft structures, the concept dates back several centuries

- **Ancient mathematicians found the circumference of a circle by approximating it by the perimeter of a polygon（Cyclotomic method, 割圆术）**
- **Each side of the polygon can be called an element**
- $S^{(l)} < S < S^{(u)}$



Historical Background

- The first development can be traced back to the work of A. Hrennikoff in 1941 and R. Courant in 1943.

Although these pioneers used different perspectives in their finite element approaches, they each identified the one common and essential characteristic: mesh discretization of a continuous domain into a set of discrete sub-domains, usually called elements

- Since mid-1950s, engineers in the aircraft industry have worked on developing approximate methods for the prediction of stresses induced in aircraft wings

- In 1956, Turner et al. presented a method for modeling the wing skin using three-node triangles.

The study by Turner et al. is considered one of the key contributions in the development of the finite element method

Historical Background

- **Although it was originally developed based mostly on intuition and physical argument, the method was recognized as a form of the classic Rayleigh-Ritz method in the early 1960s**

Once the mathematical basis of the method was recognized, the developments of new finite elements for different types of problems and the popularity of the method started to grow almost exponentially

- **Another fundamental mathematical contribution is represented by the book "An Analysis of the Finite Element Method" by Gilbert Strang and George Fix, first published in 1973**

Since then, finite element method has been generalized for the numerical modeling of physical systems in many engineering disciplines including heat transfer, fluid dynamics and electromagnetism

The digital computer provided a rapid means of performing the many calculations involved in the finite element analysis and made the method practically viable

Historical Background

Main History Milestones

- **1941-1943: Hrennikoff and Courant developed mesh discretization methods for solving elasticity and structural analysis problems in civil and aeronautical engineering**
- **1956: Ray W. Clough published the first paper on the finite element method. The term "finite elements" was coined in a 1960 article**
- **1959: General Motors and IBM build the computer system DAC-1 (Design Augmented by Computers) to facilitate the design of cars**
- **1960: William Fetter from Boeing coins the term "computer graphic" for his human factors cockpit drawings**

Historical Background

- 1965: NASTRAN (NASA Structural Analysis) is developed as structural analysis solver tool
- 1977: FIESTA, the first professional FEM p-version code, was initiated by Alberto Peano from ISME
- 1982: PROBE, developed by Barna Szabo and Kent Myers, was the first 'industrial' implementation of p-version FEA for research and aerospace applications
- 1987: MECHANICA was developed by RASNA Corp
- 2006: ASME Guide for Verification and Validation in Computational Solid Mechanics is released
- 2008: NASA released first standard for development of models and simulations

The finite element technique has been so well established that today, it is considered one of the best methods to solve a wide variety of practical problems efficiently

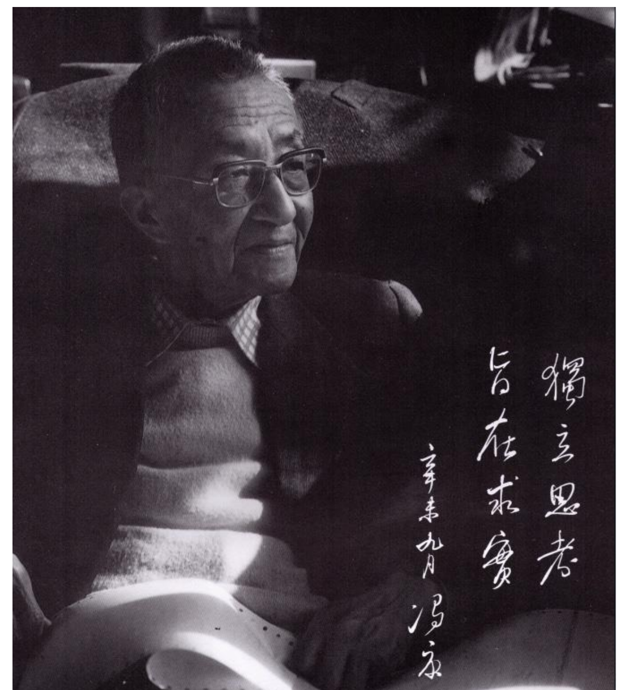
FEM in China

- **Feng Kang (冯康)**

In the late 1950s, Feng turned his attention to applied mathematics, where his most important contributions lie. Independently of parallel developments in the West, he created a theory of the finite element method——Peter Lax

- 中国近代数学能够超越西方或与之并驾齐驱的主要原因有三个，一个是陈省身教授在示性类方面的工作，一个是华罗庚在多复变函数方面的工作，一个是冯康在有限元计算方面的工作。

  ——丘成桐



FEM Application

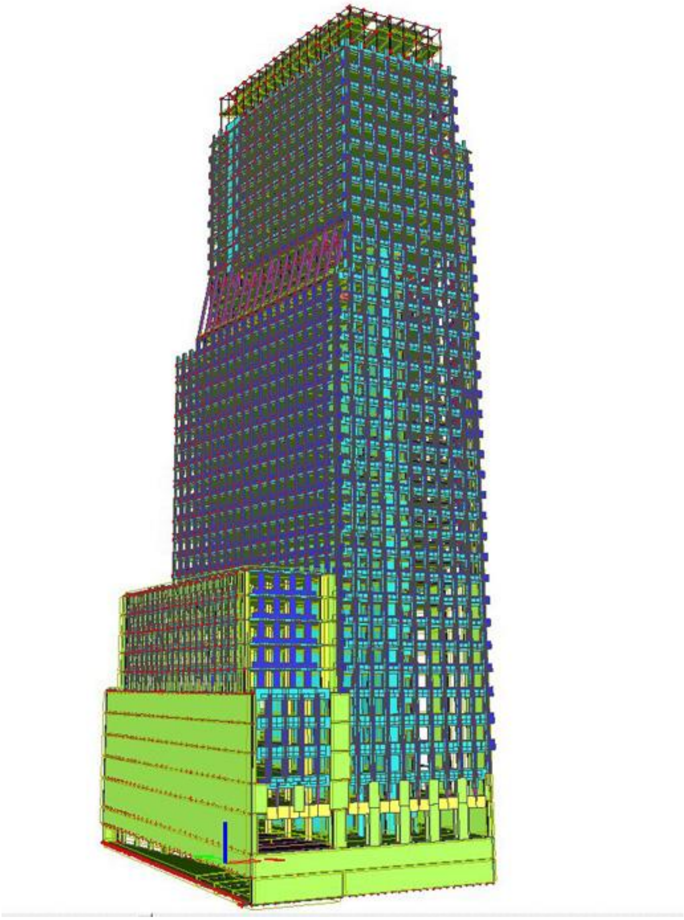
Engineering application of finite element analysis

Area of study	Equilibrium problems	Eigenvalue problems	Propagation problems
1. Civil engineering structures	Static analysis of trusses, frames, folded plates, shell roofs, shear walls, bridges, and prestressed concrete structures	Natural frequencies and modes of structures; stability of structures	Propagation of stress waves; response of structures to aperiodic loads
2. Aircraft structures	Static analysis of aircraft wings, fuselages, fins, rockets, spacecraft, and missile structures	Natural frequencies, flutter, and stability of aircraft, rocket, spacecraft, and missile structures	Response of aircraft structures to random loads; dynamic response of aircraft and spacecraft to aperiodic loads
3. Heat conduction	Steady-state temperature distribution in solids and fluids	—	Transient heat flow in rocket nozzles, internal combustion engines, turbine blades, fins, and building structures
4. Geomechanics	Analysis of excavations, retaining walls, underground openings, rock joints and soil–structure interaction problems; stress analysis in soils, dams, layered piles, and machine foundations	Natural frequencies and modes of dam–reservoir systems and soil–structure interaction problems	Time-dependent soil–structure interaction problems; transient seepage in soils and rocks; stress wave propagation in soils and rocks
5. Hydraulic and water resources engineering; hydrodynamics	Analysis of potential flows, free surface flows, boundary layer flows, viscous flows, transonic aerodynamic problems; analysis of hydraulic structures and dams	Natural periods and modes of shallow basins, lakes, and harbors; sloshing of liquids in rigid and flexible containers	Analysis of unsteady fluid flow and wave propagation problems; transient seepage in aquifers and porous media; rarefied gas dynamics; magneto-hydrodynamic flows

Engineering application of finite element analysis

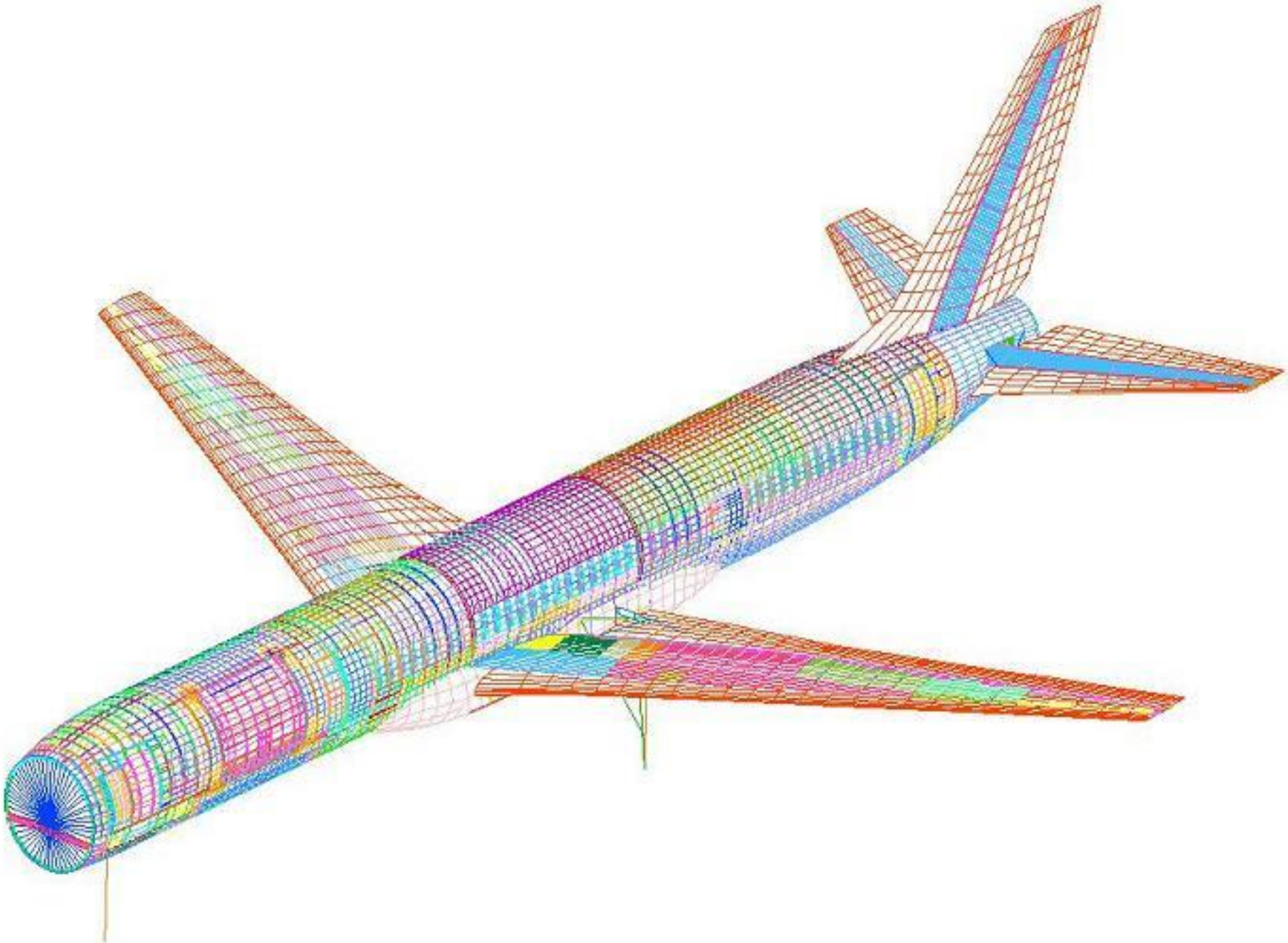
Area of study	Equilibrium problems	Eigenvalue problems	Propagation problems
6. Nuclear engineering	Analysis of nuclear pressure vessels and containment structures; steady-state temperature distribution in reactor components	Natural frequencies and stability of containment structures; neutron flux distribution	Response of reactor containment structures to dynamic loads; unsteady temperature distribution in reactor components; thermal and viscoelastic analysis of reactor structures
7. Biomedical engineering	Stress analysis of eyeballs, bones, and teeth; load-bearing capacity of implant and prosthetic systems; mechanics of heart valves	—	Impact analysis of skull; dynamics of anatomical structures
8. Mechanical design	Stress concentration problems; stress analysis of pressure vessels, pistons, composite materials, linkages, and gears	Natural frequencies and stability of linkages, gears, and machine tools	Crack and fracture problems under dynamic loads
9. Electrical machines and electromagnetics	Steady-state analysis of synchronous and induction machines, eddy current, and core losses in electric machines, magnetostatics	—	Transient behavior of electromechanical devices such as motors and actuators, magnetodynamics

FEM Application



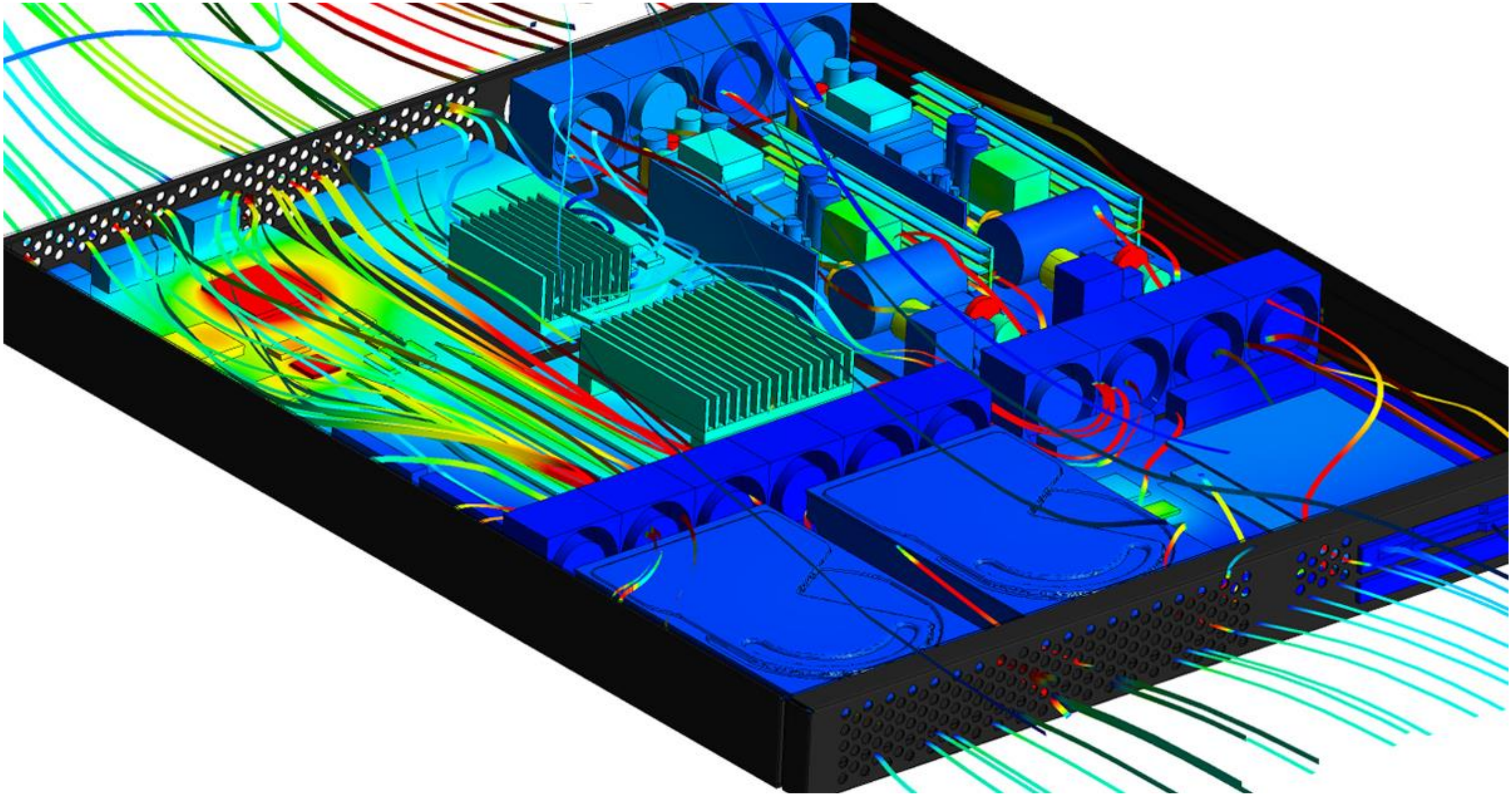
Application in civil engineering

FEM Application



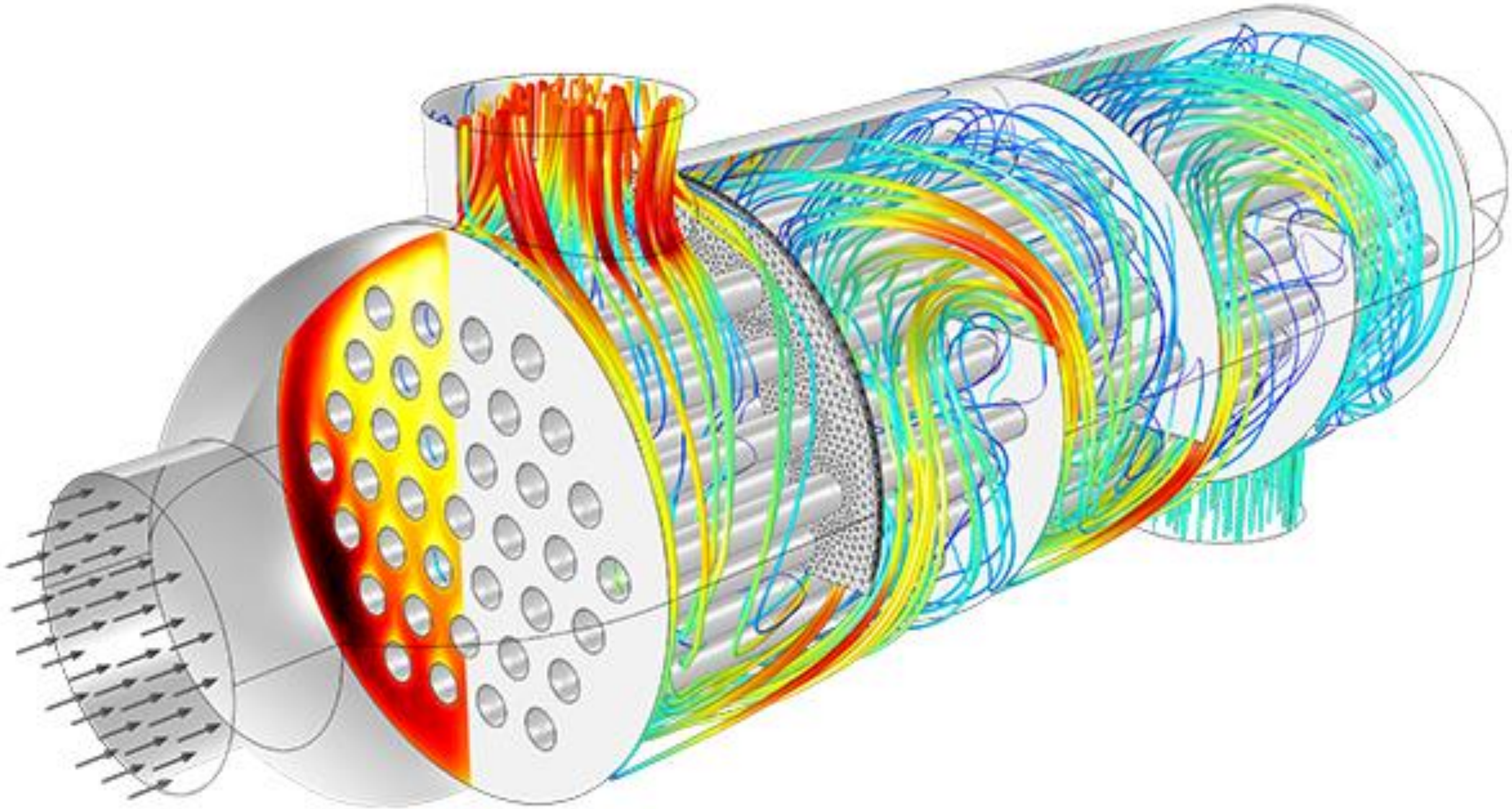
Application in the assessment of Aircraft structures

FEM Application



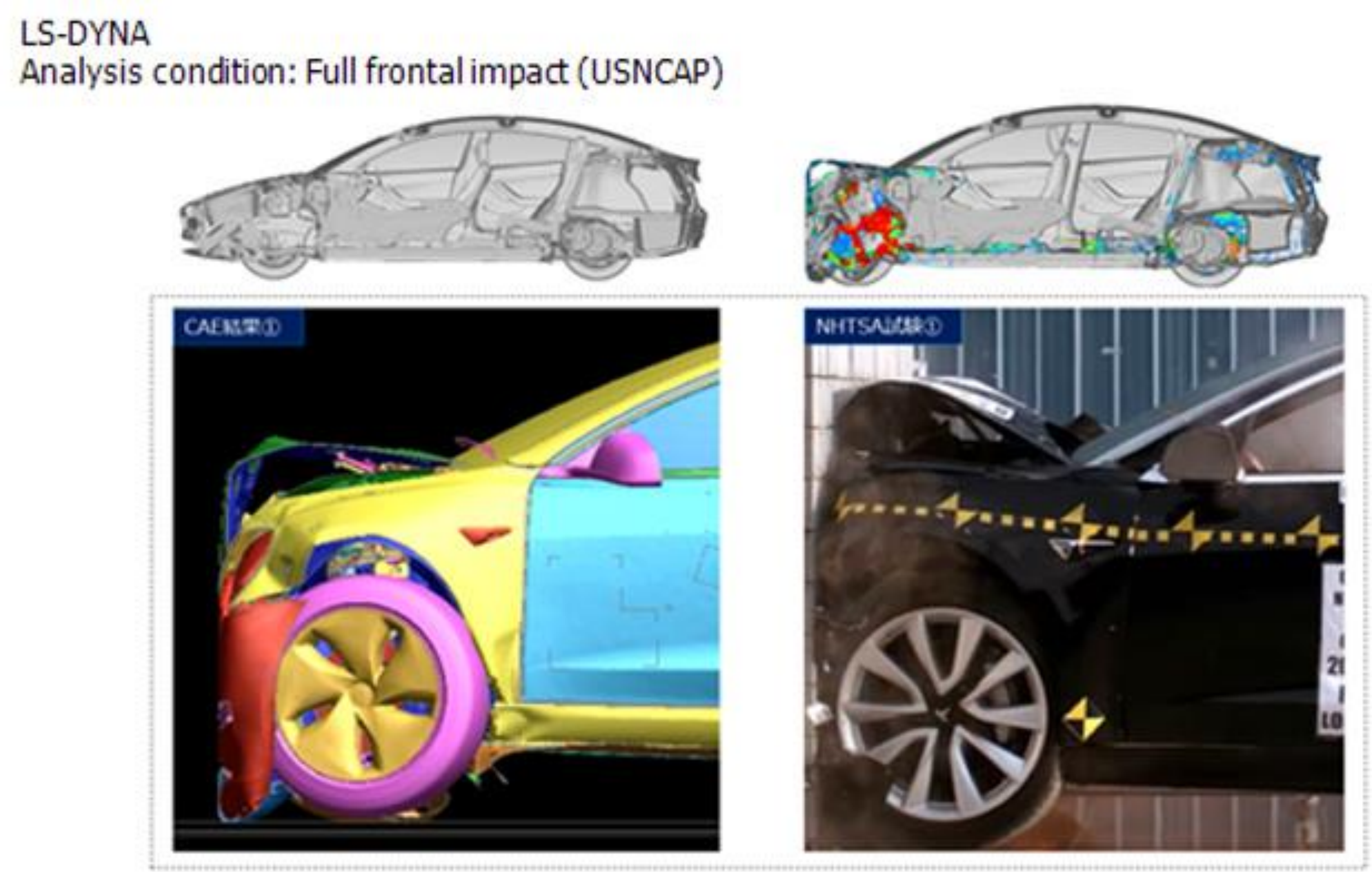
Application in the heat conduct of a computer

FEM Application



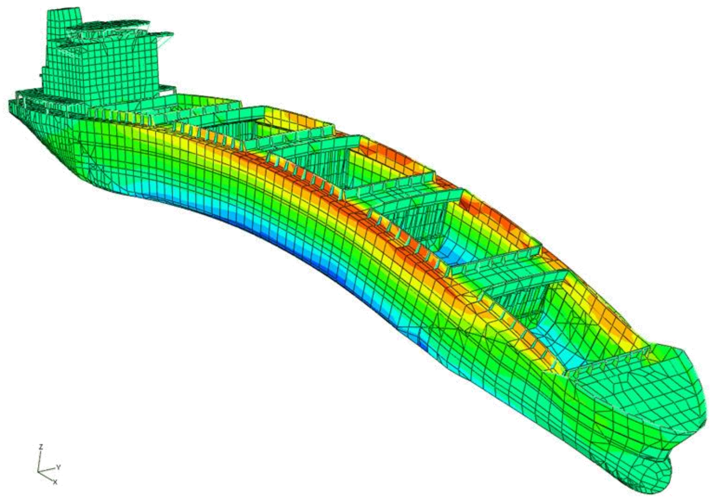
Application of a heat exchanger with water flowing in the inner tubes

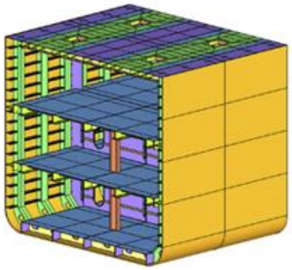
FEM Application

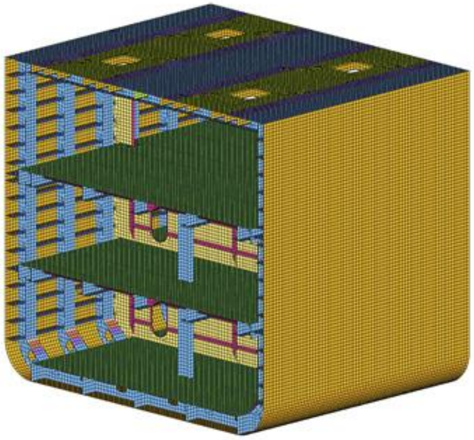


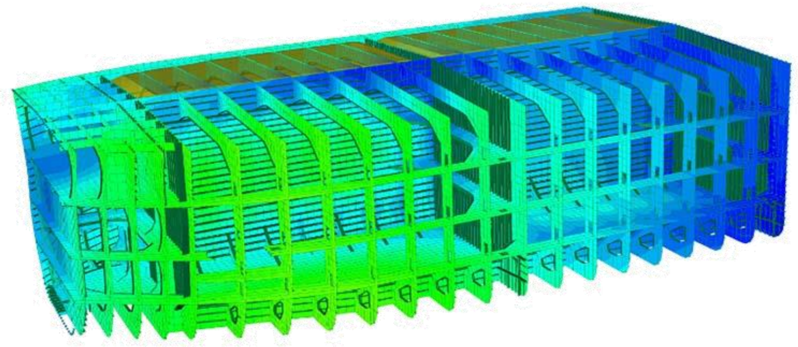
Application of a car crash test using FEM (LS-DYNA)

FEM Application









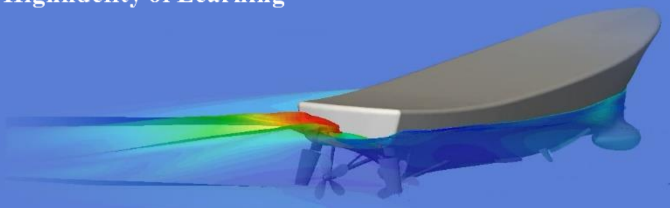
Application in naval architecture analysis

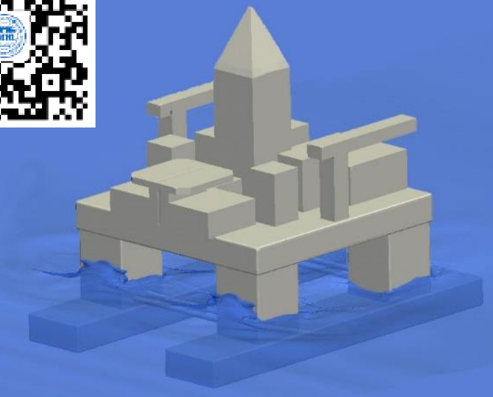
谢 谢！

CMHL SJTU COMPUTATIONAL MARINE HYDRODYNAMICS LAB
上海交大船舶与海洋工程计算水动力学研究中心

创新创智·求真求实

Creation of Mind, Highfidelity of Learning





http://dcwan.sjtu.edu.cn