A ONE-WAY COUPLING STRATEGY OF THE GREEN-NAGHDI EQUATIONS AND THE OPENFOAM

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Highlights:

- A numerical wave tank is built based on the one-way coupling of the fully nonlinear Green-Naghdi equations and the OpenFOAM.
- The regular waves and Freak waves are simulated to give a comparative study between the Green-Naghdi equations and the one-way coupling strategy.

1. Introduction

The viscous effects usually need not to be considered on the free surface in a numerical wave tank. This has a great benefit for decreasing the computing cost. However, it could not be ignored on a solid boundary and for breaking waves because of the existence of vortex. The Navier-Stokes equations are most commonly used for a viscous numerical wave tank. It is usually accompanied with an expensive computing cost. So it should be attractive to develop a coupling method for the viscous and non-viscous flow. There are two methodologies to achieve the coupling work. One is “Velocity decomposition” and the other is “Domain decomposition”.

An exhaustively description on the “velocity decomposition” strategy is given in Edmund (2012). He divides the velocity to irrotational and rotational parts. On the other hand, Ferrant et al. (2002) propose another “velocity decomposition” strategy named Spectral Wave Explicit Navier-Stokes Equations (SWENSE). It avoids the computational cost of refining the mesh on the free surface.

Campana et al (1992) propose a “domain decomposition” strategy to calculate the wing in steady flow. They state that the information exchanging is mainly problem for this strategy. Hamilton and Yeung (2011) used a cylindrical surface to separate the viscous domain from the potential domain. The prediction-correction scheme is utilized to solve the pressure discontinues on the match surface. Kim et al. (2012) develop an Euler Overlay Method (EOM) to analysis the ringing loads on a vertical cylinder in the Ocean wave. In the overlay zone, an added source term related with the outer Euler solution is introduced in the Navier-Stokes equations. Moreover, a damping function is utilized to avoid reflecting effect to the CFD boundary. Paulsen et al. (2014) developed a one-way coupling strategy by using relaxation zone. A fully nonlinear potential flow solver combined with a Navier-Stokes solver is carried out.

Herein, we provided a coupling strategy of the fully nonlinear Green-Naghdi (GN) equations and OpenFOAM. The GN equations (see e.g., Demirbilek and Webster, 1992) have different levels for representation of the horizontal and vertical variations in the wave kinematics. This level theory is subject to degree of polynomial representation. The free surface boundary conditions are always satisfied and the irrotationality assumption is not required in the derivation of the GN equations. So the GN equations are derived from Euler Equation. Owing to the assumption on the velocity variation in the vertical direction across the fluid sheet, the GN equations can be easily utilized in deep water (Zheng et al., 2016) and wave-current interactions (Duan et al., 2016).

In this paper, a couple work of GN equations and OpenFOAM is carried out. The governing equations and coupling strategy are described in Section 2, test cases are in Section 3 and some conclusions are in Section 4.

2. Numerical model and coupling strategy

2.1 The Green Naghdi Equations

The governing equations of GN theory are developed from the continuity equation and the Euler's equations for an incompressible fluid:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \cdot \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x}$$
$$\frac{\partial w}{\partial t} + u \cdot \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \cdot (\frac{\partial p}{\partial z} + \rho \cdot g) \tag{2}$$

The fully nonlinear free surface conditions are always satisfied. The shape function in the finite depth GN equations is a polynomial depends upon *z* only:

$$u(x, z, t) = \sum_{n=0}^{K-1} u_n(x, t) \cdot z^n \quad ; \quad w(x, z, t) = \sum_{n=0}^{K} w_n(x, t) \cdot z^n \tag{3}$$

For more details, the reader is referred to Zhao et al. (2014). The Green-Naghdi equations are used as the solver of non-viscous domain.

2.2 The OpenFOAM

In OpenFOAM, each application implements has a specific mathematical model. For the water wave issues, the momentum and continuity equations as well as the Volume of Fluid (VOF) scheme are solved in a standard implementation in OpenFOAM.

$$\nabla \cdot \vec{u} = 0 \tag{4}$$

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla \cdot \left(\rho\vec{u} \cdot \vec{u}^T\right) = -\nabla p + g\left(x - x_r\right) \cdot \nabla\rho + \nabla \cdot (\mu_{tot}\nabla\vec{u}) \tag{5}$$

A Finite Volume Method (FVM) is used in OpenFOAM to discretize the computational domain.

2.3 Coupling Strategy

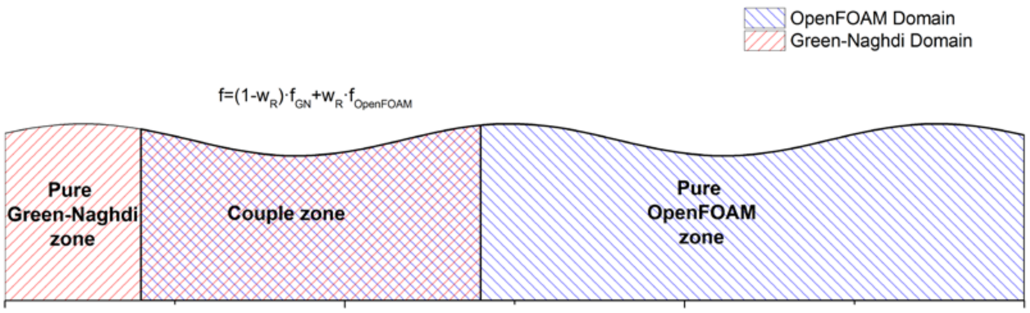

Figure 1. The sketch of computational domain

The strategy introduced here can be classified as the domain decompositions. An open-source toolbox named Waves2Foam is used as the solver of CFD domain. We use the relaxation scheme developed by Jacobsen et al. (2012) to achieve a one-way coupling strategy. In Fig. 1 the computational domain is presented. The relaxation scheme is used in the coupling zone shown in Fig. 1. The target quantities denoted by f_{GN} are obtained from the Green-Naghdi equations. w_R is the weighting function.

3. Test cases

3.1 Regular waves

Regular wave is simulated by using the Green-Naghdi results as target information. The description of the regular wave is displayed in Table 1. The length of coupling zone is 2λ. The time history of two wave probe (x/λ=2, x/λ=4) are shown in Fig. 2. The CFD wave agrees with the target Green-Naghdi wave. However, significant decreasing is observed in wave elevation during the propagation. We have to state that more exhaustively convergence study will be given in the workshop.

Table 1: Case description

Items	**Water depth (d)**	**Wave period (T)**	**Wave length (λ)**	**Wave amplitude (A)**
Values	0.6 m	0.7018 s	0.808 m	0.0288 m

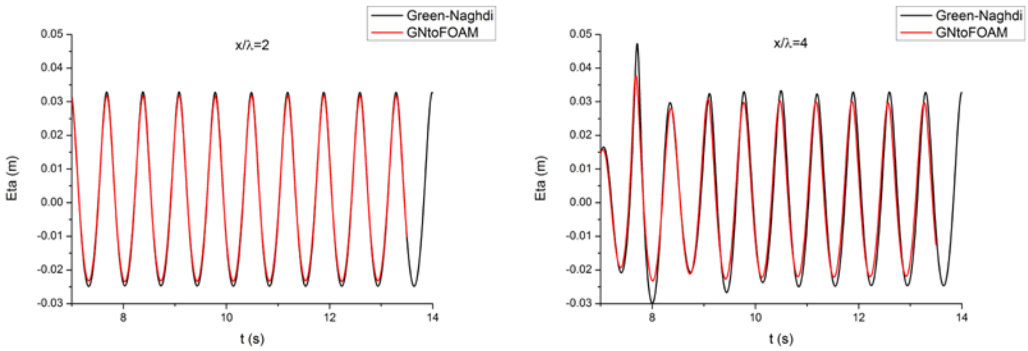

Figure 2. Time history of two wave probes (x/λ=2, x/λ=4)

3.2 Freak waves

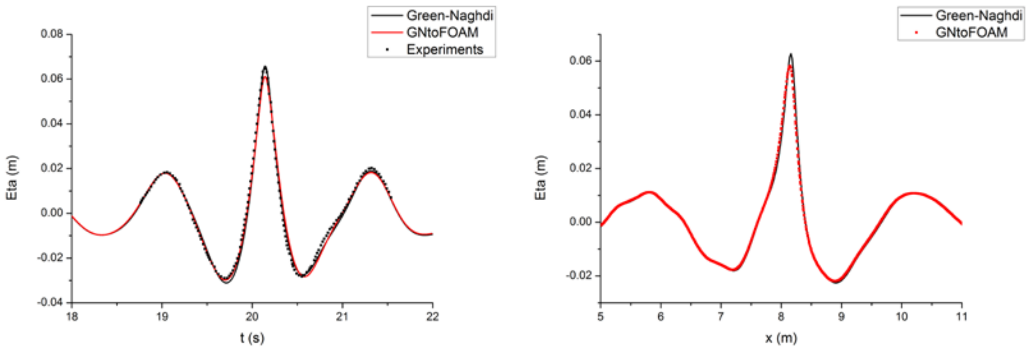

Figure 3. Time history (8.32 m) and snapshot (20 s) of focusing freak wave

In Fig. 3, the freak wave generated from wave focusing is simulated. The period ranges are (0.6 s, 1.4 s) and the wave group consisted of 29 individual wave components which has equal amplitude and equal period spacing. The input amplitude A is 55 mm. The focus location and time is set as 8.0 m and 20 s, respectively. This focus time is long enough for the high frequency component go through the focus location. The same focusing wave group is generated in a physical wave flume by Baldock et al. (1996). In the numerical simulation, the focus location and time is shifted to 8.32 m and 20.14 s because of the nonlinearity. It is shown that the target Green-Naghdi wave has better agreement with the experimental data than the focusing CFD wave even though the later has finer mesh in the horizontal direction. The computational cost is 111 s for GN and 489560 s for GNtoFOAM. Similarly, we have to states that more convergence studies will be presented in the workshop.

4. Conclusions

This paper provided a one-way coupling strategy of the fully nonlinear Green-Naghdi equations and the OpenFOAM. Two cases including the regular wave and focusing freak wave are shown in this work. In both of them, the waves generated from the coupling strategy agree with the target Green-Naghdi waves. This one-way coupling strategy provides a probability to shorten the computational domain and has good performance in accuracy. However, it is shown that the Green-Naghdi equations do well in efficiency. More exhaustively convergence studies will be presented in the workshop. A study on the two-way coupling strategy of Green-Naghdi equations and OpenFOAM is undergoing.

Acknowledgments

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