DEVELOPMENT OF AN ARBITRARY LAGRANGIAN-EULERIAN FINITE VOLUME METHOD FOR METAL FORMING SIMULATION IN OPENFOAM

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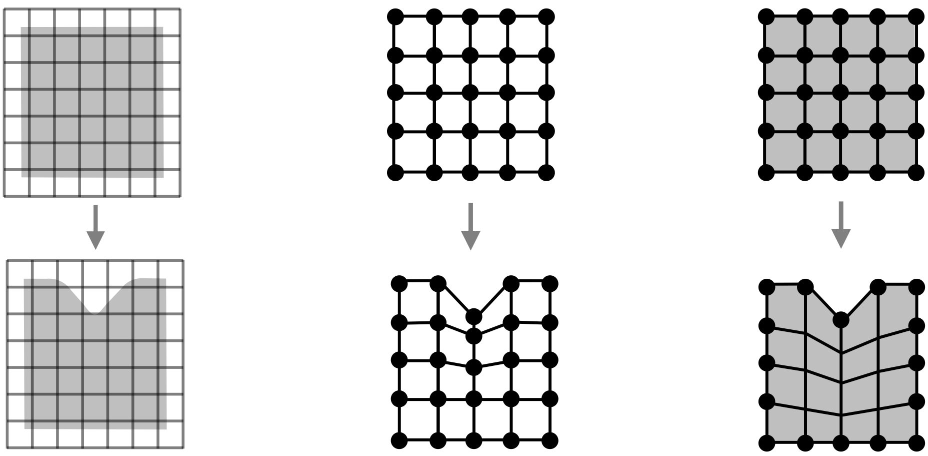
Introduction

Modelling of metal forming problems has traditionally adopted one of three approaches:

- Eulerian approach;
- Lagrangian approach;
- Arbitrary Lagrangian-Eulerian approach.

As indicated schematically in Figure 1, the Eulerian approach follows a domain as material flows through it; whereas, the Lagrangian approach follows material as it flows through a domain. The third approach, Arbitrary Lagrangian Eulerian (ALE), is a hybrid method that attempts to combine the best of both Eulerian and Lagrangian methods, where the domain boundary tracks the material boundary, and the material flows through the internal domain. Consequently, ALE approaches have the Lagrangian ability to capture the material elastic response, memory effects, and residual stresses, with the Eulerian ability to efficiently simulate extreme deformations with no concern for deteriorating mesh quality.



(a) Eulerian (fluid) formulation

(b) Lagrangian (solid) formulation

(c) Arbitrary Lagrangian-Eulerian formulation

Figure 1: Approaches to describe the initial and deformed configurations (adapted from [1])

The current project builds on the previous developments of Lagrangian finite volume methods for solid mechanics [1, 2, 3, 4, 5] to propose an ALE approach suitable for metal forming problems.

Methodology

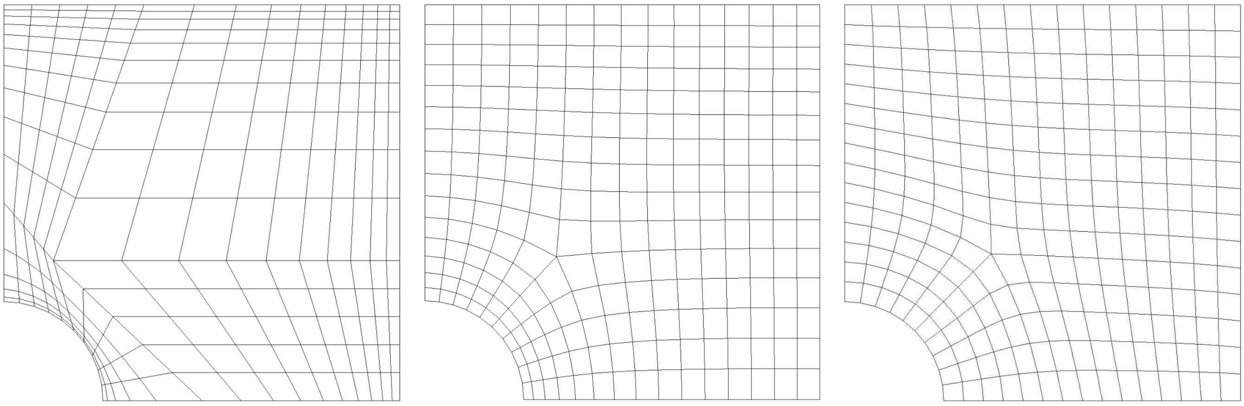
There are a number of variants of ALE methods; the approach adopted in the current work is a two step procedure:

1. an updated Lagrangian solution step,

2. followed by a mesh smoothing and field mapping step.

Updated Lagrangian solution step:
In the first step, an updated Lagrangian formulation is used to solve the governing momentum equation for the displacement field, as described recently [1]: this step provides the displacement/velocity of the deforming material, and as a Lagrangian step it also provides the motion of the mesh. To complete the updated Lagrangian step, the mesh is moved to the deformed configuration using the material displacement/velocity field.

Mesh smoothing and field remapping step:
In the second step, a mesh smoothing and field mapping procedure is applied to improve the overall mesh quality. A number of different mesh smoothing methods have been examined, including: (i) explicit point-based Laplacian smoothing with arithmetic-average weights (see Figure 2(b)); (ii) explicit point-based Laplacian smoothing with cell-volume weights (see Figure 2(c)); and (iii) implicit cell-based Laplacian smoothing method with cell-volume weights.



(a) Mesh before smoothing (b) Mesh after smoothing (arithmetic-average weights) (c) Mesh after smoothing (cell-volume weights

Figure 2: Point-based Laplacian smoothing method, comparing weight calculation method. The initial mesh is purposely distorted to examine the robustnesses of the mesh smoothing procedures.

Following mesh smoothing, the fields must be mapped to the new smoothed mesh. This field mapping can be performed in at least two ways: (a) interpolation from the initial mesh to the smoothed mesh; or (b) transport the fields using an advection equation approach. Both methods are considered here; however, the interpolation method is found to be much more expensive and difficult to efficiently parallelise. Consequently, focus is given here to the advection method. The advection mapping approach solves a conservation advection equation for each field to be mapped:

$$\frac{\partial \psi}{\partial t} + \boldsymbol{\nabla} \cdot \phi_{\mathrm{m}} \psi = 0 \tag{1}$$

where ψ represents the scalar/vector/tensor intrinsic field to be mapped, and ϕ_{m} is the mesh flux/velocity. By limiting the mesh motion to a fraction of the local cell size (*i.e.* a mesh Courant number less than unity), Equation 1 can be solved in an efficient explicit manner. The equation is discretised using the standard finite volume method, as implemented in OpenFOAM, where the temporal term employs a 1^{st} order Euler method and the advection term interpolation is performed using a 2^{nd} order van Leer scheme with Monotone Upstream-centered Schemes for Conservation Laws (MUSCL) limiter; the explicit update of a field cell value reduces to:

$$\psi^{[i+1]} = \frac{1}{\Omega^{[i+1]}} \left[\Omega^{[i]} \psi^{[i]} + \sum_{f}^{\text{nfaces}} (\Delta\Omega)_f \ \psi_f^{[i]} \right] \tag{2}$$

where $\psi^{[i]}$ is the cell field value before smoothing, $\psi^{[i+1]}$ is the cell field value after smoothing, $\Omega^{[i]}$ is the cell volume before smoothing, $\Omega^{[i+1]}$ is the cell volume after smoothing, f represents a cell face, nfaces is the number of faces in the cell, $(\Delta\Omega)_f$ is the volume swept by face f, and $\psi_f^{[i]}$ is the value of the field at face f before smoothing.
Following details of the solution algorithm, mesh smoothing and field mapping, a number of steel wire metal forming test cases will be presented, and the new ALE method compared with a traditional fully Lagrangian approach.

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