A TWO-PHASE MODEL FOR SATURATED GRANULAR-WATER INCLINED FLOWS

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1 Introduction

Gravity-driven flows of highly concentrated mixtures of granular material and water are involved in a wide variety of geophysical processes. Among them, natural flows such as debris flows, snow avalanches and submarine landslides have caused serious disasters worldwide. As compared to the relatively abundant studies of dry granular flows, study of granular-water mixture flows is more challenging due to the complex interactions between granular particles and the ambient water. The existence of ambient water has a great effect on the dynamic characteristics of dense granular flows.

This paper presents a comprehensive two-dimensional two-phase model for inclined flows of saturated granular and water mixtures over erodible and rigid beds (see Figure 1). The model is based on a general collisional-frictional law for the granular stresses. The buoyancy and drag force are considered to represent the two-phase interactions. The effects of sidewall and bottom wall are also taken into account in this model.

The present numerical model is developed via OpenFOAM® based on the solver called twoPhaseEulerFoam. The two-phase model is further applied to simulate the laboratory experiments of fully-developed granular-water mixture flows over an inclined erodible and a rigid bed. The good agreements with the measured distribution of the concentration, velocity and granular temperature confirm the capability of the model to capture the dynamic features of saturated granular-water mixture flows under different regimes.

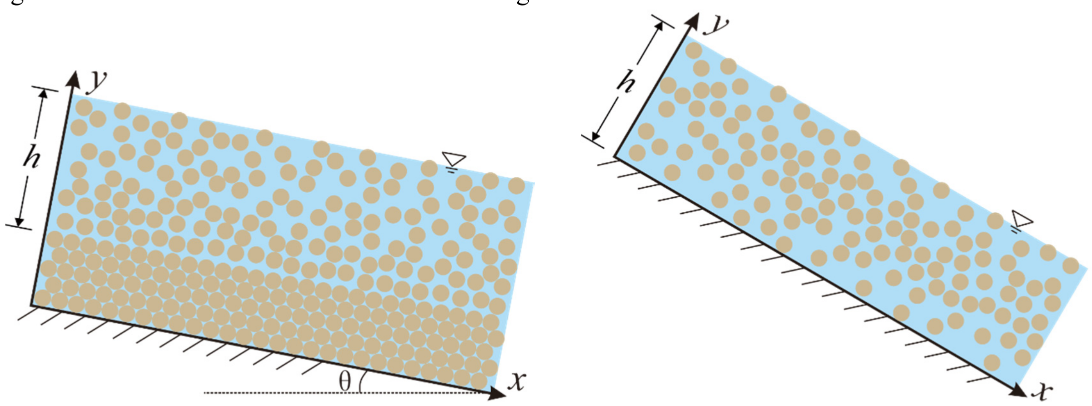

Figure 1: Sketches of saturated granular-water inclined flows over erodible (left panel) or rigid bed (right panel).

2 Model formulation

In an Eulerian-Eulerian two-phase model, the granular phase and the fluid phase are described as two interpenetrating continuums. The phase-averaged basic equations can thus be derived from an average of the mass and momentum conservation laws for the granular material and the fluid over a control volume. To consider the effects of the sidewall, through further averaging along transversal direction, the governing equations for the fluid and granular phase can be written as

$$\frac{\partial\left(\alpha_f\rho_f\right)}{\partial t}+\frac{\partial\left(\alpha_f\rho_f U_{f,i}\right)}{\partial x_i}=0 \tag{1}$$

$$\frac{\partial\left(\alpha_s\rho_s\right)}{\partial t}+\frac{\partial\left(\alpha_s\rho_s U_{s,i}\right)}{\partial x_i}=0 \tag{2}$$

$$\frac{\partial\left(\alpha_f\rho_f U_{f,i}\right)}{\partial t}+\frac{\partial\left(\alpha_f\rho_f U_{f,i}U_{f,j}\right)}{\partial x_j}=-\alpha_f\frac{\partial p_f}{\partial x_i}+\frac{\partial \tau_{f,ij}}{\partial x_j}-F_i+\alpha_f\rho_f g_i \tag{3}$$

$$\frac{\partial\left(\alpha_s\rho_s U_{s,i}\right)}{\partial t}+\frac{\partial\left(\alpha_s\rho_s U_{s,i}U_{s,j}\right)}{\partial x_j}=-\alpha_s\frac{\partial p_f}{\partial x_i}-\frac{\partial p_s}{\partial x_i}+\frac{\partial \tau_{s,ij}}{\partial x_j}+F_i+\alpha_s\rho_s g_i+\frac{2p_s\mu_w}{W}\frac{U_{s,i}}{\left|\mathbf{U}_s\right|} \tag{4}$$

where, the subscripts f and s denote quantities of fluid and granular phases, respectively; the subscripts $i, j = 1, 2$ denote streamwise and vertical directions. α is the volume fraction and satisfies $\alpha_s + \alpha_f = 1$; U is the velocity and ρ is the material density of the relevant phase; p is the pressure and τ is the deviatoric stress; F represents the granular-fluid interactive force; g is the gravitational acceleration; W is the width of channel and μ_w is the frictional coefficient between granular material and the channel sidewall.

The interaction term F_i in Eqs. (3) and (4) governs the momentum exchange between the fluid and granular phases. In dense granular problems, the lift and virtual-mass forces are insignificant when compared to the drag force. Thus, we consider only drag force in F_i, as

$$F_i = K\left(U_{f,i} - U_{s,i}\right) \tag{5}$$

where K is a generalized drag coefficient. Considering the particle group effect, the Gidaspow's (1994) formula is employed

$$K = \begin{cases} \dfrac{3}{4} C_D \dfrac{\rho_f \alpha_s \left|\mathbf{U}_f - \mathbf{U}_s\right|}{d_s} \alpha_f^{-1.65} & (\alpha_s \le 0.2) \\ \dfrac{150 \alpha_s^2 \mu_f}{\alpha_f^2 d_s^2} + \dfrac{1.75 \rho_f \alpha_s \left|\mathbf{U}_f - \mathbf{U}_s\right|}{\alpha_f d_s} & (\alpha_s > 0.2) \end{cases} \tag{6}$$

where d_s is the particle diameter and the drag coefficient C_D is given by

$$C_D = \begin{cases} \dfrac{24}{\mathrm{Re}_s}\left(1 + 0.15\,\mathrm{Re}_s^{0.687}\right) & \left(\mathrm{Re}_s < 1000\right) \\ 0.44 & \left(\mathrm{Re}_s \ge 1000\right) \end{cases} \tag{7}$$

in which, $\mathrm{Re}_s = \rho_f \left|\mathbf{U}_f - \mathbf{U}_s\right| d_s / \mu_f$ is the particle Reynolds number and μ_f is the viscosity of the fluid.

Ignoring the fluid turbulence for dense granular problems, the shear stress of the fluid phase can be expressed as

$$\tau_{f,ij} = 2\alpha_f \mu_f S_{f,ij} \tag{8}$$

with $S_{f,ij} = 1/2\left(\partial U_{f,i}/\partial x_j + \partial U_{f,j}/\partial x_i\right) - 1/3\left(\partial U_{f,k}/\partial x_k\right)\delta_{ij}$ being the tensor of the deviatoric rate of fluid strain.

Flows of granular material generally cover two contrasting regimes: the rapid regime in which intense collisions occur among granular particles, and the quasi-static regime when enduring inter-particle contacts are predominant. To accurately describe the granular stresses in various regimes, a general collisional-frictional law is adopted, including a rate-dependent collisional part and a rate-independent frictional part

$$p_s = p_s^c + p_s^f, \qquad \tau_{s,ij} = \tau_{s,ij}^c + \tau_{s,ij}^f \tag{9}$$

where the superscripts c and f represent the collisional and frictional components of the granular stress, respectively. The collisional pressure can be formulated by the kinetic theory of Lun et al. [1],

$$p_s^c = \alpha_s \rho_s \Theta\left(1 + 4\eta\alpha_s R\right) \tag{10}$$

where, Θ is so-called granular temperature, representing the kinetic energy of the granular material due to velocity fluctuations; $R = (2-\alpha_s) \big/ \left[2(1-\alpha_s)^3\right]$ is the particle radial distribution function; $\eta = (1+e)/2$ with e is the restitution coefficient of particle collisions, defined as

$$e = e_d - 2.85\,\mathrm{St}^{-0.5} \tag{11}$$

which has proved effective for various types of granular material. e_d is the restitution coefficient of dry granular particles, which is often suggested to be 0.9 for glass beads. The Stokes number adopted here is a function of the granular temperature: $\mathrm{St} = \rho_s d_s \Theta^{0.5} \big/ \left(18\mu_f\right)$.

The governing equation for the granular temperature Θ, taking into account the effect of the fluid phase [2], can be written as

$$\frac{3}{2}\left[\frac{\partial\left(\alpha_s\rho_s\Theta\right)}{\partial t} + \frac{\partial\left(\alpha_s\rho_s U_{s,i}\Theta\right)}{\partial x_i}\right] = \frac{\partial}{\partial x_i}\left(\kappa_s \frac{\partial\Theta}{\partial x_i}\right) + \left(-p_s^c\delta_{ij} + \tau_{s,ij}^c\right)\frac{\partial U_{s,i}}{\partial x_j} - \alpha_s\rho_s J_s + \Gamma \tag{12}$$

where, the first term on the right side represents the diffusion of fluctuating energy with κ_s being a diffusion coefficient; the second term is the production of fluctuating energy due to shear in the granular material; J_s is the energy dissipation due to inelastic particle collisions; and Γ represents the production or dissipation due to interaction between the granular particles and the fluid. Based on the kinetic theory of Lun et al. [1] and Gidaspow [2],

$$\kappa_s = \frac{75\rho_s d_s\sqrt{\pi\Theta}}{48\eta(41-33\eta)R}\left[\left(1 + \frac{12}{5}\eta\alpha_s R\right)\left(1 + \frac{12}{5}\eta^2(4\eta-3)\alpha_s R\right) + \frac{64}{25\pi}(41-33\eta)\eta^2\alpha_s^2 R^2\right] \tag{13}$$

$$J_s = \frac{48}{\sqrt{\pi}}\eta(1-\eta)\frac{\alpha_s R}{d_s}\Theta^{3/2}, \qquad \Gamma = -3K\Theta + \frac{81\alpha_s \mu_f^2 \left|\mathbf{u}_f - \mathbf{u}_s\right|^2}{R d_s^3 \rho_s \sqrt{\pi\Theta}} \tag{14}$$

Following Lun et al. [1], the collisional shear stress for granular material can be written as

$$\tau^c_{s,ij} = 2\mu^c_s S_{s,ij} \tag{15}$$

with $S_{s,ij} = 1/2\left(\partial U_{s,i}/\partial x_j + \partial U_{s,j}/\partial x_i\right) - 1/3\left(\partial U_{s,k}/\partial x_k\right)\delta_{ij}$ being the tensor of the deviatoric rate of granular strain. The granular viscosity μ^c_s can be determined as

$$\mu^c_s = \rho_s d_s \sqrt{\Theta}\left[\frac{5\sqrt{\pi}}{96\eta(2-\eta)R}\left(1+\frac{8}{5}\eta\alpha_s R\right)\left(1+\frac{8}{5}\eta(3\eta-2)\alpha_s R\right) + \frac{8}{5\sqrt{\pi}}\eta\alpha_s^2 R\right] \tag{16}$$

Frictional stress develops when contacts between granular particles become long-lasting and form a granular skeleton. For cohesionless granular material, the frictional stress may be generally expressed as

$$\tau^f_{s,ij} = 2\mu^f_s S_{s,ij} \tag{17}$$

where μ^f_s is the viscosity due to inter-particle friction, formulated as

$$\mu^f_s = \frac{p^f_s}{\hat{S}}\left(\sqrt{\sin^2\phi + \left(\frac{\partial U_{s,k}}{\partial x_k}\frac{\cos^2\phi}{2\hat{S}}\right)^2} + \frac{\partial U_{s,k}}{\partial x_k}\frac{\cos^2\phi}{2\hat{S}}\right) \tag{18}$$

in which, p^f_s is the frictional normal stress; $\hat{S} = \sqrt{2S_{s,ij}S_{s,ij}}$; and ϕ is the internal friction angle of the granular material. p^f_s is evaluated using an empirical relation

$$p^f_s = \begin{cases} \eta\rho_s g d_s \dfrac{\left(\alpha_s - \alpha_{\min}\right)^{\gamma_1}}{\left(\alpha_{\max} - \alpha_s\right)^{\gamma_2}} & \left(\alpha_{\max} > \alpha_s > \alpha_{\min}\right) \\ 0 & \left(\alpha_s < \alpha_{\min}\right) \end{cases} \tag{19}$$

where $\alpha_{\max}$ is the close-packed volume fraction and $\alpha_{\min}$ is the loose-packed volume fraction. The frictional pressure vanishes when the volume fraction α_s is less than $\alpha_{\min}$. The values of $\alpha_{\max}$ and $\alpha_{\min}$ depend on the arrangement pattern and size distribution of the granular particles. η, γ_1 and γ_2 are empirical constants.

3 Validation and results

Saturated granular-water inclined flows over an inclined erodible and a rigid bed [3, 4], as ideal configurations of fully developed debris flows, are tested and performed in this work. The slope inclination, granular properties and other parameters used in our model are summarized in Table 1. To obtain a steady uniform flow condition observed in experiments, periodic boundaries are used for the left and right boundaries of the computation domain (see Figure 1). A non-slip boundary condition is applied on the bottom for the erodible bed case, due to the presence of a static layer beneath the flow layer. However, for the rigid bed case, intense collisions occur between the granular particles and the bottom wall. To describe the collisional mechanism near the wall, a collisional boundary condition is employed

$$U_{s,i}\tau_{s,ij}n_j + \frac{\psi\sqrt{3}\pi\rho_s\alpha_s R\Theta^{1/2}\left|\mathbf{U}_f - \mathbf{U}_s\right|}{6\alpha_{\max}} + p^f_s \tan\phi = 0 \tag{20}$$

$$-\kappa_s n_i \frac{\partial\Theta}{\partial x_i} = \frac{\sqrt{3}\pi}{4\alpha_{\max}}\left(1-e_w^2\right)\rho_s\alpha_s R\Theta^{3/2} + \frac{\psi\sqrt{3}\pi\rho_s\Theta^{1/2}\left|\mathbf{U}_f - \mathbf{U}_s\right|^2}{6\alpha_{\max}} \tag{21}$$

Here, ψ is the roughness of the wall; e_w is the restitution coefficient between granular particles and the wall; $\mathbf{n} = \left(n_1, n_2\right)$ is the unit normal vector of the wall.

Figure 2 shows the comparisons between the numerical results and the experimental data, in terms of the concentration, velocity and granular temperature distributions. The distribution profiles appear a significant difference between erodible and rigid bed cases. For saturated granular-water flows over an erodible bed, the granular concentration decreases monotonically over depth, with maximum values close to the packed bed. The velocity profile is convex, with maximum gradients close to the free surface. The granular temperature, i.e., the fluctuation energy of the granular phase, remains nearly zero close to the bed and increases linearly over depth, reaching its maximum at the free surface. In the case of the granular-water flows over a rigid bed, the distribution profiles become quite different. The granular concentration is minimum at the bed, reaches a maximum towards the centre and reduces again near the free surface. The velocity profile is slightly concave, with its steepest gradients near the bed. The granular temperature is maximum near the bed and decreases towards the free surface, presenting a contrary variation compared to the counterpart in the erodible bed case. The overall agreements between the numerical results and experiments demonstrate the capability of our model to simulate the gravity-driven granular-water mixture flows.

Table1: Parameters for numerical simulations

Parameter	Erodible bed	Rigid bed	Parameter	Erodible bed	Rigid bed
θ	8°	22°	$\alpha_{\max}$	0.7	0.7
h	62 mm	31 mm	$\alpha_{\min}$	0.5	0.5
ρ_s	2210 kg/m^3	1540 kg/m^3	η	4×10^{-4}	4×10^{-4}
ρ_f	1000 kg/m^3	1000 kg/m^3	γ_1	2	2
d_s	6 mm	3.7 mm	γ_2	5	5
e_d	0.9	0.9	μ_w	0.4	0.4
ϕ	20°	35°	ψ	--	0.9
α_0	0.56	0.545	e_w	--	0.7

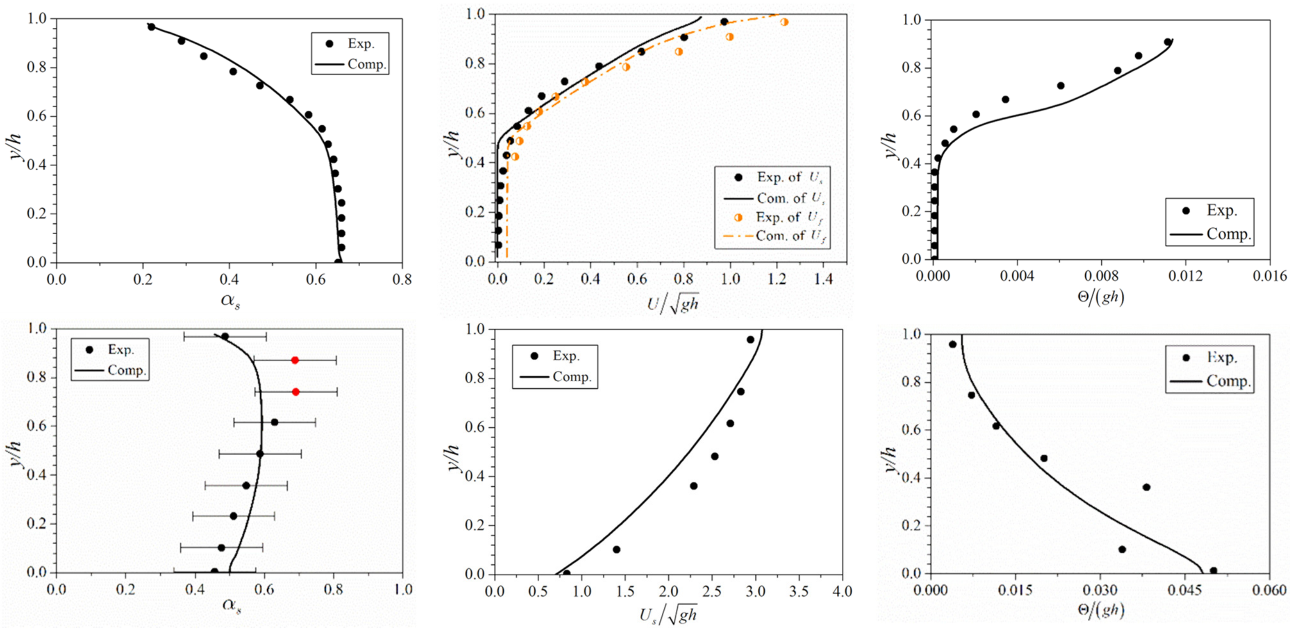

Figure 2: Depth profiles of concentration, velocity and granular temperature of the granular phase over erodible (top panel) and rigid bed (bottom panel).

4 Conclusions

An Eulerian-Eulerian two-phase model based on a general formulation of the granular stress is developed for granular-water mixture flows. In the model, the kinetic theory is extended to consider the influences of the ambient fluid and employed to compute the collisional stress in the granular phase. For the frictional stress, an improved formula derived by statistically averaging the individual contact forces among cohesionless particles is employed. The proposed two-phase model is successfully applied to the saturated granular-water inclined flows over both erodible and rigid bed. The various distribution profiles of granular concentration, velocity and granular temperature are well captured by the model.

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References

[1] C.K.K. Lun et al., "Kinetic theories for granular flow: inelastic particles in Couette flow and slightly inelastic particles in a general flowfield," J. Fluid Mech., vol. 140, 223–256, Mar. 1984.

[2] D. Gidaspow, (1994) Multiphase flow and fluidization: continuum and kinetic theory descriptions, Academic press, San Diego, 1994.

[3] A. Armanini et al., "Rheological stratification in experimental free-surface flows of granular-liquid mixtures," J. Fluid Mech., vol. 532, 269–319, Jun. 2005.

[4] A. Armanini et al., "Submerged granular channel flows driven by gravity," Adv. Water Resour., vol. 63, 1–10, Jan. 2014.