FINU TUNING OF THE γ - $R\tilde{}e_\theta$ TURBULENCEMODEL USING HISTORICAL DATA SETS

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The use of empirical turbulence models has been well documented in computational fluid dynamic simulations. The γ - $\tilde{Re}_{\theta_t}$ model, also known as the SST-transition model, proposed by Langtry and Menter, in particular has received much attention for being able to more closely replicate the pressure values on an airfoil surface as seen in experiments. The original empirical relations were based on observations by the authors, but the published relationships were developed to capture multiple geometries and experimental set-ups. This paper discusses an optimisation approach used to alter the empirical formula to match an existing data set, captured prior to the models' development. Simulations were carried out using open source CFD package openFOAM [1]. The new model coefficients are then compared to the standard formulation as well as the shear stress transport $k-\omega$ model ($k-\omega\ SST$). This work aims to show how the SST-transition model can be adapted for specific geometries using historical data sets.

Introduction

The modelling of transition from laminar to turbulent flow is a complicated task and is closely linked to aerodynamic stall. Reynolds Averaged Navier Stokes equations are not capable of fully capturing this phenomenon, while the more computationally expensive methods like Large Eddy Simulations come closer to accurate predictions. Intermittentcy models attempt to describe the transition process and are usually coupled to existing turbulence models.
Intermittentcy transport models are not new in the field of CFD, having being reported as early as the 1990s. In 2004 Vicedo et al proposed one such transport model that was applied in the modelling of separation bubbles [2]. At the time of publication the trend was to have mathematical models that were specific to certain geometries and flow parameters, thus limiting their applicability. Vicedo et al instead developed a model that required no normal-to-wall distance and related transition onset to the local momentum thickness Reynolds number. In 2005 Langrty and Menter authored a paper outlining a correlation based transition model which is based entirely on local variables[3], namely a transition momentum thickness Reynolds number ($\tilde{Re}_{\theta_t}$) and intermittentcy (γ). The model uses two transport equations for intermittentcy and transition onset criteria. Since the model is based entirely on local variables, it is compatible with unstructured meshes and well suited for parallelization. The model does not try to describe the physical process but instead is based on experimental observations and relations. Upon publication the Langtry and Menter did not release their original correlations but subsequent authors using their framework have presented their findings while using the SST-transition model.
Misaka and Obayashi applied their own correlations, based on flat plate boundary layer tests, in modelling flow around wings [4].
In 2007 Toyoda et al made use of the correlation model to predict boundary layer transition on the JAXA high-lift configuration model [5]. The authors applied their own empirical correlations to compare the lift and drag results as well as the skin friction for identifying the start of intermittentcy. Their results indicated that the model was not able to handle a large cross flow velocity. Sorensen attempted to determine the empirical relationships for the correlation model and verified the results with tests on two different aerofoils and a wind turbine rotor [6]. However the correlations supplied were also not in agreement with those previously disclosed by other authors.
In 2009 Langrty and Menter released their empirical relationships for the length of transition region (F_{length}), critical Reynolds number (Re_{θ_c}) indicating where intermittentcy first increases within the boundary layer and transition onset Reynolds (Re_{θ_t}), which is a function of pressure gradient and turbulence intensity [7]. Using previously published empirical relations, a reasonable assumption could be made on the expected relationship between all empirical relationships. Optimization was then undertaken to match the experimental values for lift and drag over a range of angles of attack, at a fixed Reynolds number. The NACA0012 airfoil was selected as it has an extensive collection of public data sets previously corroborated experimentally [8].

$\gamma - \tilde{Re}_\theta$ model

The $\gamma - \tilde{Re}_\theta$ model is coupled to the shear stress transport $k-\omega$ model. It brings in two more transport equations for intermittentcy (γ)and transition momentum thickness Reynolds number ($\tilde{Re}_{\theta_t}$). The equation for intermittentcy is

$$\frac{\partial(\rho\gamma)}{\partial t} + \frac{\partial \rho U_j \gamma}{\partial x_j} = P_\gamma - E_\gamma + \frac{\partial}{\partial x_j}\left[(\mu + \frac{\mu_t}{\sigma_f})\frac{\partial \gamma}{\partial x_j}\right] \tag{1}$$

where transition source is P_γ and the destruction source is E_γ. The transport equation for transition momentum thickness Reynolds number is

$$\frac{\partial(\rho \tilde{Re}_{\theta_t})}{\partial t} + \frac{\partial \rho U_j \tilde{Re}_{\theta_t}}{\partial x_j} = P_{\theta_t} + \frac{\partial}{\partial x_j}\left[\sigma_{\theta_t}(\mu + \mu_t)\frac{\partial \tilde{Re}_{\theta_t}}{\partial x_j}\right] \tag{2}$$

The source term P_{θ_t} is used to ensure the transport variable $\tilde{Re}_{\theta_t}$ matches the locally determined Re_{θ_t}. γ is used as a trigger for transition while $\tilde{Re}_\theta$ takes into account the non local effects of turbulence intensity. These non local effects include the decay of turbulence kinetic energy in the freestream and changes in velocity outside of the boundary layer. This equation is important as it brings together the empirical relationships which are used in the γ equation. Full details of the model are provided in [7], where the empirical relationships are given a piecewise definition. Re_{θ_t} is a function of turbulence intensity (Tu) and pressure gradient (λ_θ), while F_{length} and Re_{θ_c} are functions $\tilde{Re}_{\theta_t}$

Optimisation process

The aim of this work is to utilise the SST-transition model in CFD code such that the results for Lift and Drag match previously documented experimental results. These results were for various angles of attack and Reynolds numbers. It was decided to focus on a single Reynolds number and to tune the model to fit a single angle of attack. This work used the angle of attack with the minimum lift coefficient (C_L) found post stall for optimisation. The parameters available for optimisation are the empirical relationships. However their definition as per [7] would require the simultaneous optimisation of 37 coefficients. While the open source nature of openFOAM allows for direct coupling with outside code, coupling the CFD runs to an optimiser was not feasible due to the run time. As such a meta model was chosen to replace the CFD during optimisation. A breeder genetic algorithm (BGA) was used for optimisation. The objective function being minimized during optimisation was the difference between the experimental data sets' C_L and that reported by the simulation.

Approximating empirical relationships

An effective meta model requires a good training set. When fitting a quadratic polynomial to data, at least 1.5 times the number of coefficients are required in terms of training samples. As we increase the dimensions of the problem we expect an increased complexity. For 37 coefficients a conservative approach would be to require a minimum of 55 samples to fit a simple polynomial. The relationship being modelled, our empirical parameters versus C_L, are assumed to be more complex thus an effort was made to reduce the number of coefficients. The parametric curves described by Langrty and Menter were replaced with more complex mathematical functions with reduced coefficients. Looking at [6] and [7] it was determined that the transition length was approximated by a Gaussian curve:

$$F_{length} = \frac{A}{\sqrt{2\pi B^2}} \cdot e^{\frac{-\tilde{Re}_{\theta_t}^2}{2\sigma^2}} + C \tag{3}$$

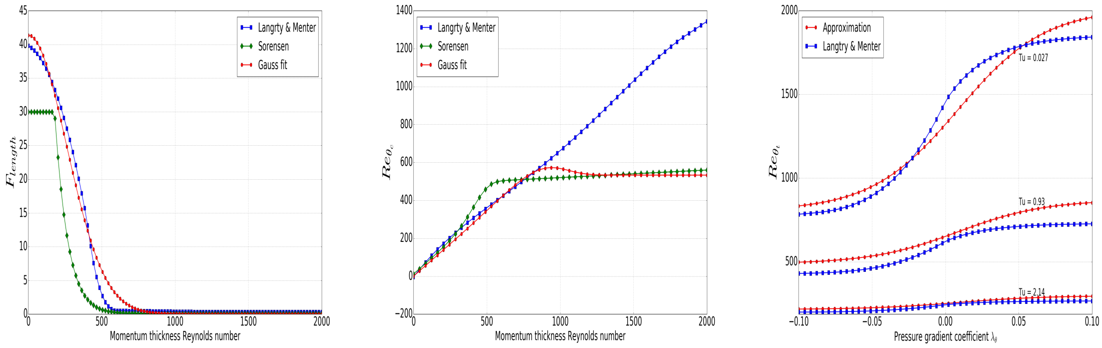
while the critical Reynolds number, should more closely follow the blended function as laid out by [6]:

$$\begin{aligned} \beta &= \tanh(\tilde{Re}_{\theta_t}/D)^E \\ Re_{\theta_c} &= \beta \cdot F + (1-\beta)\cdot(0.68 \cdot \tilde{Re}_{\theta_t}) \end{aligned} \tag{4}$$

In equation 4 the gradient of the straight line portion is equal to the gradient of a straight line fit to [7] definition. This allowed the capture of both types of relationships. Transition onset Reynolds number was represented as a hyperbolic tangent overlaid on an exponential function for λ_θ and an exponential function for Tu.

$$Re_{\theta_t} = G\exp(-H \cdot Tu)\tanh(I \cdot \lambda_\theta \cdot 900 - J) + K \tag{5}$$

The new mathematical functions are plotted in figure 1 along with the original formulations. These equations were tested on the flat plate example, used by Langtry and Menter in the model development, and showed a good correlation with expected results thus the number of variables was reduced from 37 to 16.



(a) Curve fit for F_{length} (b) Curve fit for Re_{θ_c} (c) Curve fit for Re_{θ_t}

Figure 1: The simplified curves used in the meta mode compared with the original formulation

Meta Model

Having reduced the number of variables, latin hyper cube sampling was used to generate testing and training data points. The relationships being modelled were not linear so various regression techniques were tested. Support vector regression (SVR), multi-layered perceptons (MLP) and random forest (RF) methods were all tested with an increasing number of sample points.The random forest approach showed the most promise, as the number of samples increased the in and out of sample error decreased and levelled off at the lowest error out of the 3 models. Hyper tuning the RF model achieved a coefficient of determination (R^2) value for in sample training of 0.92 and out of testing sets value of 0.45.

Results

Figure 2 shows C_L as reported by Sheldahl and Klimas [8] at a Reynolds number of 1e6 over a range of angles of attack. The figure also shows the results of various turbulence models for the same conditions. The standard $k - \omega$ SST model captures the trend with a noticeable delay. The original $\gamma - \tilde{Re}_\theta$ model is the worst performing with large over predictions in lift after separation has occurred. Finally the optimised $\gamma - \tilde{Re}_\theta$ is plotted showing an improvement. Table 1

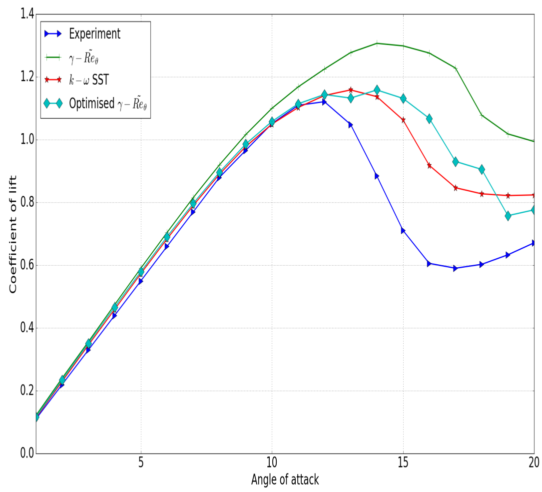


Figure 2: NACA 0012 lift coefficients at Reynolds number of 1e6 for various turbulence models

compares each model using the Pearson correlation coefficient (r). each turbulence model was evaluated for correlation to the data set. The optimisation started with a seed vector that had the coefficients approximating the original formulation, producing an objective function value of 0.58. The meta model reported the value as 0.57. Optimisation was complete after 75 generations with a final value of 0.37. The new coefficients were then run through CFD for all angles of attack and a final check of the difference between C_L showed a value of 0.34. This tells substantiated that the meta model was built correctly and provided accurate predictions.

Table 1: Pearson correlation coefficient, r, for reported lift coefficient versus experimental data set

Model	r
$\gamma - \tilde{Re}_\theta$	0.78
$k - \omega$ SST	0.92
Opt. $\gamma - \tilde{Re}_\theta$	0.88

Conclusion

This paper presented an approach to adjusting empirical relationships in a turbulence model. The adjustments were made in an attempt to correlate CFD reported lift coefficients with historical experimental data sets. The phenomena being modelled is unsteady but due to time constraints steady state simulations were used. The building of a surrogate model was important to reduce computational cost and the time needed for data collection. The reduction in number of variables for these relationships has proven effective and valid. The optimised model did not outperform the standard $k - \omega$ SST model in lift predictions, however further testing will be conducted to see if the location of transition point has been improved through this work. It is recommended to adjust the boundaries for the 16 variables and run the optimisation again.

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