AN IMMERSED BOUNDARY WALL FUNCTION FOR SMOOTH WALL SHEAR STRESS

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Keywords: *immersed boundary method, wall function, wall shear stress*

An immersed boundary (IB) wall function for turbulent flows using the $k-\epsilon$ model is developed in this work. The IB method is based on the existing method in foam-extend-3.0 and foam-extend-4.0 [1, 2]. In this method the whole computational mesh is classified into three categories, i.e., live cells, IB cells, and dead cells. No-slip boundary condition on the IB surface is imposed through manipulating the values in IB cells. The coefficient matrices of the resulted linear equation system are manipulated by eliminating the corresponding off-diagonal elements and modifying the relevant diagonal elements and source term. Thus, the values in IB cells are prescribed. The values in dead cells are fixed (usually as zero) and not included in the computation. In essence, the key of this IB method is imposing boundary conditions through IB cells.
However, we found that the current implementation of turbulence models and in particular the wall functions sometime cause issues in the simulation results. An important quantify for many fields is the wall shear stress. In man y cases, current implementation gives noisy wall shear stress distribution on the immersed boundary, which makes further calculation based on the wall shear not reliable. Examples of such further calculation include heat transfer, sediment transport, and surface wear.
To address this problem, we developed a new wall function with the use of the $k-\epsilon$ model. It is modified from the method introduced in previous work [1, 2]. It enforces the wall law by changing the values in IB cells. However, the original wall function implemented does not behave well when y^+ is in the buffer layer or the viscous sublayer range.
As shown in Fig. 1a, in order to evaluate the value at the IB cell center, an image point, which is further away from the immersed wall and into the fluid region, is defined to reconstruct the flow information from surrounding live cells. For the image point and the IB cell center, y^+ is calculated respectively according to

$$y^+_{\mathrm{IP}} = \frac{C_\mu^{1/4}\sqrt{k_{\mathrm{IP}}}y_{\mathrm{IP}}}{\nu}, \quad y^+_{\mathrm{IB}} = y^+_{\mathrm{IP}}\frac{y_{\mathrm{IB}}}{y_{\mathrm{IP}}} \tag{1}$$

where y_{IP} denotes the distance from image point to IB surface. y_{IB} denotes the distance from IB cell center to IB surface. In the original implementation, the ratio of $y_{\mathrm{IP}}/y_{\mathrm{IB}}$ is usually set as 2. Shear velocity (u_τ) for the image point, which is assumed to be equal to the shear velocity calculated at IB cell center since both are on the same velocity profile, is calculated as

$$u_\tau = \begin{cases} C_\mu^{1/4}\sqrt{k_{\mathrm{IP}}} & \text{if } y^+_{\mathrm{IP}} > y^+_{\mathrm{Laminar}} \\ \sqrt{\nu|u^{old}_{\mathrm{tan,IP}}|/y_{\mathrm{IP}}} & \text{if } y^+_{\mathrm{IP}} \leqslant y^+_{\mathrm{Laminar}} \end{cases} \tag{2}$$

where $u^{old}_{\mathrm{tan,IP}}$ denotes the interpolated tangential velocity at image point, $|\;|$ denotes its magnitude. $y^+_{\mathrm{Laminar}} = 11$. Thus, wall shear stress can be calculated as

$$\tau_w = u_\tau^2 \tag{3}$$

Then, based on the wall law, new tangential velocity at IB cell center can be evaluated as

$$u^{\mathrm{new}}_{\mathrm{tan,IB}} = \begin{cases} \dfrac{u_\tau \kappa}{\log\left(E y^+_{\mathrm{IB}}\right)} & \text{if } y^+_{\mathrm{IB}} > y^+_{\mathrm{Laminar}} \\ u_\tau y^+_{\mathrm{IB}} & \text{if } y^+_{\mathrm{IB}} \leqslant y^+_{\mathrm{Laminar}} \end{cases} \tag{4}$$

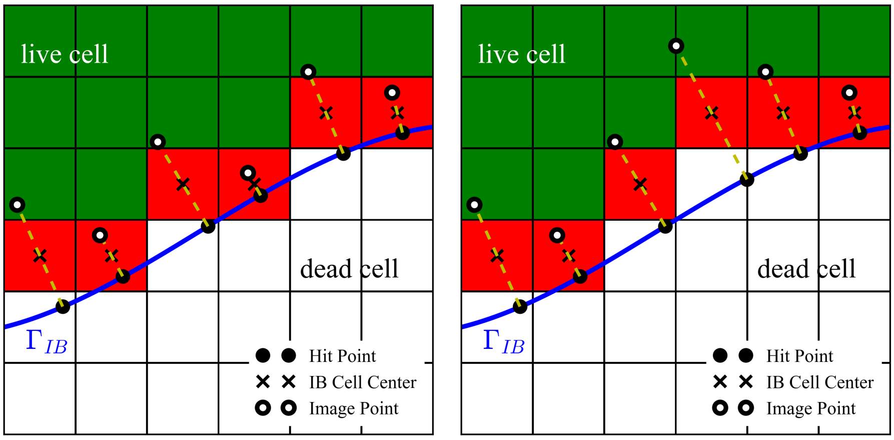
where E is the roughness coefficient, usually set as 9.8 for smooth wall. Afterwards, the eddy viscosity ν_t, k, and ϵ at IB cell center can be calculated respectively as follows

$$\nu_t = \begin{cases} \dfrac{y^+_{\mathrm{IB}}\kappa}{\log\left(E y^+_{\mathrm{IB}}\right)}\nu & \text{if } y^+_{\mathrm{IB}} > y^+_{\mathrm{Laminar}} \\ 0 & \text{if } y^+_{\mathrm{IB}} \leqslant y^+_{\mathrm{Laminar}} \end{cases} \tag{5}$$

$$k^{\mathrm{new}}_{\mathrm{IB}} = \begin{cases} (\nu_T + \nu)\dfrac{u^{\mathrm{old}}_{\mathrm{tan,IP}}}{y_{\mathrm{IP}}}C_\mu^{-0.5} & \text{if } y^+_{\mathrm{IB}} > y^+_{\mathrm{Laminar}} \\ k_{\mathrm{IP}} & \text{if } y^+_{\mathrm{IB}} \leqslant y^+_{\mathrm{Laminar}} \end{cases} \tag{6}$$

$$\epsilon_{\mathrm{IB}}^{\mathrm{new}} = \begin{cases} \dfrac{C_\mu^{0.75}(k_{\mathrm{IB}}^{\mathrm{new}})^{1.5}}{\kappa y_{\mathrm{IB}}} & \text{if } y_{\mathrm{IB}}^+ > y_{\mathrm{Laminar}}^+ \\ \epsilon_{\mathrm{IP}} & \text{if } y_{\mathrm{IB}}^+ \leqslant y_{\mathrm{Laminar}}^+ \end{cases} \tag{7}$$

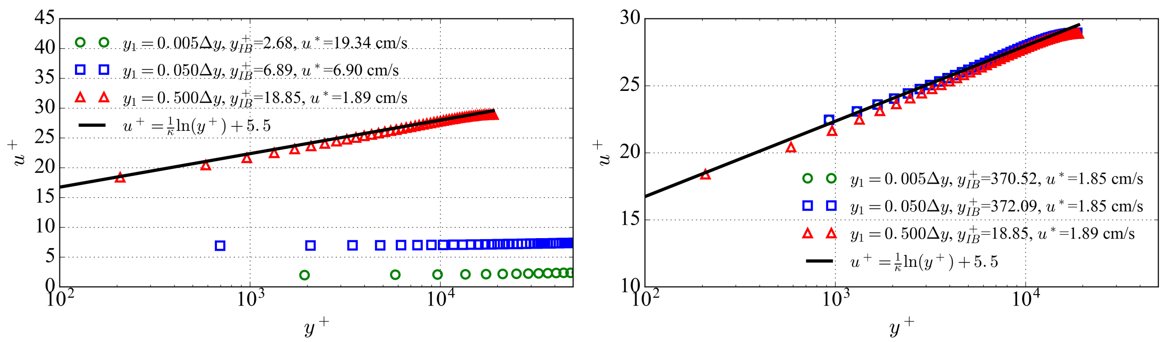
where in the laminar region, k_{IB} is treated as the same value as at image point.
As seen in Fig. 2a, when the IB wall distance (y_{IB}^+) falls in the laminar region, the original wall function can not reproduce the log-law. Unfortunately, in immersed boundary method, it is hard to control the distance between the IB cells and the immersed surface. This distance can even change when the immersed boundary is moving. In order to overcome this, a new algorithm is developed. The key idea is to adaptively increase y^+ by changing the IB cells (see Fig. 1). For any IB cell, if y_{IB}^+ falls in the laminar region, the IB cell will be replaced by an adjacent live cell slightly away from the IB surface to ensure the new IB cell is located in the log-law layer. This replacement of IB cell will not have significant impact on the accuracy since the mesh around the immersed boundary is usually refined. After all, the immersed boundary method is only an approximation of the wall effect to the flow.



(a) before y^+ adaptation **(b) after y^+ adaptation**

Figure 1: 2D schematic of y^+ adaptation process. Red-filled cells are IB cells. Green-filled cells are live cells. White-filled cells are dead cells.

As shown in Fig. 2 (b), the result of 1D channel flow validation case shows the new y^+ adaptation algorithm captures the wall law regardless the IB wall distances. One can also observe that the new algorithm makes sure y_{IB}^+ is always in the log-law region (> 11).
To further demonstrate the new algorithm, Fig. 3 shows the wall shear stress distribution over a dune-like bathymetry



(a) before y^+ adaptation (original method) **(b) after y^+ adaptation**

Figure 2: Log-law results for 1D validation case with different y_1.

using immersed method. In this case, the background 3D mesh is structured and the immersed interface is dune-like. It can be observed that without y^+ adaptation, the wall shear stress is not smooth as it should be. In the back of the dune, there are some "hot" spots of high wall shear caused by the small IB wall distance. By applying the new algorithm, the wall shear stress distribution is much smoother and more reasonable.

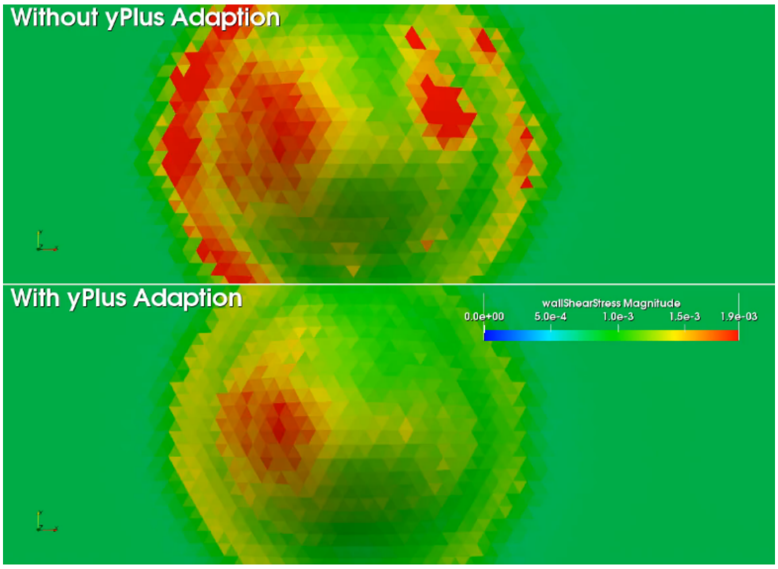


Figure 3: Wall shear stress distribution over a dune-like bathymetry. Flow is from left to right.

Acknowledgments

The authors wish to thank Dr. Hrvoje Jasak for his discussions with us on the topic of this paper.

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