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Numerical Investigation of Motion Response of Two Model Ships in Regular Waves
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Abstract
In this paper, the sea keeping performances of two model ships in regular waves are studied by our in-house solver naoe-FOAM-SJTU based on OpenFOAM code package. Volume of Fluid (VOF) method is used to capture the free interface and Finite Volume Method (FVM) is adopted as the discretization scheme. Different wave conditions are set by the wave generation and damping module in the solver. The heave and pitch are simulated, and green water is found during the ship motion. The function of bulbous bow for that is discussed.

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Keywords: viscous-flow; ship motion response; regular waves; naoe-FOAM-SJTU solver

1. Introduction
The seakeeping feature is one important indicator of a ship’s hydrodynamic performance. When a ship is sailing in stormy waves, the ship motions with six DOFs bring about a series of problems, such as the added resistance, reduction of propelling efficiency, structure strength, stability and comfort quality. As a result, it has always been

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of great concern of the ship designers and users. In this paper, the accurate prediction of ship motions to study the seakeeping performance and compare the influence of different hull forms is chosen as the main task.

Experimental method [1] is considered as the most precise way to study the seakeeping issue. However, the requirement on the devices and site and the high cost limit its application. In recent decades, a lot of studies are based on potential flow theory, such as the Rankine source method [2] and ship strip theory [3]. Computational Fluid Dynamics (CFD) method is also applied to solve related hydrodynamic problems frequently. The traditional methods using potential flow theory have advantages such as simplicity and high computational efficiency, while CFD method is able to deal with the strong nonlinear factors caused by large amplitude motions, breaking free surface and waves, with full consideration of flow velocity. Moreover, the application of CFD has benefited from advancing computer science and numerical methods. Especially the RANS (Reynolds-Averaged Navier-Stokes) simulation [4] has widely been used to compute the ship resistance and motion responses under various sea conditions.

Yang et al (2006) [5] used finite element method and unstructured-grid to simulate a LNG ship with liquid tanks sailing in head and oblique waves. The coupled motion between the hull and tank with the inner and outer flow filed is considered about. Chengsheng Wu et al (2008) [6] built a numerical wave tank based on N-S method. The study on the ship motion of a Wigley sailing in different wave conditions was conducted. Overset was also applied to the computation of ship motion and added resistance of a catamaran in head waves by Castiglione et al (2011) [7]. Hamid et al (2013) [8] carried on the CFD verification and validation by simulating the added resistance and ship motions of KVLCC2 sailing in short and long head waves using CFDShip-Iowa.

In this paper, RANS simulation was adopted to predict the ship motion in different wave conditions to study the seakeeping property of two special guard ships. Moreover, the motion responses of typical points on ship were simulated. Compared with using commercial software directly, the independent development based on open source code package is more flexible and extensible for multi-aspect issues. Based on OpenFOAM, the solver naoe-FOAM-SJTU [9] was used as the computational tool as its applicability for various kinds of complex hydrodynamic issues. On the foundation of original solver codes, a module for solving six DOFs motion equations was added, along with deforming mesh technology and numerical wave generating & damping modules. Previous work had made many progresses about developing and validating this solver applied in this paper. In 2011, Zhirong Shen et.al [10] simulated the viscous flow acting on KVLCC2, KCS, and DTMB5415 and compared with the measurement results by solver naoe-FOAM-SJTU. Haixuan Ye et.al (2012) [11] computed the added resistance and vertical ship motions in regular head waves, and validated the ability of solver naoe-FOAM-SJTU to solve the strong nonlinear problems.

2. Mathematic Model and Numerical Methods

2.1. Introduction to naoe-FOAM-SJTU solver

The incompressible RANS equations are applied in this paper as governing equations. The equation of continuity and momentum equation can be written as Eqs. (1) and (2):

\[ \nabla \cdot U = 0 \quad (1) \]

\[ \frac{\partial \rho U}{\partial t} + \nabla \cdot (\rho (U - U_g) U) = -\nabla p_d + g \cdot \nabla \rho + \nabla \cdot (\mu_{eff} \nabla U) + (\nabla U) \cdot \nabla \mu_{eff} + f_\sigma + f_s \quad (2) \]

\[ \mu_{eff} = \rho (\nu + \nu_t) \quad (3) \]

Here, \( U \) stands for the velocity field while \( U_g \) means velocity of mesh nodes, \( p_d \) is dynamic pressure, \( \rho \) is mixture density of the two phases, \( g \) is the acceleration vector of gravity. In Eq. (3), \( \mu_{eff} \) is the effective dynamic viscosity coefficient, where \( \nu \) is kinematic viscosity coefficient and \( \nu_t \) is eddy viscosity coefficient. \( f_\sigma \) is surface tension term, which impacts the free surface. To protect the flow field from the interface of echo wave, the source term \( f_s \) is added for sponge layer to absorb the generated wave [12].

The SST k-\omega model [13] is applied as the turbulence model in this paper. This turbulence model
synthesizes the advantages of standard $k-\omega$ model and $k-\varepsilon$ model. The former one is appropriate to simulate the turbulent flow far way the wall plane, while the other one is good at solve the boundary flow near the wall under different pressure gradients. The application of SST $k-\omega$ model would improve the accuracy of numerical computation results.

VOF (Volume of Fluid) method with artificial bounded compression technique is adopted for simulating free surface in this paper. The VOF transport equation is formulated as below:

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot [(U - U_y)\alpha] + \nabla \cdot [U_r (1 - \alpha)\alpha] = 0$$

in which, $\alpha$ represents the volume of fraction, the relative proportion of the two phase fluid. The value of $\alpha$ is:

$$\begin{cases}
\alpha = 0 & \text{air} \\
\alpha = 1 & \text{water} \\
0 < \alpha < 1 & \text{interface}
\end{cases}$$

Moreover, the first two terms on the left hand side of Eq. (4) stand for traditional volume of fluid transport equation and the other term represents the artificial compression term. The term $(1-\alpha)\alpha$ makes the compression term take effect only on the interface without affecting the numerical computation out of the transition layer. $U_r$ is the velocity field for compressing the interface. The velocity field can be computed at cell faces by the maximum velocity magnitude at the interface region, defined as:

$$U_{r,f} = n_f \min \left\{ C_\alpha \frac{\Phi}{S_f}, \max \left( \frac{\Phi}{S_f} \right) \right\}$$

where $\Phi$ is face volume flux, including the flux of grid velocities from the PISO algorithm; $C_\alpha$ is a compression coefficient indicating the degree of compression. It is set 1.0 in this paper based on previous experience. The smaller $C_\alpha$ is, the less enhanced the compression of the interface is. $C_\alpha = 0$ means no effect on interface while $C_\alpha = 1$ leads to a conservative compression.

Finite Volume Method (FVM) is used to discretize both the RANS equations and VOF transport equation in naoe-FOAM-SJTU. Moreover, different interpolation schemes are applied. In detail, a second-order TVD limited linear scheme is used to discretize the convection terms in Eq. (2); the diffusion terms is discretized by a second-order central difference scheme; Van Leer scheme is applied for VOF equation discretization; A second-order backward method is applied for temporal discretization. For the discretized RANS equations with coupled pressure and velocity, PISO (pressure-implicit-split-operator) algorithm is adopted to solve the coupled equation of velocity and pressure.

A complete 6DOF motion solver was developed for numerical computations to predict sinkage and trim of ship. The details of the solving processes can be obtained by related thesis. In naoe-FOAM-SJTU solver, two coordinate systems are adopted to solve 6DOF equations including earth-fixed and ship-fixed coordinate system. The linear and angular velocity in two systems can be transformed by matrices. The shear stress and total pressure can be calculated by the velocity field, and then the forces and moments in ship-fixed coordinate can be obtained. By rigid-body equations the linear and angular accelerations are educed. The velocity and motions can be calculated by integrating method. The dynamic deformation mesh is applied. The mesh topology would remain unchanged with the moving hull. By stretching and squeezing the cell shapes and changing the spacing between nodes, the hull motions can be simulated well. The location of grid nodes in the flow field can be solved according to the Laplace equation:

$$\nabla \cdot (\gamma \nabla x_g) = 0$$

in which, $x_g$ is the node’s displacement and $\gamma$ means the diffusivity coefficient. In this paper, the value of $\gamma$ can be determined as:

$$\gamma = \frac{1}{r^2}$$

thereinto, $r$ stands for the distance between cell center and hull surface. The deformation of the grids near the hull surface would be small enough to ensure the high quality of the mesh around the hull.
In addition, the solver also includes both wave generation and damping modules [14]. They have been used in past studies maturely. Different conditions of linear regular wave can be simulated in this numerical tank. In this paper, the first order stoke wave in deep water is adopted and the sponge layer is used as the damping area after the hull stern in the flow field.

2.2. Fourier series analysis and transfer function

According to the periodicity of the force and motion responses, Fourier series analysis has been widely applied to the CFD analysis of seakeeping performance. The Fourier series of any time-dependent variable \( \phi(t) \) can be written as:

\[
\phi(t) = \frac{\phi_0}{2} + \sum_{n=1}^{N} \phi_n \cos(n\omega t + \gamma_n), n = 1, 2, \ldots
\]

(9)

\[
\phi_n = \sqrt{a_n^2 + b_n^2}, \gamma_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)
\]

(10)

\[
a_n = \frac{2}{T} \int_0^T \phi(t) \cos(w_nt)dt, b_n = -\frac{2}{T} \int_0^T \phi(t) \sin(w_nt)dt
\]

(11)

The subscript \( n \) means the order of different responses. In Eq. (9) and (10), the parameter \( \phi_n \) stands for the motion amplitude while \( \gamma_n \) represents for the phase. The amplitude with \( n=1 \) indicates the linear component of the variable, while the terms with \( n \geq 1 \) reflect the nonlinear extent. They can be computed in accordance with Eq. (11). Generally, the nonlinear terms are considered as \( n = 2 \) because the higher order terms are small quantities which can be neglected.

In addition, the method of Transfer Function is applied to analyze the frequency response of each motion. The transfer functions of heave and pitch is defined as:

\[
TF_3 = \frac{x_3}{a}, TF_5 = \frac{x_5}{ak}
\]

(12)

The subscript 3 means the third DOF of hull motion namely heave, while 5 means the fifth DOF of hull motion namely pitch. The numerator stands for the first order amplitude of the Fourier series of each motion response; \( a \) means the amplitude of incident wave and \( k \) means the wave number.

2.3. The simulation of vorticity field

In this paper, the tensor invariant \( Q \) is used to indicate the vorticity field [15]. The velocity field of three dimensions can be indicated as:

\[
u_i = A_{ij}x_j
\]

(13)

where the \( A_{ij} \) is the velocity gradient tensor. \( Q \) can be written as:

\[
Q = \frac{1}{2}(P_{ij} - S_{ij}S_{ji} - W_{ij}W_{ji})
\]

(14)

in which, \( S_{ij} \) is the rate of strain tensor and \( W_{ij} \) is the rate of rotation tensor. These parameters are defined as:

\[
P = A_{ii}
\]

\[
S_{ij} = \frac{A_{ij} + A_{ji}}{2}, W_{ij} = \frac{A_{ij} - A_{ji}}{2}
\]

(15)

The vorticity field of three dimensions and its change can be clearly shown by the distribution of \( Q \) according to the velocity field.
3. Mathematic Model and Numerical Methods

3.1. Hull model

The main dimensions and particulars of the two patrol ships are shown in Table 1. The related data of full scale ships are set in the columns of Hull 1 and Hull 2. The model scale of the two patrol ships are chosen for the numerical computation in this paper. Compared with the full scale, the main dimensions and particulars of model scale are set in the columns of Model 1 and Model 2. The 3D models are displayed and compared in Fig. 1., including the details of both bow and stern. In particular, the size and location of the bulbous bow of model 2 are shown clearly.

<table>
<thead>
<tr>
<th>Main Features</th>
<th>Symbol</th>
<th>Hull 1</th>
<th>Hull 2</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale ratio</td>
<td>$\lambda$</td>
<td>30.270</td>
<td>30.270</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Length between perpendiculars</td>
<td>$L_{pp}$ (m)</td>
<td>90.800</td>
<td>90.800</td>
<td>3.000</td>
<td>3.000</td>
</tr>
<tr>
<td>Length of waterline</td>
<td>$L_{wl}$ (m)</td>
<td>94.000</td>
<td>94.000</td>
<td>3.106</td>
<td>3.106</td>
</tr>
<tr>
<td>Breadth</td>
<td>$B$ (m)</td>
<td>12.200</td>
<td>12.200</td>
<td>0.403</td>
<td>0.403</td>
</tr>
<tr>
<td>Draught</td>
<td>$d$ (m)</td>
<td>3.400</td>
<td>3.400</td>
<td>0.112</td>
<td>0.112</td>
</tr>
<tr>
<td>Block Coefficient</td>
<td>$C_B$</td>
<td>0.486</td>
<td>0.504</td>
<td>0.486</td>
<td>0.504</td>
</tr>
<tr>
<td>Wetted surface</td>
<td>$S$ (m$^2$)</td>
<td>1079.130</td>
<td>1119.400</td>
<td>1.178</td>
<td>1.222</td>
</tr>
<tr>
<td>Longitudinal center of gravity</td>
<td>$L_{CG}$ (m)</td>
<td>48.620</td>
<td>48.620</td>
<td>1.606</td>
<td>1.606</td>
</tr>
<tr>
<td>Vertical center of gravity</td>
<td>$K_{G}$ (m)</td>
<td>1.810</td>
<td>1.810</td>
<td>0.060</td>
<td>0.060</td>
</tr>
<tr>
<td>Longitudinal inertia radius</td>
<td>$K_{yy}/L_{ww}$</td>
<td>0.230</td>
<td>0.230</td>
<td>0.230</td>
<td>0.230</td>
</tr>
<tr>
<td>Transverse inertia radius</td>
<td>$K_{xx}/B$</td>
<td>0.380</td>
<td>0.380</td>
<td>0.380</td>
<td>0.380</td>
</tr>
<tr>
<td>Vertical inertia radius</td>
<td>$K_{zz}/L_{ww}$</td>
<td>0.230</td>
<td>0.230</td>
<td>0.230</td>
<td>0.230</td>
</tr>
<tr>
<td>Vertical inertia moment</td>
<td>$K_{yy}$ (kg·m$^2$)</td>
<td>$7.98 \times 10^8$</td>
<td>$7.98 \times 10^8$</td>
<td>31.359</td>
<td>31.359</td>
</tr>
<tr>
<td>Longitudinal inertia moment</td>
<td>$K_{xx}$ (kg·m$^2$)</td>
<td>$7.98 \times 10^8$</td>
<td>$7.98 \times 10^8$</td>
<td>31.359</td>
<td>31.359</td>
</tr>
<tr>
<td>Transverse inertia moment</td>
<td>$I_{xy}$ (kg·m$^2$)</td>
<td>$3.93 \times 10^7$</td>
<td>$3.93 \times 10^7$</td>
<td>1.545</td>
<td>1.545</td>
</tr>
</tbody>
</table>

According to the 6-DOF motion simulation module of naoe-FOAM-SJTU, two coordinate systems are built for solving the motion equations, as what mentioned before. In Table 3 1, the longitudinal or vertical center of gravity is calculated in the earth-fixed system, while all of the inertia radius and moment are calculated in the ship-fixed coordinate system.

By comparing the two ships, it can be found that the main dimensions of them are identical except the block coefficient and the wetted surface area. The main differences of them are bow shape and arrangement. Model 1 adopts the raked bow rather than the bulbous bow which is applied to model 2. Moreover, the structure of bulwark is only equipped on model 1. These characteristic differences lead to the unlike seakeeping performance of each hull.

3.2. Case conditions

Two conditions are considered in this paper, including the different wave states and sailing velocity conditions. As shown in Table 2, $V$ stands for the sailing velocity of the hull in model scale, which corresponds to 22kn and 18kn of the hull in full scale. $h$ means the wave height; $A$
means the wave length; \(ak\) is the wave steepness, in which \(a\) represents for the wave amplitude and \(k\) stands for the wave number. The column of Motion shows the unfixed DOFs of the hull motion. Under the conditions of sailing in head waves, pitch and heave should be considered, because the trim and sinkage of the hull have a significant influence on the resistance and seakeeping performance. \(T_e\) means the period of encounter in the ship-fixed coordinate system, calculated as,

\[
T_e = \frac{2\pi}{w_e} = \frac{1}{\left(\frac{U_0}{\lambda} + \sqrt{\frac{g}{2\pi\lambda}}\right)} \tag{16}
\]

For all of the cases in this paper, the regular cosine wave is set as the flow condition. At the beginning of calculation, the hull is under upright condition and the X-Y plane is set as the water plane. Due to the working condition of these ships, the liquid density is set 1024 kg/m\(^3\) as the sea-water density of 20\(^\circ\)C.

<table>
<thead>
<tr>
<th>case</th>
<th>model</th>
<th>(V) (m/s)</th>
<th>(Fr)</th>
<th>(Re)</th>
<th>(h) (m)</th>
<th>(ak)</th>
<th>motion</th>
<th>(T_e) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>model 1</td>
<td>1.683</td>
<td>0.31</td>
<td>4.79(\times)10(^6)</td>
<td>0.1652</td>
<td>2.21</td>
<td>0.235</td>
<td>pitch, heave</td>
</tr>
<tr>
<td>2</td>
<td>model 1</td>
<td>2.057</td>
<td>0.38</td>
<td>5.86(\times)10(^6)</td>
<td>0.1320</td>
<td>4.50</td>
<td>0.092</td>
<td>pitch, heave</td>
</tr>
<tr>
<td>3</td>
<td>model 2</td>
<td>2.057</td>
<td>0.38</td>
<td>5.86(\times)10(^6)</td>
<td>0.1320</td>
<td>4.50</td>
<td>0.092</td>
<td>pitch, heave</td>
</tr>
<tr>
<td>4</td>
<td>model 2</td>
<td>1.683</td>
<td>0.31</td>
<td>4.79(\times)10(^6)</td>
<td>0.1652</td>
<td>2.21</td>
<td>0.235</td>
<td>pitch, heave</td>
</tr>
</tbody>
</table>

### 4. Computational Domain and Mesh Generations

Due to the symmetrical structure of hull by X-Z plane, a half of the flow field with a half hull can be used as the computational domain. The motion response of the half hull is the same as the actual integrated ship. As a result, it helps reduce the amount of computations and improve the efficiency.

*SnappyHexMesh*, the mesh generation utility of OpenFOAM is used to create the mesh in this paper. Firstly, a structural background mesh with hexahedral grid is generated. Then the homogeneous grid is divided into the octree structure by *SnappyHexMesh*. The boundary mesh is added near the hull surface in order to simulate the complex flow in the boundary layer of the hull. In this paper the meshes of all four cases are generated by the same size and setting. In the earth-fixed coordinate system, the computational domain is determined as \(-1.0L_{pp} < x < 3.0L_{pp}\), \(0 < y < 1.0L_{pp}\), \(-1.0L_{pp} < z < 1.0L_{pp}\). The mesh details of model 1 and model 2 are displayed in Figure Error! No text of specified style in document.-1, along with the overall view of the domain.
As shown in Figure 4-1(a), the interface grids are refined to better simulate the free surface and wave-making situation. Furthermore, the refinement regions also cover the boundary layer of the hull, the region near the bow and stern structure, and the region consists of a big variation of curvature and skewness. It is necessary to refine the important region so that a higher accuracy of the numerical computation can be achieved. The number of cell of the computational mesh is about 1.23×10^6.

5. Results Analysis

5.1. The pitch and heave of two hulls

After amounts of time steps, it can be found that the motion responses have cyclical fluctuations. In the earth-fixed coordinate system, the pitch and heave variation with time are shown in Figure 5.1. The horizontal axis is the dimensionless time duration. The vertical axis is the motion amplitude. The positive result of pitch stands for trim by the stern while the positive result of heave represents for the motion forward the positive direction of Z-axis.

From Figure 5.1, for all of the cases, the duration curves of pitch are regular sinusoid or cosinusoid, showing the obvious linear characteristic of these responses. The non-linear component cannot be observed. The response amplitude of small wave steepness equaling to 0.092 is larger than that of big wave steepness equaling to 0.235. It is inferred that the influence of velocity is beyond that of wave intensity of the conditions in this paper. The increase of ship velocity of Froude number from 0.31 to 0.38 makes the hull pitch change much more heavily. In addition, the regularity of heave is just the same as pitch of the different conditions.
For quantity analysis, the Fourier series expansion method is adopted to process the data of pitch and heave. The zero order amplitude of pitch or heave stands for the value of time-average. The first order amplitude means the linear intensity, while the second order is the greatest nonlinear intensity which can be used to represent the nonlinear intensity. Due to the slight influence of the responses above second order, these high order terms can be ignored. The zero, first and second order of amplitude and phase are concerned in this paper, which are shown in Table 3.

<table>
<thead>
<tr>
<th>Hull No.</th>
<th>Motion</th>
<th>( ak )</th>
<th>0(^{th}) amp. (m)</th>
<th>1(^{st}) amp. (m)</th>
<th>phase (deg)</th>
<th>2(^{nd}) amp. (m)</th>
<th>phase (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>hull 1</td>
<td>heave</td>
<td>0.092</td>
<td>-0.0053</td>
<td>0.0552</td>
<td>-127.8</td>
<td>0.0010</td>
<td>-9.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.235</td>
<td>-0.0073</td>
<td>0.0130</td>
<td>72.0</td>
<td>0.0011</td>
<td>170.7</td>
</tr>
<tr>
<td></td>
<td>pitch</td>
<td>0.092</td>
<td>0.67</td>
<td>4.96</td>
<td>-71.5</td>
<td>0.16</td>
<td>37.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.235</td>
<td>-0.57</td>
<td>1.20</td>
<td>98.5</td>
<td>0.10</td>
<td>-164.2</td>
</tr>
<tr>
<td>hull 2</td>
<td>heave</td>
<td>0.092</td>
<td>-0.0045</td>
<td>0.0550</td>
<td>-126.1</td>
<td>0.0011</td>
<td>22.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.235</td>
<td>-0.0037</td>
<td>0.0149</td>
<td>73.6</td>
<td>0.0012</td>
<td>163.4</td>
</tr>
<tr>
<td></td>
<td>pitch</td>
<td>0.092</td>
<td>0.75</td>
<td>5.05</td>
<td>-71.0</td>
<td>0.15</td>
<td>49.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.235</td>
<td>-0.35</td>
<td>1.26</td>
<td>97.1</td>
<td>0.10</td>
<td>-167.1</td>
</tr>
</tbody>
</table>

As shown in Table 3, the 0\(^{th}\) and 2\(^{nd}\) harmonic amplitudes of both heave and pitch motions are similar of these conditions. Their values are all smaller than the 1\(^{st}\) harmonic components. It indicates that the linearity plays the main role in the motion response. For both hulls, the 1\(^{st}\) harmonic amplitudes of heave and pitch are reduced under the combined action of increasing wave steepness and decreasing of velocity. Also the change of phase of the 1\(^{st}\) harmonic components is more or less as \( \pi \). The velocity augment help improve the nonlinearity of the motion response of both hulls. Furthermore, comparing hull 1 with hull 2, the time average value of the pitch of hull 2 is larger than that of hull 1 when \( ak=0.092 \). However, when \( ak=0.235 \), the time average value of the pitch of hull 2 becomes smaller than that of hull 1. It means that the bulbous bow has a damping function on the pitch motion only under special conditions.

To better analyze the motion responses, the method of Transfer Function is used in this paper. \( TF_3 \) stands for the transfer function of heave and \( TF_5 \) represents for the transfer function of pitch. According to Eq. (12), the calculation result is listed in Table 4.

<table>
<thead>
<tr>
<th>Hull No.</th>
<th>( \lambda/L )</th>
<th>( TF_3 )</th>
<th>( TF_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>hull 1</td>
<td>0.74</td>
<td>0.157</td>
<td>5.123</td>
</tr>
<tr>
<td></td>
<td>1.50</td>
<td>0.836</td>
<td>53.913</td>
</tr>
<tr>
<td>hull 2</td>
<td>0.74</td>
<td>0.180</td>
<td>5.348</td>
</tr>
<tr>
<td></td>
<td>1.50</td>
<td>0.833</td>
<td>54.867</td>
</tr>
</tbody>
</table>

In Table 4, for the wave condition of \( \lambda/L \) equaling to 1.5, the transfer functions of both heave and pitch are larger than that of \( \lambda/L \) equaling to 0.74. It can be inferred that when \( \lambda/L =1.5 \), the encounter frequency is closer to the natural frequency of both hulls. As a result, it brings about a more intensive hull motion. The difference of the bow form has less impact on the natural frequency and the body motion.
5.2. The motion response of typical locations

In this paper, three typical locations are studied to discuss the seakeeping of hulls in head wave conditions, including the center of gravity, the control room and the helicopter lifting platform. The coordinates of corresponding point of the typical locations are provided in Table 5. For more convenient comparison, the dimensionless terms including time $t'$, relative displacement $\delta'$, velocity $U'$ and acceleration $a'$ are gained by the Eq. (17).

$$
\begin{align*}
    t' &= \frac{t}{T_e}, \\
    \delta' &= \frac{\delta}{h}, \\
    U' &= \frac{U}{h \cdot \omega_e}, \\
    a' &= \frac{a}{h \cdot \omega_e^2}
\end{align*}
$$

in which, $T_e$ is the period of encounter; $h$ stands for the wave height; $\omega_e$ is the frequency of encounter. Due to the similar regular patterns, only one case is displayed here. For instance, the motion response case 1 is shown in Figure 3.

<table>
<thead>
<tr>
<th>Typical Point</th>
<th>Coordinate (in full scale)</th>
<th>Coordinate (in model scale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center of gravity</td>
<td>(0, 0, 0)</td>
<td>(0, 0, 0)</td>
</tr>
<tr>
<td>Control room</td>
<td>(-17.82, 0, 13.9)</td>
<td>(-0.5887, 0, 0.4593)</td>
</tr>
<tr>
<td>Helicopter lifting platform</td>
<td>(35.5, 0, 3)</td>
<td>(1.1729, 0, 0.0991)</td>
</tr>
</tbody>
</table>

Figure 3 The motion response of typical points of case 1
Four complete periods of the motion response have been shown in Figure Error! No text of specified style in document.-3. Due to the only two unfixed motion degrees of freedom of pitch and heave, the center of gravity only have the motion response in Z direction. While the points of control room and helicopter lifting platform have two motion responses in X direction and Z direction respectively. The control room is at the fore part of this hull and on the contrary, the helicopter lifting platform is at the aft part of this hull. With the action of wave motion, the relative displacement, velocity and acceleration in each direction have changed regularly and periodically.

For all of the cases, if the relative displacement in a certain direction is equal to zero, it means that this point has returned to its initial location in this direction. The positive value of each motion response stands for the motion vector is along the positive direction of that axis. For the center of gravity, the change of \( \delta \) is the same as the heave motion response of whole hull. The amplitude of \( \delta' \), velocity \( U' \) and acceleration \( a' \) can reflect the intensity of wave condition. As a result, the response amplitude of case 2 and case 3 is smaller than case 1 and case 4. In addition, the acceleration response of helicopter lifting platform in X direction is not a regular sine or cosine curve, which illustrates the intensive nonlinear motion of the motion.

5.3. Free surface

The simulation of free surface is an ignorable aspect to study the seakeeping performance of the two ships in this paper. Some strong nonlinear phenomena can be observed.

In order to show the different wave patterns at different moments, the graphics of the free surface at four typical moments in one period are displayed. The moments including \( t/T=0, 0.25, 0.50, 0.75 \). Take case 1 for example as shown in Figure Error! No text of specified style in document.-4.

![Figure Error! No text of specified style in document.-4 The free surface of hull 1 in one period (fn=0.31)](image)

The free surface of case 1 is well shown in Figure Error! No text of specified style in document.-4. With the function of the regular wave, the heave motion of the hull can be obviously observed by the freeboard height at bow. Meanwhile the change of trim angle reflects the pitch motion of the hull. Some intensive nonlinear phenomenon can be simulated when the wave passes the hull surface. For instance, in Figure Error! No text of specified style in document.-4(b), the wave breaking of the heavy wave crest at bow was found. The hull
wetness would grow heavily. If it is sailing under a sea condition of higher level, the phenomenon of green water and slamming may happen, which is bad for the resistance and seakeeping performance. In addition, from these four graphics, the wake field after the stern is also displayed. Apparent wave trough and crest are generated, spread from the stern to far field.

The free surfaces of other cases are similar to case 1. However, when velocity increases, the phenomenon of green water of both hulls is observed because of the larger amplitude of hull motion and deficient freeboard. The free surface of case 3 is shown in Figure Error! No text of specified style in document.-5.

Figure Error! No text of specified style in document.-5 The free surface of hull 2 in one period (fn=0.38)

The appearance of green water would deteriorate the seakeeping performance of both hull1 and hull 2. Also the hull bow would be impacted, which affect the strength of the whole hull. The water may damage the equipment on the ship and bring about a bad comfort performance. It results from the severe pitch motion caused by high sailing velocity and large wave height. To avoid this disadvantageous phenomenon, a higher free board is necessary. An appropriate bulbous bow can reduce the wave making height near the bow, and then reduce the possibility of green water.

6. Conclusions

In this paper, RANS simulation on the motion responses in regular head waves and seakeeping performance of two typical patrol ships with different bow forms were studied by our in-house solver naoe-FOAM-SJTU. The powerful ability of solver naoe-FOAM-SJTU to solve strong nonlinear problems was verified and intensified.

By Fourier expansion and the transfer function, the numerical results show that the motion fluctuations either in vertical direction or around Y axle present typical linearity. Non-linearity is aggravated due to the increase of hull velocity and wave height. But in this paper, the change of hull velocity is dominated. When the encounter frequency is approximately equal to the natural frequency of the hull, the motion responses are more severe. Moreover, the motion responses in different directions of typical locations are calculated by
rigid-body motion equations. These motions with time durations are obtained. The motion of control room has a larger variation in X direction, and the motions of helicopter lifting platform are relatively small, which is conductive to the operation of helicopter.

The free surfaces of two hulls were captured to show the seakeeping performance. Unfortunately when sailing in high speed, the green water and wave breaking should never be ignored. In this paper these strong nonlinear phenomena can be simulated and observed for both hull 1 and hull 2. Furthermore, the vorticity fields were shown well, especially at the location of bow and stern. It brings about the strenuous vibration and makes seakeeping performance worse.

Finally, the comparison between hull 1 without bulbous bow and hull 2 with bulbous bow showed that, the design of bulbous of hull 2 did not help to improve the seakeeping performance well. The motion responses seem to be limited affected by the bow shape at least under the conditions in this paper. And when sailing at high speed, the green water and wave breaking also took place for both two hulls. It is not sure that the bulbous bow would damp the motion and the height of wave generation under other conditions. It is founded that the bulbous bow certainly can help improve the seakeeping performance, but the effect is limited by wave condition and other factors such as the location and size of bulbous bow, which should be considered comprehensively. As a result, more studies should be carried on.

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